

# DIAGNOSIS OF DISCRETE EVENT SYSTEMS WITH PETRI NETS AND CODING THEORY

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Abstract: Event sequences estimation is an important issue for fault diagnosis of DES, so far as fault events cannot be directly measured. This work is about event sequences estimation with Petri net models. Events are assumed to be represented with transitions and firing sequences are estimated from measurements of the marking variation. Estimation with and without measurement errors are discussed in  $n$  – dimensional vector space over alphabet  $Z_3 = \{-1, 0, 1\}$ . Sufficient conditions and estimation algorithms are provided. Performance is evaluated and the efficiency of the approach is illustrated on two examples from manufacturing engineering.

## 1 INTRODUCTION

Modern technological processes include complex and large-scale systems, where faults in a single component have major effects on the availability and performances of the system as a whole. For example manufacturing systems consists of many different machines, robots and transportation tools all of which have to correctly satisfy their purpose in order to ensure and fulfil global objectives. In this context, a failure is any event that changes the behaviour of the system such that it does no longer satisfy its purpose (Rausand et al., 2004). Faults can be due to internal causes as to external ones, and are often classified into three subclasses: plant faults that change the dynamical input – output properties of the system, sensor faults that results in substantial errors during sensors reading, and actuator faults when the influence of the controller to the plant is disturbed. In order to limit the effects of the faults on the system, diagnosis is used to detect and isolate the failures. Diagnosis includes distinct stages: the fault detection decides whether or not a failure event has occurred; the fault isolation find the component that is faulty; the fault identification identifies the fault and estimates also its magnitude. Model-based and data-based methods have been investigated for diagnosis (Blanke et al., 2003).

The motivations for the diagnosis of discrete event system (DES) are obvious as long as DES occur naturally in the engineering practice. Many actuators like switches, valves and so on, only jump between discrete states. Binary signals are mainly

used with numerical systems and logical values “true” and “false” are often used as input and output signals. Alarm sensors that indicate that a physical quantity exceeds a prescribed bound are typical systems with only two logical states. Moreover, in several systems also the internal state is discrete valued. As an example, robot encoders are discrete valued even if the number of discrete state is large enough to produce smooth trajectories. At last, one must keep in mind that a given dynamical system can always be considered as a DES system or as a continuous variable system according to the purpose of the investigation. As long as supervision problems are considered, a rather broad view on the system behaviour can be adopted that is based on discrete signals. On the contrary, if signals have to remain in a narrow tolerance band, the following approaches do no longer fit and one has to adopt a continuous point of view (Blanke et al., 2003).

The behaviour of DES is described by sequences of input and output events. In contrast to the continuous systems only abrupt changes of the signal values are considered with DES. In that case, the problem has been originally investigated with observation methods for automata developed in connection with the supervisory control theory (Ramadge et al., 1987). Concerning model-based methods automata (Sampath et al., 1995) or Petri nets (Ushio et al. 1998) models can be used. This article focus on diagnosis of DES modelled with Petri nets (PN) where failures are represented with some particular transitions. The problem is to detect and isolate the firing of the failure transitions in a

given firing sequence. The firings of the failure transitions are assumed to be unobservable and must be estimated according to complete or partial marking measurements that are eventually disturbed by measurement errors. As a consequence a method based on coding theory is proved to be suitable for sensor faults diagnosis. The article is divided into six sections. Section two is about Petri nets states. Section three states the diagnosis problem for DES and is about the usual state space methods for PN. Section four details the event estimation with coding theory that can be combined with state space approach. Both methods are presented in a framework in the conclusion.

