DISCRETE-TIME ADAPTIVE REPETITIVE CONTROL
Internal Model Approach

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Abstract: Repetitive control is known as one of the most effective methods to reduce repetitive errors with a known period in various practical control systems performing repetitive tasks. The application of Internal Model Control (IMC) structure for repetitive control is introduced. Two IMC-based repetitive control configurations are proposed together with their adaptive versions. A comparative simulation study is carried out for the model of a first link of the robot.

1 INTRODUCTION

Many computer-controlled control systems perform repetitive (periodic) tasks thus being subjected to repetitive as well as nonrepetitive disturbances. Rejecting of periodic disturbances or tracking a periodic reference signal can be considered as the original aim of the repetitive controller. In last years much effort has been devoted to the development of discrete-time repetitive control systems which may be considered to be very powerful tools to regulate the repetitive errors whose fundamental frequencies are priori known (Hillerström and Walgama, 1996; Chang et al., 1995; Kempf et al., 1993; Hu and Yu, 1996). The case of uncertain period time is analyzed in (Steinbuch, 2002). Usually, the repetitive errors containing only one fundamental frequency and its harmonics are taken for consideration. A discrete-time repetitive controller for odd harmonic reference and disturbance signals is proposed in (Griñó and Costa-Castelló, 2005). This type of signals appear for example in power electronics systems. Usually, the period of repetitive signals is assumed to be known. In (Steinbuch, 2002), a new structure for repetitive control is proposed which is robust for changes in period-time. The problem of tracking arbitrary periodic reference signals is discussed in (Ledwich and Bolton, 1993), where the compensator design is proposed to give zero steady-state error. The robustness issues of repetitive control are for example examined in (Chang et al., 1995; Hu and Yu, 1996; Tenney and Tomizuka, 1996). The problem of adaptive repetitive control is not much discussed in the literature.

In this paper, two structures of the adaptive repetitive IMC system are presented and simulated using the model of one link of the robot.

2 THE INTERNAL MODEL PRINCIPLE

A block diagram of the conventional discrete-time repetitive control system based on the Internal Model Principle (IMP) for a single fundamental frequency of repetitive errors is shown in Fig.1.

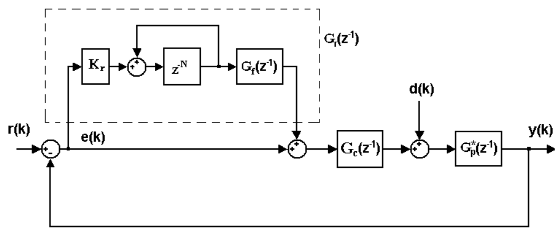

Figure 1: IMP-based repetitive control system.

In this block diagram $r(k)$ and $d(k)$ represent the unknown periodic reference and disturbance with known period, respectively. Typically, the disturbance is assumed to have one fundamental frequency f_o and higher harmonics. The gain K_r is an adjustable parameter of the repetitive controller $G_r(z^{-1})$.

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The IMP implies a use of the repetitive signal generator which is a N step delay chain with positive feedback around it (Hillerström and Walgama, 1996) having the transfer function

$$G_{im}(z^{-1}) = \frac{z^{-N}}{1 - z^{-N}} \quad (1)$$

This generator represents simply the model of a periodic disturbance. If T_s denotes the sampling period then NT_s is chosen to be equal to the period of the fundamental component of the repetitive errors, i.e. $NT_s = T_o = \frac{1}{f_o}$ so $N = \frac{T_o}{T_s}$. A harmonic signal has only one component at $\frac{2\Pi k}{NT_s}$ rad s^{-1} for $k = 1, 2, \cdots$.
Let the plant be given by the transfer function $G_p^*(z^{-1})$. It is known (Kempf et al., 1993) that for the repetitive control system design a parametric model of the plant is required. The nominal plant is characterized by the transfer function

