OBTAINING MINIMUM VARIABILITY OWA OPERATORS UNDER A FUZZY LEVEL OF ORNESS

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Abstract: Finding the optimal OWA (ordered weighted averaging) operators is important in many decision support problems. The OWA-operators enables the decision maker to model very different kinds of aggregator operators. The weights need to be, however, determined under some criteria, and can be found through the solution of some optimization problems. The important parameter called the level of orness may, in many cases, be uncertain to some degree. Decision makers are often able to estimate the level using fuzzy numbers. Therefore, this paper contributes to the current state of the art in OWA operators with a model that can determine the optimal (minimum variability) OWA operators under a (unsymmetrical triangular) fuzzy level of orness.

1 INTRODUCTION

Information aggregation is used in many applications. Some fields of research that takes advantage of aggregation may be found in Neural Networks, fuzzy logic controllers, multi-criteria optimisation and more. Aggregation is necessary to logically split up entities onto several units. A very eminent way of doing aggregators is the OWA operators, originally described by (Yager, 1988). He defined a weight, w_i, to be associated with an ordered position of the aggregate. The weights are often ordered such that the best criterion is associated with the first weight and so on. Given the weights for each object, Yager defined a level of orness, which will represent a major characteristic of the weighting structure. An orness-value of zero represents a situation that the weakest criterion has the full weight, whereas an orness-value of one represents the opposite, i.e. the strongest criterion has the full weight.

Finding the optimal distribution of the weights under a certain level of orness has obtained some interest during the last decade. The weights can be optimal in many ways; O'Hagan, for instance (1988), presented a numerical method to find the maximum entropy OWA operators under a crisp level of orness. Quite recently (Fuller and Majlender, 2001), (Fuller and Majlender, 2003) and (Carlsson et al., 2003) extended those results with both a analytical model for the maximum entropy problem as well as an analytical solution to the minimum variability problem. These contributions are interesting and sound theoretical findings. They did not, however, consider a fuzzy level of orness. The level of orness is often estimated from expert opinions and can be inherent fuzzy. Therefore, this paper contributes with a fuzzy orness level, minimum variability, OWA operator model. This paper does not use the Lagrange multiplier method, used by (Fuller and Majlender, 2001, 2003), but instead the constraints for the minimum variability problem are substituted in the objective function. Afterwards, the objective function is assumed to have a triangular fuzzy level of orness. This paper uses the signed distance method (Yao and Wu, 2000) to defuzzify the objective function, where after the optimisation problem is checked for convexity and solved numerically to the optimal solution. Other contributions using the signed distance method to defuzzify fuzzy numbers are (Salameh and Jaber, 2000) and (Yao and Chiang, 2003), for instance.

The paper is organised as follows: first the minimum variability OWA operator problem is formulated. Then the problem is altered to contain only an objective function, where after the level of orness is allowed to be fuzzy, but defuzzified in

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order to obtain a crisp optimal solution to the problem. Finally, a small problem is solved and compared with the solution obtained by (Fuller and Majlender, 2003).

2 THE MINIMUM VARIABILITY OWA OPERATOR PROBLEM

According to (Fuller and Majlender, 2003), the minimum variability problem is the following:

$$
\begin{aligned}
\min \quad & \frac{1}{n}\sum_{i=1}^{n} w_i^2 - \frac{1}{n^2} \\
s.t. \quad & orness(w) = \sum_{i=1}^{n} \frac{n-i}{n-1} w_i = \alpha, \quad 0 \le \alpha \le 1 \qquad (1) \\
& \sum_{i=1}^{n} w_i = 1
\end{aligned}
$$

where w_i is the positive weigths (the variables in the optimisation problem) and n is the total number of weights. α is the level of orness (parameter in the optimisation problem). This model can be solved analytically to optimum using a Lagrange multiplier method as in Fuller and Majlender (2003).