## 2 ORDINARY PETRI NETS

An ordinary PN with  $n$  places and  $q$  transitions is defined as  $\langle P, T, \text{Pre}, \text{Post} \rangle$  where  $P = \{P_i\}$  is a non-empty finite set of  $n$  places,  $T = \{T_j\}$  is a non-empty finite set of  $q$  transitions, such that  $P \cap T = \emptyset$ .  $\text{Pre}: P \times T \rightarrow \{0, 1\}$  is the pre-incidence application and  $W_{PR} = (w_{ij}^{PR}) \in \{0, 1\}^n \times q$  with  $w_{ij}^{PR} = \text{Pre}(P_i, T_j)$  is the pre-incidence matrix.  $\text{Post}: P \times T \rightarrow \{0, 1\}$  is the post-incidence application and  $W_{PO} = (w_{ij}^{PO}) \in \{0, 1\}^n \times q$  with  $w_{ij}^{PO} = \text{Post}(P_i, T_j)$  is the post-incidence matrix. The PN incidence matrix  $W$  is defined as  $W = W_{PO} - W_{PR} \in Z_3^{n \times q}$  with  $Z_3 \in \{-1, 0, 1\}$  and  $w_i$  stands for the  $i^{\text{th}}$  column of  $W$  (Askin et al., 1993; Cassandras et al., 1999; David et al., 1992).  $M = (m_i) \in (Z^+)^n$  is defined as the marking vector and  $M_1 \in (Z^+)^n$  as the initial marking vector, with  $Z^+$  the set of non negative integer numbers. A firing sequence  $\sigma = T_1 T_2 \dots T_k$  is defined as an ordered series of transitions that are successively fired from marking  $M$  to marking  $M'$  (i.e.  $M[\sigma > M']$ ) such that equation (1) is satisfied:

$$\sigma: M \xrightarrow{T_1} M_1 \xrightarrow{T_2} M_2 \rightarrow \dots \xrightarrow{T_k} M' \quad (1)$$

A sequence  $\sigma$  can be represented by its characteristic vector (i.e. Parikh vector)  $X = (x_j) \in (Z^+)^q$  where  $x_j$  stands for the number of times  $T_j$  has occurred in sequence  $\sigma$  (David et al., 1992). Marking  $M'$  resulting from marking  $M$  with the execution of sequence  $\sigma$  is given by (2) where  $X$  is the characteristic vector for sequence  $\sigma$ :

$$\Delta M = M' - M = W.X \quad (2)$$

The reachability graph  $R(\text{PN}, M_1)$  is the set of markings  $M$  such that a firing sequence  $\sigma$  exists from  $M_1$  to  $M$ . A sequence  $\sigma$  is said to be executable

for marking  $M_1$  if there exists a couple of markings  $(M, M') \in R(\text{PN}, M_1)$  such that  $M[\sigma > M']$ .

## 3 DIAGNOSABILITY AND DIAGNOSER DESIGN FOR DES

### 3.1 Problem Statement

In the context of diagnosis, it is commonly assumed that no inspection of the process is possible. As a consequence the diagnosis is only based on available measurement data. Basically, the diagnosis problem for a dynamical system with input  $u$ , output  $y$  and subject to some faults  $f$ , is to detect and isolate the faults from a given sequence of input – output couples  $(U, Y)$  with:

$$U = (u(0), u(1), \dots, u(k)) \quad (3)$$

$$Y = (y(0), y(1), \dots, y(k))$$

where  $k$  stands for time  $t = k\Delta t$ , and  $\Delta t$  represents the sampling period of sensors. The main issues are (1) to decide the diagnosability of the faults; (2) to detect, isolate and identify the faults that are diagnosable. In case of model - based diagnosis, the input – output couples  $(U, Y)$  are usually compared with the behaviour of a reference model. Fault indicators like residuals are worked out from this comparison. It is often convenient to separate actuator, system and sensor faults.

As long as DES are considered the inputs and faults are usually considered as events and the outputs are related to the states of the DES. A reference model (automata, finite state machines, Petri nets, and so on) can be used for diagnosis purpose and sequences of estimated outputs obtained thanks to the model are compared with the measured outputs of the system. Indicators of the faults result from this comparison. According to the traces generated by the system, faults are :

- (1) strongly diagnosable if they result in immediate abnormal behaviours (no intermediate event is required for diagnosis);
- (2) weakly diagnosable if they result in abnormal behaviours after a finite number of intermediate events;
- (3) non diagnosable if no abnormal behaviour occurs whatever the future evolution of the system.

Let us notice that the notion of strong or weak diagnosability for DES is related to the question of persistent excitation in temporal systems.