$$G_p(z^{-1}) = z^{-d} \frac{B(z^{-1})}{A(z^{-1})} \quad (2)$$

with $B(z^{-1}) = b_1 z^{-1} + \cdots + b_{nb} z^{-nb}$, $A(z^{-1}) = 1 + a_1 z^{-1} + \cdots + a_{na} z^{-na}$ and $d \geq 0$.
A nominal feedback controller $G_c(z^{-1})$, typically a lag-lead compensator or PD controller is designed so that for the nominal open-loop transfer function $G_o(z^{-1}) = G_c(z^{-1}) G_p(z^{-1})$, the nominal closed-loop transfer function

$$G(z^{-1}) = \frac{G_o(z^{-1})}{1 + G_o(z^{-1})} \quad (3)$$

is asymptotically stable and minimumphase. To assure the stability of the control system with repetitive controller the filter $G_f(z^{-1})$ such that

$$G_f(z^{-1}) G(z^{-1}) = 1 \quad (4)$$

is usually introduced (Chang et al., 1995; Kempf et al., 1993; Chang et al., 1998).

3 THE MULTIPLE REPETITIVE CONTROL SYSTEM

The purpose of the multiple repetitive controller is to regulate multiple repetitive errors which contain multiple dominant fundamental frequencies and their harmonics (Chang et al., 1998). The multiple repetitive discrete-time control system is depicted in Fig.2. It is worthy to note that all repetitive control systems can be augmented by multiple repetitive loops.

Consider again the unmodelled dynamics in the form of a multiplicative modelling uncertainty given by $G^*(z^{-1}) = G(z^{-1})[1 + \Delta(z^{-1})]$. Then from (4), a relationship between $G_f(z^{-1})$ and $G^*(z^{-1})$ can be obtained in terms of modelling uncertainty

$$G_f(z^{-1}) G^*(z^{-1}) = 1 + \Delta(z^{-1})] \quad (5)$$

From (3),(4) and (5), a modelling uncertainty can be derived as

$$\Delta(z^{-1}) = \frac{G_p^*(z^{-1}) - G_p(z^{-1})}{G_p(z^{-1})(1 + G_o^*(z^{-1}))} \quad (6)$$

where $G_o^*(z^{-1}) = G_c(z^{-1}) G_p^*(z^{-1})$.

Assuming that $|\Delta(z^{-1})| \leq \varepsilon$ for each z such that $|z| \geq 1$, the robust stability can be demonstrated (Chang et al., 1998) provided that the gains K_{ri} satisfy the condition

$$\sum_{i=1}^{n} K_{ri} < \frac{2}{1 + \varepsilon}. \quad (7)$$

4 THE INTERNAL MODEL CONTROL STRUCTURE

4.1 The Main IMC Configuration

The discrete-time IMC (Internal Model Control) system structure is shown in Fig.3. This structure is a counterpart of the continuous-time IMC controller given in (Datta, 1998). It is known that every stabilizing controller $G_c(z^{-1})$ is given by

$$G_c(z^{-1}) = \frac{Q(z^{-1})}{1 - G_p(z^{-1}) Q(z^{-1})} \quad (8)$$

where $Q(z^{-1})$ varies over the set of all stable rational transfer functions. This structure may also yield a stable closed-loop performance for unstable plant

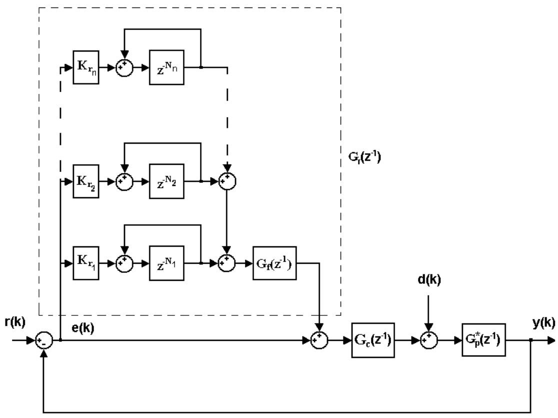

Figure 2: Multiple repetitive control system.