The model in eq. (1) can be reformulated by substituting each of the two constraints into the objective function. The second constraint in (1) will give us the following relationship:

$$w_1 = 1 - \sum_{i=2}^{n} w_i \qquad (2)$$

Subsituting (2) into (1) yields

$$
\begin{aligned}
\min \quad & \frac{1}{n}\sum_{i=2}^{n} w_i^2 - \frac{1}{n^2} + \frac{1}{n}\left(1 - \sum_{i=2}^{n} w_i\right)^2 \\
s.t. \quad & \sum_{i=2}^{n} \frac{n-i}{n-1} w_i + \left(1 - \sum_{i=2}^{n} w_i\right) = \alpha,
\end{aligned}
\qquad (3)
$$

The constraint in (3) will give us the following relationship:

$$w_2 = -\sum_{i=3}^{n} (i-1) w_i + (n-1)(1-\alpha) \qquad (4)$$

Using (4) in (3) will give us the simplified optimisation problem, containing only an objective function as follows (after some simplifications)

$$
\begin{aligned}
\min \quad & \frac{1}{n}\sum_{i=3}^{n} w_i^2 - \frac{1}{n^2} \\
& + \frac{1}{n}\left(n + \alpha - n\alpha - 1 - \sum_{i=3}^{n} (i-1) w_i\right)^2 \\
& + \frac{1}{n}\left(-n - \alpha + n\alpha + 2 + \sum_{i=3}^{n} (i-2) w_i\right)^2
\end{aligned}
\qquad (5)
$$

First of all it is worth noticing that the optimisation problem in eq. (5) is convex. The convexity can be established by examining the terms and since classical convexity theory states that a function $f(x) = (\sum_i c_i x_i + k)^2$, is always convex, where c_i and k are constants.

The next step is to manipulate (5) to remove the squares (in order to be able to defuzzify it with the signed distance method). This will result (after some simplifications and rearrangements) in the following problem (i.e. to an equivalent problem to the one found in eq. 5):

$$
\begin{aligned}
\min \quad & -\frac{1}{n^2} + 2n + 2\frac{\alpha^2}{n} + 2n\alpha^2 + \frac{5}{n} \\
& + 10\alpha - 4\alpha^2 - 4n\alpha - 6\frac{\alpha}{n} - 6 \\
& + \frac{1}{n}\sum_{i=3}^{n} (2i^2 - 6i + 6) w_i^2 - \sum_{i=3}^{n} (4i-6) w_i \\
& - \frac{\alpha}{n}\sum_{i=3}^{n} (4i-6) w_i + \frac{2}{n}\sum_{i=3}^{n} (3i-5) w_i \\
& + \alpha \sum_{i=3}^{n} (4i-6) w_i \\
& + \frac{2}{n}\sum_{i=3}^{n-1} \sum_{j=1+i}^{n} (i-1)(j-1) w_i w_j \\
& + \frac{2}{n}\sum_{i=3}^{n-1} \sum_{j=1+i}^{n} (i-2)(j-2) w_i w_j
\end{aligned}
\qquad (6)
$$

There are some intermediate steps between eqs. (5) and (6) that are left out since it would require some additional pages of formulas. The reformulation of eq. (5) into the problem in eq. (6) may seem to complicate the problem structure, but in fact it helps the defuzzification step, since α will only be found in separate terms.

3 DEFUZZIFICATION OF THE ORNESS VALUE

If the α value (the orness value) is triangular fuzzy, denoted as $\tilde{\alpha}$, the optimization problem becomes simply the following:

$$\begin{aligned}
\min \quad & -\frac{1}{n^2}+2n+2\frac{\tilde{\alpha}^2}{n}+2n\tilde{\alpha}^2+\frac{5}{n} \\
& +10\tilde{\alpha}-4\tilde{\alpha}^2-4n\tilde{\alpha}-6\frac{\tilde{\alpha}}{n}-6 \\
& +\frac{1}{n}\sum_{i=3}^{n}\left(2i^2-6i+6\right)w_i^2-\sum_{i=3}^{n}(4i-6)w_i \\
& -\frac{\tilde{\alpha}}{n}\sum_{i=3}^{n}(4i-6)w_i+\frac{2}{n}\sum_{i=3}^{n}(3i-5)w_i \\
& +\tilde{\alpha}\sum_{i=3}^{n}(4i-6)w_i \\
& +\frac{2}{n}\sum_{i=3}^{n-1}\sum_{j=1+i}^{n}(i-1)(j-1)w_iw_j \\
& +\frac{2}{n}\sum_{i=3}^{n-1}\sum_{j=1+i}^{n}(i-2)(j-2)w_iw_j
\end{aligned} \tag{7}$$

Some basics from fuzzy set theory need to be introduced in order to make the following model development self-contained.