The figure 1 is an example of diagnosis with finite state machine. The system has 5 states {A, B, C, D, E}, 4 outputs {1, 2, 3, 4}, 5 inputs {a, b, c, f<sub>1</sub>, f<sub>2</sub>} (3 normal events {a, b, c} and 2 fault events {f<sub>1</sub>, f<sub>2</sub>}). The reference model (full lines only) and the system (full and dashed lines) evolve according to the figure 1. Diagnosability analysis and diagnosers design result from the simulation with automata in figure 1.

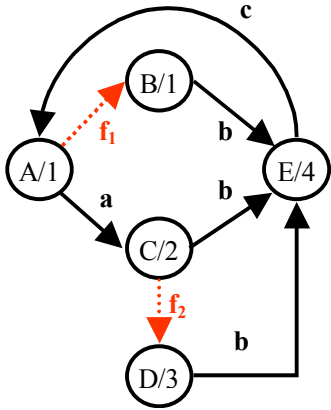


Figure 1: Example of diagnosis with finite state machine.

If the state of the system is measured, then the faults f<sub>1</sub> and f<sub>2</sub> are both strongly diagnosable as long as the fault events lead to an immediate difference between system state (S) and estimated one (S<sub>est</sub>) (table 1, grey cells). If only the output is measured then the fault f<sub>2</sub> is strongly diagnosable but the fault f<sub>1</sub> is weakly diagnosable in the sense that intermediate event “b” must occur so that the system output (O) and estimated output (O<sub>est</sub>) become different. If state “E” results in output “1” instead of “4” then fault f<sub>2</sub> is non diagnosable.

### 3.2 Diagnosis with Petri Nets

The previous approach can be applied to Petri net models with finite reachability graph to prove the diagnosability of the faults and to design diagnosers based on Petri net models. The basis idea is to investigate the indeterminate cycles in partial expansion of the reachability graph (Ushio et al., 1998). The considered PN are live (i.e. for any T<sub>j</sub> ∈ T, and for all M ∈ R(PN, M<sub>1</sub>) there exists a sequence σ executable from M that includes transition T<sub>j</sub>) and safe (i.e. for all M ∈ R(PN, M<sub>1</sub>), M ∈ {0, 1}<sup>n</sup>). Some places are assumed to be observable and other not, and transitions, that are associated with events, are usually assumed to be unobservable. A cycle is called “determined” if it contains at least one observable state that results with no ambiguity from

a normal firing sequence, or from a firing sequence with a fault. The fault is diagnosable if and only if there is no indeterminate cycle in partial expansion of the reachability graph that correspond to the observable part of the system. For a diagnosable fault, the detection and isolation can be obtained according to the finite state machine that corresponds to partial expansion of the reachability graph. Let us notice that the method is different from the diagnosis with finite state machines in the sense that knowledge of inputs is not required and that definition of outputs is restricted to marking projection.

Let consider the system PN1 in figure 2 as an example. The reachability graph of PN1 is the finite state machine of figure 1. If the set of observable places is given by P<sub>O1</sub> = {P<sub>1</sub>, P<sub>4</sub>, P<sub>5</sub>}, the observable part of the labelled reachability graph R(PN1, {T<sub>1</sub>}, (1, 0, 0, 0, 0)<sup>T</sup>, P<sub>O1</sub>) is worked out as in figure 3a. This diagnoser has an indetermined cycle so the system is not diagnosable (figure 3a, left cycle). If P<sub>O2</sub> = {P<sub>1</sub>, P<sub>3</sub>}, the observable part of the labelled reachability graph R(PN1, {T<sub>1</sub>}, (1, 0, 0, 0, 0)<sup>T</sup>, P<sub>O2</sub>) is worked out as in figure 3b. This diagnoser has no indetermined cycle so the system is diagnosable.