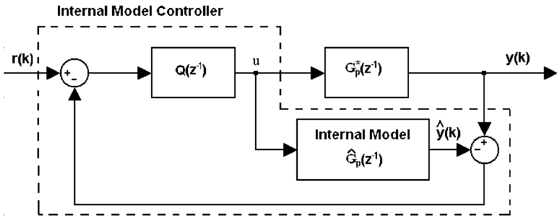

Figure 3: IMC structure.

provided that a plant model $\hat{G}_p(z^{-1})$ is stable, however in this case $Q(z^{-1})$ must not only be stable but must also satisfy certain constraints imposed by unstable poles of the plant.

Suppose that the (possibly proper) $\hat{G}_p(z^{-1}), Q(z^{-1})$ are stable so that the IMC structure is stable for $\hat{G}_p(z^{-1}) = G_p^*(z^{-1})$. Let the uncertainty modelling have the following multiplicative form

$$G_p^*(z^{-1}) = \hat{G}_p(z^{-1})[1 + \Delta_p(z^{-1})] \tag{9}$$

where $\Delta_p(z^{-1})$ is stable strictly proper uncertainty. From the IMC structure (Fig.3) the following equation can be derived

$$u(k) = -\hat{G}_p(z^{-1})Q(z^{-1})\Delta_p(z^{-1})u(k) + Q(z^{-1})r(k) \tag{10}$$

so

$$||u(k)||_2 \le ||\hat{G}_p(z^{-1})Q(z^{-1})\Delta_p(z^{-1})||_\infty ||u(k)||_2 + \\ + ||Q(z^{-1})||_\infty ||r(k)||_2 \tag{11}$$

This shows that if

$$||\hat{G}_p(z^{-1})Q(z^{-1})\Delta_p(z^{-1})||_\infty < 1 \tag{12}$$

then

$$||u(k)||_2 \le [1 - ||\hat{G}_p(z^{-1})Q(z^{-1})\Delta_p(z^{-1})||_\infty]^{-1} \times \\ \times ||Q(z^{-1})||_\infty ||r(k)||_2 \tag{13}$$

so the condition (12) gives the sufficient condition for L_2 stability, thus the IMC structure is robust with respect to modelling errors in the plant. Note that the closed-loop transfer function is

$$\frac{y(z)}{r(z)} = \frac{\hat{G}_p(z^{-1})Q(z^{-1})\Delta_p(z^{-1}) + \hat{G}_p(z^{-1})Q(z^{-1})}{1 + \hat{G}_p(z^{-1})Q(z^{-1})\Delta_p(z^{-1})} \tag{14}$$

For similar approach in continuous-time IMC structure see (Datta, 1998).

4.2 The Pole-placement IMC Configuration

The standard RST controller has a form

$$R(z^{-1})u(k) = -S(z^{-1})y(k) + T(z^{-1})r(k+d+1) \tag{15}$$

and is the solution of

$$A(z^{-1})R(z^{-1}) + z^{-d}B(z^{-1})S(z^{-1}) = A(z^{-1})P(z^{-1}) \tag{16}$$

where $P(z^{-1})$ is the stable polynomial the roots of which are assumed to be the closed-loop poles. The above equation implies that

$$S(z^{-1}) = A(z^{-1})S'(z^{-1}), \tag{17}$$

i.e.(16) is replaced by

$$R(z^{-1}) + z^{-d}B(z^{-1})S'(z^{-1}) = P(z^{-1}) \tag{18}$$

and this allows the controller to be characterized by

$$R(z^{-1}) = P(z^{-1}) - z^{-d}B(z^{-1})S'(z^{-1}). \tag{19}$$

Polynomial $S'(z^{-1})$ is assumed to be stable. For example, if $R(z^{-1})$ contains an integrator then