Definition 1. Consider the fuzzy set $\tilde{A}=(a,b,c)$ where $a<b<c$ and defined on R, which is called a triangular fuzzy number, if the membership function of $\tilde{A}$ is given by

$$\mu_{\tilde{A}}(x)=\begin{cases}(x-a)/(b-a), & a\le x\le b,\\ (c-x)/(c-b), & b\le x\le c\\ 0, & \textit{otherwise.}\end{cases}$$

Definition 2. Let $\tilde{B}$ be a fuzzy set on R and $0\le\alpha_c\le 1$. The α_c-cut of $\tilde{B}$ is all the points x such that $\mu_{\tilde{B}}(x)\ge\alpha_c$, i.e.

$$B(\alpha_c)=\{x \mid \mu_{\tilde{B}}(x)\ge\alpha_c\}$$

In order to find non-fuzzy values for the model we need to use some distance measures and we will use the signed distance (Yao and Wu, 2000).

Definition 3. For any a and $0\in R$, the signed distance from a to 0 is $d_0(a,0)=a$. And if $a<0$, the distance from a to 0 is $-a=-d_0(a,0)$.

Let Ω be the family of all fuzzy sets $\tilde{B}$ defined on R for which the α-cut $B(\alpha_c)=[B_L(\alpha_c),B_U(\alpha_c)]$ exists for every $\alpha_c\in[0,1]$, and both $B_L(\alpha_c)$ and $B_U(\alpha_c)$ are continuous functions on $\alpha_c\in[0,1]$. Then, for any $\tilde{B}\in\Omega$, we have (see Chang, 2004, for instance)

$$\tilde{B}=\bigcup_{0\le\alpha_c\le 1}\left[B_L(\alpha_c)_{\alpha_c},B_U(\alpha_c)_{\alpha_c}\right]$$

From Chang (2004) it can be finally stated (originally by results from Yao and Wu, 2000) how to calculate the signed distances.

Definition 4. For $\tilde{B}\in\Omega$ define the signed distance of $\tilde{B}$ to $\tilde{0}_1$ as

$$d(\tilde{B},\tilde{0}_1)=\frac{1}{2}\int_0^1\left[B_L(\alpha_c)+B_U(\alpha_c)\right]d\alpha_c.$$

The Definition 3 will give us several properties of which the most important is

Property 1. Consider the triangular fuzzy number $\tilde{A}=(a,b,c)$: the α-cut of $\tilde{A}$ is $A(\alpha_c)=[A_L(\alpha_c),A_U(\alpha_c)]$, for $\alpha_c\in[0,1]$, where $A_L(\alpha_c)=a+(b-a)\alpha_c$ and $A_U(\alpha_c)=c-(c-b)\alpha_c$, the signed distance of $\tilde{A}$ to $\tilde{0}_1$ is

$$d(\tilde{A},\tilde{0}_1)=\frac{1}{4}(a+2b+c).$$

Let us assume that we have a triangular fuzzy orness level, i.e.

$$\tilde{\alpha}=(\alpha-\Delta_l,\alpha,\alpha+\Delta_h) \tag{8}$$

(Note that the orness value, α , should not be mixed up with the α-cut, called α_c.) Then we will defuzzify $\tilde{\alpha}$ in two different ways, depending on whether $\tilde{\alpha}$ is squared or not. From Property 1 we will get directly that the signed distance of $\tilde{\alpha}$ is

$$\begin{aligned} d(\tilde{\alpha},\tilde{0}) &= \frac{1}{4}\left[(\alpha-\Delta_l)+2\alpha+(\alpha+\Delta_h)\right] \\ &= \alpha+\frac{1}{4}\Delta_h-\frac{1}{4}\Delta_l \end{aligned} \tag{9}$$