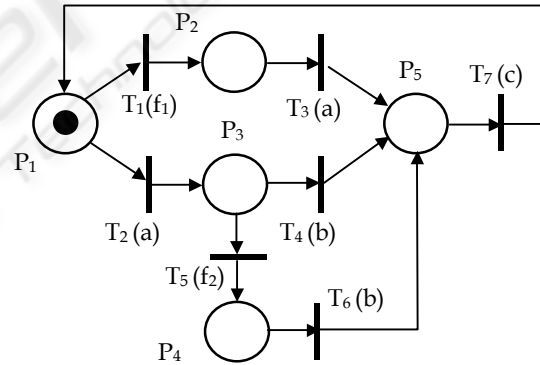
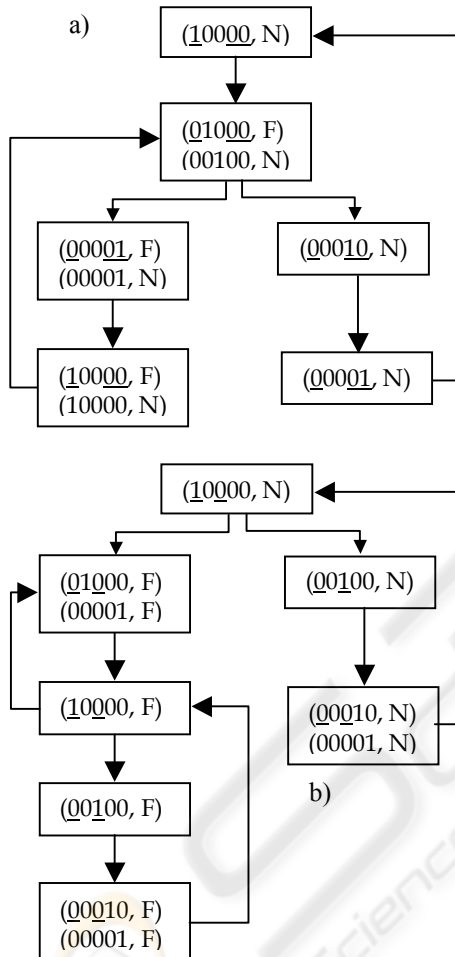


Figure 2: Example PN1 of Petri net.

Let us mention that other approaches have been developed for diagnosis based on event detectability (Ramirez – Trevino et al., 2007) and structural properties (Lefebvre et al., 2007). All above mentioned approaches require complete or partial measurements of the marking vector. Thus, they are sensitive to measurement errors. As a consequence, it is important to detect and eventually correct the errors that disturb the measurements of marking variation in order to obtain an exact estimation of the occurrence of events. The next section concerns events estimation and can be introduced as a diagnosis method for sensor faults.

Table 1: Example of input sequence (I), state sequence (S), output sequence (O), estimated state sequence ( $S_{est}$ ) and estimated output sequence ( $O_{est}$ ) for the final state machine in figure 1

I	a	b	c	a	$f_2$	b	C	$f_1$	b	c	a	b	...
S	C	E	A	C	D	E	A	B	E	A	C	E	...
O	2	4	1	2	3	4	1	1	4	1	2	4	...
$S_{est}$	C	E	A	C	C	E	A	A	A	A	C	E	...
$O_{est}$	2	4	1	2	2	4	1	1	1	1	2	4	...


 Figure 3 : Two partial expansions of the reachability graph for PN1 a)  $R(PN1, \{T_1\}, (1, 0, 0, 0, 0)^T, P_{O1})$ ; b)  $R(PN1, \{T_1\}, (1, 0, 0, 0, 0)^T, P_{O2})$ .

#### 4 SENSOR FAULTS DIAGNOSIS BASED ON CODING THEORY

Event sequences estimation is an important issue for fault diagnosis of DES, so far as fault events cannot be directly measured. This section is about event sequences estimation with PN models. Events are assumed to be represented with transitions and firing

sequences are estimated from measurements of the marking variation. Estimation with and without measurement errors can be discussed in  $n$ -dimensional vector space over alphabet  $Z_3 = \{-1, 0, 1\}$  (Lefebvre, 2008). The basis idea to correct measurement errors by projecting measurements in orthogonal subspace of  $\text{Vect}(W)$  where  $\text{Vect}(W)$  stands for the subspace generated by the columns of  $W$ . This method is inspired from linear coding theory (Van Lint, 1999) and extends the results presented for continuous PN in (Lefebvre et al., 2001).

Our contribution can be compared to another method that incorporates redundancy into Petri nets to detect and identify faults (Li et al., 2004; Wu et al., 2002, 2005) and uses algebraic decoding techniques as the Berlekamp – Massey decoding (Berlekamp, 1984). The marking of the original PN is embedded into a redundant one and the diagnosis of faults is performed by mean of linear parity checks. In comparison with the method developed in (Wu et al., 2005), our approach does not require additive places, but is less efficient for faults correction.