$$S'(1) = \frac{P(1)}{B(1)} \tag{20}$$

yielding

$$R(z^{-1}) = P(z^{-1}) - z^{-d}\frac{B(z^{-1})P(1)}{B(1)} \tag{21}$$

and

$$S(z^{-1}) = A(z^{-1})\frac{P(1)}{B(1)} \tag{22}$$

Using the controller equation (15) and (18) one obtains

$$\frac{P(z^{-1})B(1)}{A(z^{-1})P(1)}u(k) = -[y(k) - z^{-d}\frac{B(z^{-1})}{A(z^{-1})}u(k)] + \\ + \frac{T(z^{-1})B(1)}{A(z^{-1})P(1)}r(k+d+1) \tag{23}$$

which is the IMC scheme as shown in Fig.4 where

$$G_T(z^{-1}) = \frac{T(z^{-1})B(1)}{A(z^{-1})P(1)}. \tag{24}$$

and using the notation from Fig.3

$$Q(z^{-1}) = \frac{A(z^{-1})P(1)}{P(z^{-1})B(1)}. \tag{25}$$

It is easy to see that taking $T(z^{-1}) = \frac{P(1)A(1)}{B(1)}$ guarantees the zero steady-state error in the case of perfect matching.

4.3 The Repetitive IMC Configuration

The proposed repetitive IMC system structure is represented in Fig.5. This is a combination of the IMC structure (Figs.3,4) and the standard repetitive controller (or multiple repetitive controller). The aim of this control system is reject the repetitive errors by the repetitive controller and to improve the robustness by a proper choice of $Q(z^{-1}$.

From (3), (4), (9) and (13) the following relation between uncertainties $\Delta_p(z^{-1})$ and $\Delta(z^{-1})$ can be found

$$\Delta(z^{-1}) = \frac{\Delta_p(z^{-1})}{1+G_o^*(z^{-1})}. \quad (26)$$

Taking into account that $|\Delta(z^{-1})| \leq \varepsilon$ as in (6) the following condition can be derived

$$|\frac{\Delta_\mu(z^{-1})}{1+G_o^*(z^{-1}))}| \leq \varepsilon \quad (27)$$

This means that under this condition the robust stability of the repetitive IMC structure will be assured if additionally the uncertainty $\Delta(z^{-1})$ is stable. The inequality (27) can not practically be checked out because $G_o^*(z^{-1})$ is not known, however using (8) and (9) the inequality $|\Delta(z^{-1})| \leq \varepsilon$ takes a form

$$|\frac{\Delta_p(z^{-1})(1-G_p(z^{-1})Q(z^{-1}))}{1+G_p(z^{-1})Q(z^{-1})\Delta_p(z^{-1})}| \leq \varepsilon \quad (28)$$

so the (multiple) repetitive IMC system is robustly stable if the uncertainty $\Delta_p(z^{-1})$ is such that the above condition is fulfilled.

4.4 The Adaptive Repetitive IMC Structure

The proposed adaptive repetitive IMC system structure is represented in Fig.6, where the parameter estimation is realized using the standard recursive least-squares algorithm. The adaptation is realized in an indirect way, i.e. the model parameters are first estimated, and subsequently the obtained parameter estimates $\hat{\theta}(k) = (\hat{a}_1(k), \ldots, \hat{a}_{na}(k), \hat{b}_1(k), \ldots, \hat{b}_{nb}(k))^T$ are used for tuning the parameters of both repetitive and internal model controllers.

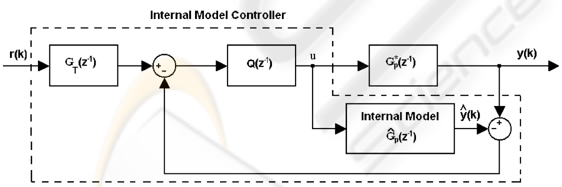

Figure 4: Pole-placement IMC structure.

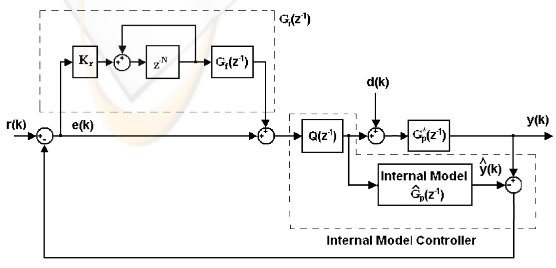

Figure 5: Repetitive IMC structure.