And according to Definition 4 we will get that the signed distance for $\tilde{\alpha}^2$ will be

$$
\begin{aligned}
d(\widetilde{\alpha}^2,\widetilde{0}) &= \frac{1}{2}\int_0^1\left[\left(\alpha^2\right)_L(\alpha_c)+\left(\alpha^2\right)_U(\alpha_c)\right]d\alpha_c \\
&= \frac{1}{2}\int_0^1\begin{bmatrix}\left(\alpha-\Delta_l+\Delta_l\alpha_c\right)^2 \\ +\left(\alpha+\Delta_h-\Delta_h\alpha_c\right)^2\end{bmatrix}d\alpha_c \\
&= \frac{1}{2}\int_0^1\begin{bmatrix}\alpha^2-\alpha\Delta_l+\alpha\Delta_l\alpha_c-\alpha\Delta_l+\Delta_l^2 \\ -\Delta_l\alpha_c+\alpha\Delta_l\alpha_c-\Delta_l^2\alpha_c+\Delta_l^2\alpha_c^2 \\ +\alpha^2+\alpha\Delta_h-\alpha\Delta_h\alpha_c+\alpha\Delta_h \\ +\Delta_h^2-\Delta_h^2\alpha_c-\alpha\Delta_h\alpha_c+\Delta_h^2\alpha_c \\ +\Delta_h^2\alpha_c^2\end{bmatrix}d\alpha_c
\end{aligned}
\tag{10}
$$

Finally we will get the signed distance value for $\widetilde{\alpha}^2$ as

$$
d(\widetilde{\alpha}^2,\widetilde{0}) = \alpha^2-\frac{1}{2}\alpha\Delta_l+\frac{1}{2}\alpha\Delta_h+\frac{1}{6}\Delta_l^{\,2}+\frac{1}{6}\Delta_h^{\,2} \tag{11}
$$

The defuzzified objective function will be

$$
\begin{aligned}
\min\ & -\frac{1}{n^2}+2n+2\frac{d(\widetilde{\alpha}^2,\widetilde{0})}{n}+2n\cdot d(\widetilde{\alpha}^2,\widetilde{0}) \\
& +\frac{5}{n}+10d(\widetilde{\alpha},\widetilde{0})-4d(\widetilde{\alpha}^2,\widetilde{0})-4n\cdot d(\widetilde{\alpha},\widetilde{0}) \\
& -6\frac{d(\widetilde{\alpha},\widetilde{0})}{n}-6+\frac{1}{n}\sum_{i=3}^{n}\left(2i^2-6i+6\right)w_i^2 \\
& -\sum_{i=3}^{n}(4i-6)w_i-\frac{d(\widetilde{\alpha},\widetilde{0})}{n}\sum_{i=3}^{n}(4i-6)w_i \\
& +\frac{2}{n}\sum_{i=3}^{n}(3i-5)w_i+d(\widetilde{\alpha},\widetilde{0})\sum_{i=3}^{n}(4i-6)w_i \\
& +\frac{2}{n}\sum_{i=3}^{n-1}\sum_{j=1+i}^{n}(i-1)(j-1)w_iw_j \\
& +\frac{2}{n}\sum_{i=3}^{n-1}\sum_{j=1+i}^{n}(i-2)(j-2)w_iw_j
\end{aligned}
\tag{12}
$$

And putting the signed distances (to defuzzify) of $\widetilde{\alpha}$ and $\widetilde{\alpha}^2$ respectively (Equations 9 and 11), into eq. (7) will give us the final defuzzified objective function as