Let us assume that measurement  $\Delta \hat{M}$  of marking variation  $\Delta M \in (Z_3)^n$  may be affected by additive error vector  $E \in (Z_3)^n$ :  $\Delta \hat{M} = \Delta M + E$  where “+” stand for the sum endowed over  $Z_3$ . Error vector will be characterized according to the Hamming distance  $d(W)$  of the considered PN that is defined with the Hamming distance of the columns of incidence matrix :

$$d(W) = \min\{\min\{d(w_i, w_j), i \neq j\}, \min\{d_0(w_i)\}\} \quad (4)$$

where  $d(w_i, w_j)$  stands for the Hamming distance between columns  $w_i$  and  $w_j$  of matrix  $W$  and  $d_0(w_i) = d(w_i, 0)$  stands for the weight of vector  $w_i$ .

It is assumed that error vector  $E$  verifies the following conditions:

- $\Pr(d_0(E) = 0) > \Pr(d_0(E) = 1) > \dots > \Pr(d_0(E) = n)$  where  $\Pr(d_0(E) = i)$  is the probability that weight of  $E$  equals  $i$ ;
- An error in position  $i$  does not influence other positions;
- A symbol in error can be each of the remaining symbols with equal probability.

A short estimation algorithm easy to use and to implement when state measurement is complete (i.e. all entries of  $\Delta\hat{M}$  are measured), and error free (i.e. measurement equals actual marking variation  $\Delta M$ ), is based on the comparison of measurement with respect to columns of  $W$  and zero vector (this corresponds to the condition of event-detectability in case that all places are observable). When this measurement equals a single column of  $W$ , the algorithm decides that the corresponding transition fired. When it equals the zero vector, the algorithm decides that no transition fired.

When measurement is perturbed by non zero error  $E$ , two problems must be mentioned :

- a) A miss estimation may occur when  $\Delta\hat{M}$  is non zero and different from any columns of  $W$ . The estimation algorithm is not able to decide if a transition fired or not and which transition fired. As consequence the algorithm does not give any decision.
- b) A wrong estimation may occur when  $\Delta\hat{M}$  does not equal actual marking variation  $\Delta M$  but equals zero vector or another column of  $W$ . The estimation algorithm decides if a transition fired or not and which transition fired, but the decision is wrong due to the measurement error.

To overcome these difficulties and to improve estimation, diagnosis can be reformulated as a linear problem in  $((Z_3)^n, +, *)$ , with the Smith transformation of  $W$ , where “+” and “\*” stand for the sum and product endowed over  $Z_3$ . The Smith transformation results from elementary operations (i.e. row or column permutations, linear combinations and external products), summed up in matrices  $P \in (Z_3)^{n \times n}$  and  $Q \in (Z_3)^{q \times q}$  such that:

$$P * W * Q = \begin{pmatrix} I_r & 0 \\ 0 & 0 \end{pmatrix} \quad (5)$$

$I_r$  is the identity matrix of dimension  $r \times r$ , and  $r$  is the rank of matrix  $W$ . The Smith transformation leads to reduced incidence matrix  $W'$  :

$$\begin{aligned} W' &= (I_r \ 0) * Q^{-1} = (I_r \ 0) * P * W \\ &= F * W \in (Z_3)^{r \times q} \end{aligned} \quad (6)$$

Necessary and sufficient conditions for firing sequences estimation can be stated when measurement is error free and basic assumption in section 2.b is satisfied : columns of incidence matrix  $W'$  defined by equation (6) are distinct and non zero (Lefebvre, 2008). In case of measurement errors that

satisfy assumptions a to c, sufficient conditions inspired from coding theory can be stated. These conditions are based on Hamming distance, cosets investigation, parity check matrices, and syndromes (Van Lint, 1999). Cosets characterise the structure of  $(Z_3)^n$  according to the sum and product over  $Z_3$  (the coset  $C(u)$  of  $u$  is defined as  $C(u) = \{x \in (Z_3)^n \text{ such that } x = u + y \text{ with } y \in \text{Vect}(W)\}$ , for any vector  $u \in (Z_3)^n$ ). Parity check matrices are introduced to work out syndromes that can be considered as the signatures of the faults in  $(Z_3)^n$ . Two conditions for firing sequences estimation are proposed (Lefebvre, 2008):

- a) Columns of incidence matrix  $W$  are distinct, non zero and errors  $E$  that disturb satisfy  $d_0(E) \leq (d(W) - 1) / 2$  (i.e. the number of disturbed entries of measurement is no larger than  $(d(W) - 1) / 2$ ).
- b) Columns of reduced incidence matrix  $W'$  are distinct and non zero, and considered errors  $E$  belong to distinct cosets different from  $C(0)$ .