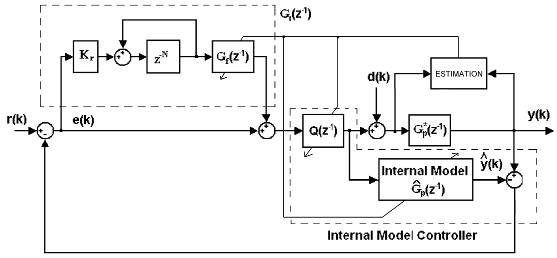

Figure 6: Adaptive repetitive IMC structure.

5 SIMULATIONS

Often robotic manipulators are required to execute repetitive tasks. Then the desired trajectory to be followed by the manipulator is bounded and periodic with known period. Below a first link of the AdeptOne robot (Tenney and Tomizuka, 1996) is taken as an example for simulations. The link considered as a plant is approximated by the nominal ARX model

$$G_p(z^{-1}) = \frac{0.000242z^{-1}}{1-1.9788z^{-1}+0.9789z^{-2}} \quad (29)$$

obtained at $\frac{1}{T_s} = 1kHz$ sampling rate. The nominal compensator has a form of PD-type

$$G_c(z^{-1}) = 119.5\frac{1-0.925z^{-1}}{1-0.65z^{-1}}. \quad (30)$$

The main IMC repetitive controller has been tested for

$$Q(z^{-1}) = \frac{119.5-347z^{-1}+335.7z^{-2}-108.2z^{-3}}{1-2.6z^{-1}+2.238z^{-2}-0.6363z^{-3}} \quad (31)$$

that has been obtained according to (8) for a stable plant model (29). In turn, the filter $G_f(z^{-1})$ was derived according to (4) as

$$G_f(z^{-1}) = \frac{1-4.55z^{-1}+8.278z^{-2}-7.529z^{-3}}{0.02892z^{-1}-0.08397z^{-2}+0.08124z^{-3}-0.02619z^{-4}} + \frac{3.424z^{-4}-0.6229z^{-5}}{0.02892z^{-1}-0.08397z^{-2}+0.08124z^{-3}-0.02619z^{-4}}. \quad (32)$$

The disturbance $d(k)$ with amplitude of 5 units contains the fundamental and harmonic frequencies of $f_{o1} = 5Hz$ $(10Hz, 15Hz)$, $f_{o2} = 7Hz$ $(14Hz, 21Hz)$, $f_{o3} = 9Hz$ $(18Hz, 27Hz)$ thus $N_1 = 200, N_2 = 143$,

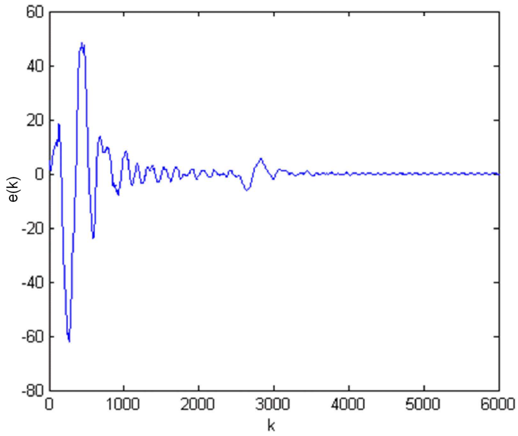

Figure 7: Adaptive repetitive IMC, disturbance attenuation.

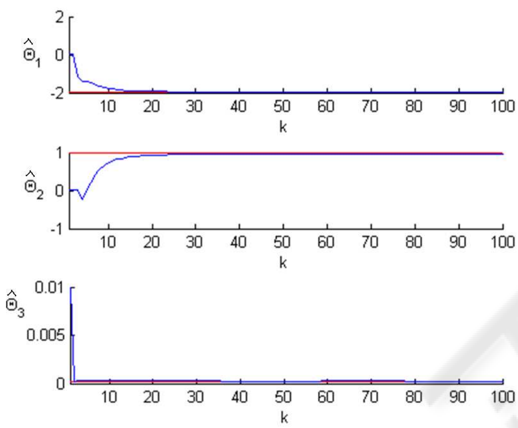

Figure 8: Adaptive repetitive IMC, parameter estimates.