$$
\begin{aligned}
\min\ & -\frac{1}{n^2}+2n+ \\
& 2\frac{\alpha^2-\frac{1}{2}\alpha\Delta_l+\frac{1}{2}\alpha\Delta_h+\frac{1}{6}\Delta_l^{\,2}+\frac{1}{6}\Delta_h^{\,2}}{n} \\
& +2n\cdot\left(\alpha^2-\frac{1}{2}\alpha\Delta_l+\frac{1}{2}\alpha\Delta_h+\frac{1}{6}\Delta_l^{\,2}+\frac{1}{6}\Delta_h^{\,2}\right) \\
& +\frac{5}{n}+10\left(\alpha+\frac{1}{4}\Delta_h-\frac{1}{4}\Delta_l\right) \\
& -4\left(\alpha^2-\frac{1}{2}\alpha\Delta_l+\frac{1}{2}\alpha\Delta_h+\frac{1}{6}\Delta_l^{\,2}+\frac{1}{6}\Delta_h^{\,2}\right) \\
& -4n\cdot\left(\alpha+\frac{1}{4}\Delta_h-\frac{1}{4}\Delta_l\right) \\
& -6\frac{\alpha+\frac{1}{4}\Delta_h-\frac{1}{4}\Delta_l}{n}-6+\frac{1}{n}\sum_{i=3}^{n}\left(2i^2-6i+6\right)w_i^2 \\
& -\sum_{i=3}^{n}(4i-6)w_i-\frac{\alpha+\frac{1}{4}\Delta_h-\frac{1}{4}\Delta_l}{n}\sum_{i=3}^{n}(4i-6)w_i \\
& +\frac{2}{n}\sum_{i=3}^{n}(3i-5)w_i+\left(\alpha+\frac{1}{4}\Delta_h-\frac{1}{4}\Delta_l\right)\sum_{i=3}^{n}(4i-6)w_i \\
& +\frac{2}{n}\sum_{i=3}^{n-1}\sum_{j=1+i}^{n}(i-1)(j-1)w_iw_j \\
& +\frac{2}{n}\sum_{i=3}^{n-1}\sum_{j=1+i}^{n}(i-2)(j-2)w_iw_j
\end{aligned}
\tag{13}
$$

The fuzzy minimum variability OWA operator problem can thus be solved by minimizing eq. (13). The first and second weight can there-after be obtained from eqs. (4) and (2), respectively (with the defuzzified value of $\widetilde{\alpha}$, and not the crisp one, c.f. eq. 14).

$$
w_2=-\sum_{i=3}^{n}(i-1)w_i+(n-1)\left(1-d(\widetilde{\alpha},\widetilde{0})\right) \tag{14}
$$

It is worth noticing that the convexity will remain (from eq. 5) through the operations, since the effect of a fuzzy orness-value (α-value) will only affect the constant in the optimization problem. (I.e. it will only affect the parameter k in the functions of the form $f(x)=(\sum_i c_ix_i+k)^2$ and, thus, not affect the convexity. In addition, the operations in eqs. 6-13 will not change the convexity assumption.) The optimization problem in eq. (13) can be solved numerically with any local nonlinear optimization methods, which can guarantee local optimal convergence. The method need not be able to handle constraints, since there are no constraints involved in eq. (13), except for the non-negativity constraint of the variables. A method that can handle simple constraints is, however, advisable so that the substituted constraints in eqs. (2) and (4) will always get non-negative values.

4 EXAMPLE

In this section, a test problem is solved and compared to the crisp solution by Fuller and Majlender (2003). This problem contains 5 weights and it is calculated for a level of orness (α-value) of 0.1, 0.2, 0.3, 0.4 and 0.5. First, the problem is compared to the crisp solution for an α-value of 0.3 and different values of the Δ-parameters (i.e. different fuzziness values). The solution is obtained by using a standard local search method on the problem in eq. (13). The problem in this paper is solved with the extended Newton method found in the standard solver available in Microsoft Excel.

Table 1: The optimal OWA-operators for different fuzziness values (α=0.3).

α	Δ_l	Δ_h	w_1	w_2	w_3	w_4	w_5	Obj
0.300	0.000	0.000	0.040	0.120	0.200	0.280	0.360	0.013
0.300	0.050	0.050	0.040	0.120	0.200	0.280	0.360	0.018
0.300	0.100	0.050	0.030	0.115	0.200	0.285	0.370	0.027
0.300	0.050	0.100	0.050	0.125	0.200	0.275	0.350	0.024

In Table 1 it should be noted that the crisp case (i.e. when the Δ's are 0) collapses to the same solution as reported in Fuller and Majlender (2003). It should also be noted that the optimal solution (in this example) remained the same as the crisp solution if $\Delta_l = \Delta_h$. In order to illustrate the behaviour of the weights for different Δ-values (as well as the objective function), Figure 1 and Figure 2 are included. In these figures, the α-value is set to 0.3, but one of the Δ-values is allowed to change. One can see in Figure 1 that if Δ_h is increased from 0 to 0.3 the objective value increases from 0.013 to 0.065 and the weights get more similar to each other. In a similar manner when Δ_l increasing from 0 to 0.3, the objective value will increase from 0.013 to 0.084 and the weights become more diverse.