Moreover, the use of the Smith transformation of incidence matrix is also helpful to define the parity check matrix  $H^T = (0 \ I_{n-r}) * P \in (Z_3)^{(n-r) \times n}$ , and to work out the syndrome of marking variation measurements  $S(\Delta\hat{M}) = H^T * \Delta\hat{M}$  and to compare it with the syndrome of errors  $S(E) = H^T * E$ . As a consequence the method leads to a less complex and more efficient diagnosis algorithm (algorithm b) in comparison with usual method based on Hamming distance (algorithm a) (Lefebvre, 2008).

#### Algorithm a

1. For each time  $k$ , measure  $\hat{M}(k)$  the current state of DES
2. Compute  $\Delta\hat{M}(k) = \hat{M}(k) - \hat{M}(k-1)$
3. Compute weight  $d_0(\Delta\hat{M}(k))$ . If  $d_0(\Delta\hat{M}(k)) \leq (d(W) - 1) / 2$ , then no event occurs between two consecutive state measurements. Go to step 6.
4. Compute Hamming distance  $d(\Delta\hat{M}(k), w_j)$  for each column  $w_j$  of  $W$ . If  $d(\Delta\hat{M}(k), w_j) \leq (d(W) - 1) / 2$  then  $T_j$  fired. Go to step 6.
5. If for all  $j = 1, \dots, q$ ,  $d(\Delta\hat{M}(k), w_j) > (d(W) - 1) / 2$  then measurement is too much disturbed by errors (i.e.  $d_0(E) > (d(W) - 1) / 2$ ) and no decision is provided (i.e. a miss estimation occurs).
6. Wait until time  $k + 1$ . Go to step 1.

**Algorithm b**

1. For each time  $k$ , measure  $\hat{M}(k)$  the current state of DES
2. Compute  $\Delta\hat{M}(k) = \hat{M}(k) - \hat{M}(k-1)$
3. Compute  $H^T * \Delta\hat{M}(k)$ . If  $H^T * \Delta\hat{M}(k) = 0$  then measurement is not disturbed by errors:  $\Delta M(k) = \Delta\hat{M}(k)$ . Go to step 5.
4. If syndrome  $H^T * \Delta\hat{M}(k) \neq 0$ , compute coset leader  $E(k)$  and  $\Delta M(k) = \Delta\hat{M}(k) - E(k)$ . Go to step 5.
5. Compute  $\Delta M'(k) = F * \Delta M(k)$ .
6. If  $\Delta M'(k) = 0$  then no event occurs between 2 consecutive state measurements. Go to step 8.
7. If  $\Delta M'(k) = w_j$  then  $T_j$  fired. Go to step 8.
8. Wait until time  $k + 1$ . Go to step 1.

The correction capacity (i.e. number of error vectors that are corrected) of algorithm a is given by equation (7):

$$\sum_{i=1}^{(d(W)-1)/2} 2^i \cdot \left( \frac{n!}{i!(n-i)!} \right) \quad (7)$$

and its complexity results from  $2n \cdot (q+1)$  scalar comparisons or operations whereas correction capacity of algorithm b equals  $3^{n-r} - 1$ , and its complexity results from  $r \cdot (2n+q) + (n-r) \cdot (2n-1+3^{n-r})$  scalar comparisons or operations (Lefebvre, 2008). As a conclusion, algorithm b (with matrix  $W'$ ) is more efficient than algorithm a (with matrix  $W$ ) for PN with small rank  $r$  in comparison with the number of places, and for PN with few transitions in comparison with the number of places. Algorithm b will be also preferred for PN with a small Hamming distance. This result is not surprising as long as the correction capacity of algorithm a is directly related to the value of Hamming distance. The determination of reduced incidence matrix does not increase the complexity of algorithm b as long as this determination is work out off-line.