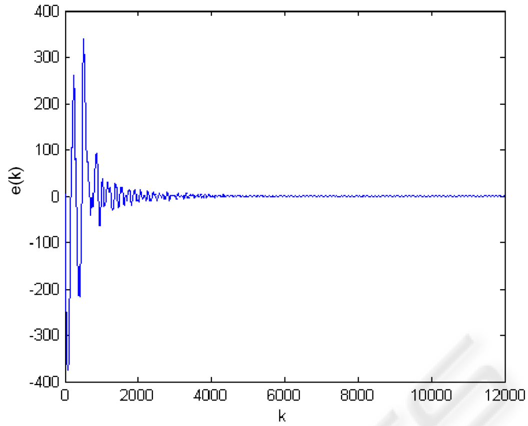

Figure 9: Adaptive repetitive pole-placement IMC.

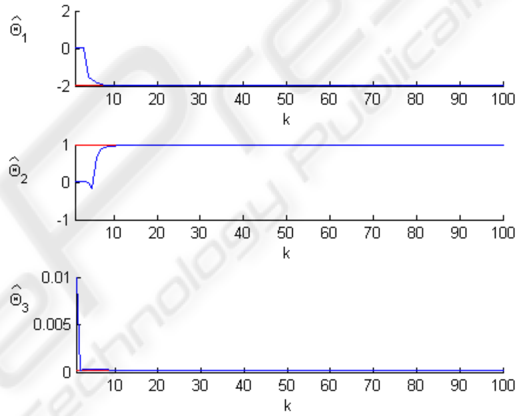

Figure 10: Adaptive repetitive pole-placement IMC.

$N_3 = 111$ with $K_{r1} = K_{r2} = K_{r3} = 0.5$. Additionally, a pulse disturbance d_p with amplitude of 15 units is also inserted to the input of the plant.

The initial conditions for parameter estimates and covariance matrix in the recursive least squares algorithm were taken as $\hat{\theta}(0) = (0.01, 0.01, 0.01)^T$ and $P(0) = 100I$.

The performance of adaptive multiple repetitive IMC control system given in Fig.7 shows the effect of disturbance attenuation. The corresponding parameter estimates are shown in Fig.8.

Finally, the adaptive pole-placement IMC structure was combined with multiple repetitive controller. For the polynomial $P(z^{-1}) = 1 - 1.8z^{-1} + 0.9z^{-2}$ one obtains from (25)

$$Q(z^{-1}) = \frac{0.1 - 0.1979z^{-1} + 0.09789z^{-2}}{0.000242 - 0.0004356z^{-1} + 0.0002178z^{-2}}, \tag{33}$$

and from (24)

$$G_T(z^{-1}) = \frac{0.0001}{1 - 1.979z^{-1} + 0.9789z^{-2}}. \tag{34}$$

The filter $G_f(z^{-1})$ was derived again from (4), however in this case the transfer function $G(z^{-1})$ is

$$G(z^{-1}) = \frac{G_T(z^{-1})Q(z^{-1})G_p(z^{-1})}{1 + G_T(z^{-1})Q(z^{-1})G_p(z^{-1})}. \tag{35}$$

The error signal is shown in Fig.9, and the corresponding parameter estimates are shown in Fig.10 for multiple harmonic disturbance attenuation.

6 CONCLUSIONS

Two structures of IMC repetitive control system are examined and their adaptive versions are simulated taking the first link of an AdeptOne robot as the example. The proposed control structures can be enlarged by the multiple repetitive controller. The adaptive loop included into the IMC repetitive control system reduces the level of parametric uncertainty thus improves the quality of disturbance attenuation. In this way the proposed configurations can be considered as the robust adaptive ones.

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