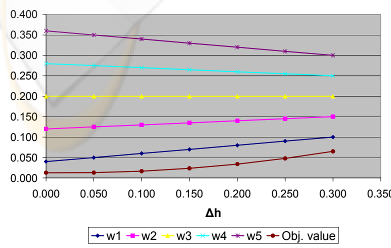

Figure 1: The sensitivity analysis of Δ_h for α=0 and Δ_l=0.

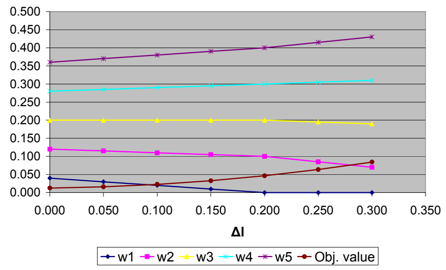

Figure 2: The sensitivity analysis of Δ_l for α=0 and Δ_h=0.

In Table 2, the optimal OWA-operators for several α-values are calculated. When the $\Delta_l = \Delta_h = 0$, (i.e. the crisp case) the operator-values are the same as the one reported by Fuller and Majlender (2003). In the case of Δ-values greater than zero (and unequal) the operator-values are different from the crisp case, except for the case of α=0.1. It is also worth noticing that the objective value for the crisp case is always better than for the fuzzy cases (in this example); when α=0.1 the increase is only about 20 %, but with bigger α-values, the bigger the increase in the objective function when fuzziness is introduced.

Table 2: The optimal OWA-operators for different α-values as well as fuzziness values.

α	Δ_l	Δ_h	w_1	w_2	w_3	w_4	w_5	Obj
0.100	0.000	0.000	0.000	0.000	0.033	0.333	0.633	0.063
0.100	0.050	0.100	0.000	0.000	0.058	0.333	0.608	0.069
0.100	0.100	0.050	0.000	0.000	0.008	0.333	0.658	0.081
0.200	0.000	0.000	0.000	0.040	0.180	0.320	0.460	0.030
0.200	0.050	0.100	0.000	0.055	0.185	0.315	0.445	0.039
0.200	0.100	0.050	0.000	0.025	0.175	0.325	0.475	0.045
0.400	0.000	0.000	0.120	0.160	0.200	0.240	0.280	0.003
0.400	0.050	0.100	0.130	0.165	0.200	0.235	0.270	0.015
0.400	0.100	0.050	0.110	0.155	0.200	0.245	0.290	0.016
0.500	0.000	0.000	0.200	0.200	0.200	0.200	0.200	0.000
0.500	0.050	0.100	0.210	0.205	0.200	0.195	0.190	0.012
0.500	0.100	0.050	0.190	0.195	0.200	0.205	0.210	0.012

5 CONCLUSIONS

This paper presents a new fuzzy minimum variability model for the OWA-operators, originally introduced by Yager (1988). Previous results in this line of research is the elegant results by Fuller and Majlender (2001, 2003), where both the minimum variability problem as well as the maximum entropy problem were solved. These results assumed, however, a crisp level of orness.

This paper added the current research track a model that could account for unsymmetrical (or symmetrical) triangular fuzzy levels of orness. This is important if the decision maker is not certain about the level of orness, but can estimate it through the proposed fuzzy numbers. The minimum variability model for the fuzzy orness level is obtained through a slightly different approach than the one used in Fuller and Majlender (2001, 2003). This paper substitutes the constraints in the problem (c.f. eq. 1) such that the variables w_1 and w_2 are eliminated out of the problem, and after some rearrangements a convex objective in smaller dimension remains of the original problem. This problem is allowed to have triangular fuzzy α-values, but in order to solve the optimisation problem, the α-values are defuzzified with the signed distance method. The defuzzified optimization problem is then solved with a numerical optimisation method that can guarantee local convergence. The first two weights are then solved from the substituted constraints.

The future research consists of analytical solutions for the optimization problem as well as extending the level of orness to contain other types of fuzzy numbers than the triangular ones. A natural extension could be to investigate the trapezoidal fuzzy numbers as well as other defuzzification methods.

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