## 5 APPLICATION

Algebraic methods have been used for the diagnosis of manufacturing and robotic systems. In order to illustrate algebraic methods, let us consider PN2 in figure 4 with incidence matrix (8), that is a simplified model of a manufacturing workshop (Silva et al., 2004). The final product is composed of two different parts that are processed in two separate machines modelled by transitions  $T_1$  and  $T_2$ , and stored in buffers  $P_4$  and  $P_6$ , respectively. Then, they are assembled by the machine  $T_3$ , and processed by

$T_4$  and  $T_5$ . During the processing, several tools are needed, modelled by places  $P_3$ ,  $P_5$  and  $P_7$ .

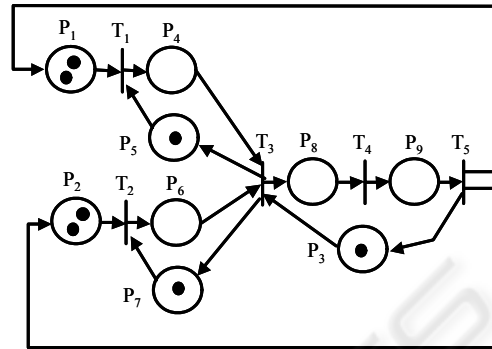


Figure 4: Model PN2 of a manufacturing system.

$$W = \begin{pmatrix} -1 & 0 & 0 & 0 & 1 \\ 0 & -1 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 & 1 \\ 1 & 0 & -1 & 0 & 0 \\ -1 & 0 & 1 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 \\ 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 1 & -1 \end{pmatrix} \quad (8)$$

PN2 has  $n = 9$  places,  $q = 5$  transitions, is of rank  $r = 4$  and incidence matrix  $W$  has a Hamming distance  $d = 2$ . Matrices  $F$  and  $H^T$ , worked out as in section 4, are given according to equations (9) and (10):

$$F = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix} \quad (9)$$

$$H^T = \begin{pmatrix} -1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 1 \end{pmatrix} \quad (10)$$

PN2 has 243 cosets and each coset has 81 vectors. The table 2 gives the relationships between syndromes and coset leaders. Let us notice that the two last syndromes correspond to two different coset leaders. As a consequence not all errors of weight 1 will be corrected by algorithms a and b (errors  $(0\ 0\ 0\ 0\ 0\ 0\ 1\ 0)^T$  and  $(0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 1)^T$  cannot be separated as errors  $(0\ 0\ 0\ 0\ 0\ 0\ 0\ -1\ 0)^T$  and  $(0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ -1)^T$ ).

Table 2: Correspondence between syndromes and coset leaders for PN2.

Syndromes	Errors of weight 1	Syndromes	Errors of weight 1
$(-1\ 0\ 0\ 1\ 0)^T$	$(1\ 0\ 0\ 0\ 0\ 0\ 0\ 0)^T$	$(1\ 0\ 0\ 0\ 0)^T$	$(0\ 0\ 0\ 0\ 1\ 0\ 0\ 0\ 0)^T$
$(1\ 0\ 0\ -1\ 0)^T$	$(-1\ 0\ 0\ 0\ 0\ 0\ 0\ 0)^T$	$(-1\ 0\ 0\ 0\ 0)^T$	$(0\ 0\ 0\ 0\ -1\ 0\ 0\ 0\ 0)^T$
$(0\ 1\ -1\ 0\ 0)^T$	$(0\ 1\ 0\ 0\ 0\ 0\ 0\ 0)^T$	$(0\ 1\ 0\ 0\ 0)^T$	$(0\ 0\ 0\ 0\ 0\ 1\ 0\ 0\ 0)^T$
$(0\ -1\ 1\ 0\ 0)^T$	$(0\ -1\ 0\ 0\ 0\ 0\ 0\ 0)^T$	$(0\ -1\ 0\ 0\ 0)^T$	$(0\ 0\ 0\ 0\ 0\ -1\ 0\ 0\ 0)^T$
$(1\ -1\ 1\ -1\ 1)^T$	$(0\ 0\ 1\ 0\ 0\ 0\ 0\ 0)^T$	$(0\ 0\ 1\ 0\ 0)^T$	$(0\ 0\ 0\ 0\ 0\ 0\ 1\ 0\ 0)^T$
$(-1\ 1\ -1\ 1\ -1)^T$	$(0\ 0\ -1\ 0\ 0\ 0\ 0\ 0)^T$	$(0\ 0\ -1\ 0\ 0)^T$	$(0\ 0\ 0\ 0\ 0\ 0\ -1\ 0\ 0)^T$
$(0\ 0\ 0\ 1\ 0)^T$	$(0\ 0\ 0\ 1\ 0\ 0\ 0\ 0)^T$	$(0\ 0\ 0\ 0\ 1)^T$	$(0\ 0\ 0\ 0\ 0\ 0\ 0\ 1\ 0)^T$ $(0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 1)^T$
$(0\ 0\ 0\ -1\ 0)^T$	$(0\ 0\ 0\ -1\ 0\ 0\ 0\ 0)^T$	$(0\ 0\ 0\ 0\ -1)^T$	$(0\ 0\ 0\ 0\ 0\ 0\ 0\ -1\ 0)^T$ $(0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ -1)^T$

Simulations for on – line estimation of the transitions firing are provided with figure 5. In these simulations, a measurement error ratio of 0.1 is supposed to be associated to each place (i.e. a probability of 0.1 that the marking variation of each place is biased). Transitions are assumed to fire with stochastic firing periods (exponential distribution) of mean value equal to 1 TU. All simulations indicate that complexity of algorithm b is not a limitation for real time applications. For the example PN2, the total CPU time for algorithm b is less than 5 TU for a simulation of 100 TU with a sampling period of 0.1 TU. This means that the average duration for each cycle of algorithm is approximately 20 times less than the sampling period. The miss estimation rate for b is about 32% in comparison with a that has a rate of 60% and the wrong estimation rate is about 8% for b in comparison with a that has a rate less than 1%. Let us mention that the large number of miss estimation (even if measurement is unbiased) is due to the small Hamming distance of  $W$  ( $d = 2$ ). For this reason numerous unbiased measurements of the marking variation are considered as suspicious and not used for estimation.

## 6 CONCLUSIONS

The investigation of diagnosis methods for discrete event systems shows that Petri nets is efficient not only to model the considered systems but also to support the diagnosis methods. Several approaches can be used in order to check diagnosability, to select sensors and to work out diagnosers. As a conclusion it is important to notice the great effort,

observed this last years to develop and improve diagnosis methods for DES. The use of the coding theory plays an important role in that development. As long as it is suitable to detect and correct measurement errors in the marking error variation. The main drawback is the strong dependence of the method to the algebraic properties of the incidence matrix.

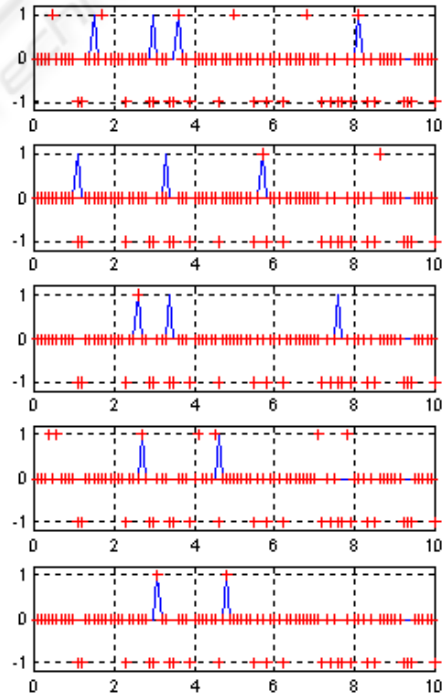


Figure 5: On – line firings estimation with algorithm b for the transitions  $T_1$  to  $T_5$  of PN2 (number of firings, full line: correct value; cross: estimated value; estimated value = -1 means miss estimation) in function of time (TU).

The method can be improved by incorporating additive places into Petri nets models. Taken into account the past sequence of events is another perspective to improve the efficiency of the method. But, the main challenge is, from our point of view, to take advantages from many important contributions that have been proposed for continuous systems. To build a bridge from continuous variable systems to DES theories remains one of the most promising issues for the next years.

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