DYNAMICAL MODELS FOR OMNI-DIRECTIONAL ROBOTS WITH 3 AND 4 WHEELS

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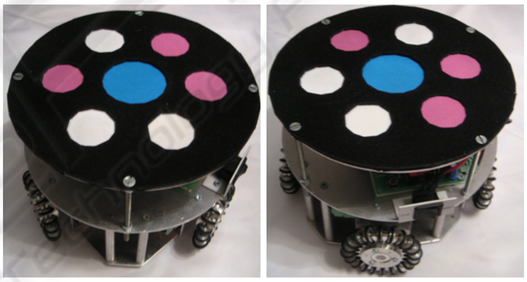
Abstract: Omni-directional robots are becoming more and more common in recent robotic applications. They offer improved ease of maneuverability and effectiveness at the expense of increased complexity. Frequent applications include but are not limited to robotic competitions and service robotics. The goal of this work is to find a precise dynamical model in order to predict the robot behavior. Models were found for two real world omni-directional robot configurations and their parameters estimated using a prototype that can have 3 or 4 wheels. Simulations and experimental runs are presented in order to validate the presented work.

1 INTRODUCTION

Omni-directional robots are becoming a much sought solution to mobile robotic applications. This kind of holonomic robots are interesting because they allow greater maneuverability and efficiency at the expense of some extra complexity. One of the most frequent solutions is to use some of Mecanum wheels (Diegel et al., 2002) and (Salih et al., 2006). A robot with 3 or more motorized wheels of this kind can have almost independent tangential, normal and angular velocities. Dynamical models for this kind of robots are not very common due to the difficulty in modeling the several internal frictions inside the wheels, making the model somewhat specific to the type of wheel being used (Williams et al., 2002).

Frequent mechanical configurations for omni-directional robots are based on three and four wheels. Three wheeled systems are mechanically simpler but robots with four wheels have more acceleration with the same kind of motors. Four wheeled robots are expected to have better effective floor traction, that is, less wheel slippage – assuming that all wheels are pressed against the floor equally. Of course four wheeled robots also have a higher costs in equipment, increased energy consumption and may require some kind of suspension to distribute forces equally among the wheels.

In order to study and compare the models of the 3 and 4 wheeled robots, a single prototype was built that can have both configurations, that is, the same mechanical platform can be used with 3 wheels and then it can be disassembled and reassembled with a 4 wheel configuration, see figure 1.



(a) Three wheeled robot. (b) Four wheeled robot.

Figure 1: Omni-directional robot.

Data from experimental runs is taken from overhead camera. The setup is taken from the heritage of the system described in (Costa et al., 2000) that currently features 25 fps, one centimeter accuracy in position(XX and YY axis) and about 3 sexagesimal degrees of accuracy in the heading of the robot.

In order to increase the performance of robots, there were some efforts on the studying their dynamical models (Campion et al., 1996)(Conceição et al., 2006)(Khosla, 1989)(Tahmasebi et al., 2005)(Williams et al., 2002) and kinematic models (Campion et al., 1996) (Leow et al., 2002)(Loh et al., 2003)(Muir and Neuman, 1987)(Xu et al., 2005). Models are based on linear and non linear dynamical systems and the estimation of parameters has been the subject of continuing research (Conceição et al., 2006)(Olsen and Petersen, 2001). Once the dynamical model is found, its parameters have to be estimated. The most common method for identification

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of robot parameters are based on the Least Squares method and Instrumental Variables.

However, the systems are naturally non-linear (Julier and Uhlmann, 1997), the estimation of parameters is more complex and the existing methods (Ghaharamani and Roweis, 1999)(Gordon et al., 1993)(Tahmasebi et al., 2005) have to be adapted to the model's structure and noise.

1.1 Structure

This paper starts by presenting the mechanical prototype and studied mechanical configurations for the 3 and 4 wheeled robots. Finding the dynamical model is discussed and then, an initial approach in estimating model parameters for each robot is done. The need for additional accuracy drives the comparative study on relative importance of the estimated parameters. An additional experiment is done for estimating final numerical values for the configurations of 3 and 4 wheels. Conclusions and future work are also presented.

2 MECHANICAL CONFIGURATIONS

Figures 2 and 3 present the configuration of the three and four wheeled robots respectively, as well as all axis and relevant forces and velocities of the robotic system. The three wheeled system features wheels separated by 120 degrees.

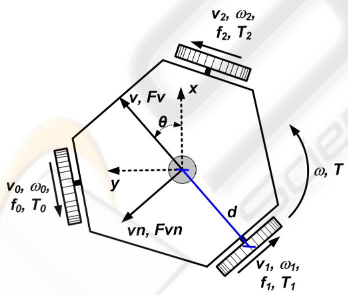


Figure 2: Three Wheeled Robot.

Figures 2 and 3 show the notation used throughout this paper, detailed as follows:

- x, y, θ - Robot's position (x,y) and θ angle to the defined front of robot;
- d $[m]$ - Distance between wheels and center robot;
- v_0, v_1, v_2, v_3 $[m/s]$ - Wheels linear velocity;
- $\omega_0, \omega_1, \omega_2, \omega_3$ $[rad/s]$ - Wheels angular velocity;
- f_0, f_1, f_2, f_3 $[N]$ - Wheels traction force;

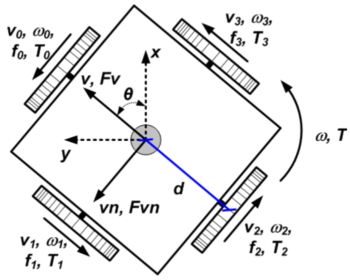


Figure 3: Four Wheeled Robot.

- T_0, T_1, T_2, T_3 $[N \cdot m]$ - Wheels traction torque;
- v, vn $[m/s]$ - Robot linear velocity;
- ω $[rad/s]$ - Robot angular velocity;
- F_v, F_{vn} $[N]$ - Robot traction force along v and vn;
- T $[N \cdot m]$ - Robot torque (respects to ω).

3 MODELS

3.1 Kinematic

The well known kinematic model of an omni-directional robot located a (x, y, θ) can be written as $v_x(t) = dx(t)/dt$, $v_y(t) = dy(t)/dt$ and $\omega(t) = d\theta(t)/dt$ (please refer to figures 2 and 3 for notation issues). Equation 1 allows the transformation from linear velocities v_x and v_y on the static axis to linear velocities v and vn on the robot's axis.

$$X_R = \begin{bmatrix} v(t) \\ vn(t) \\ \omega(t) \end{bmatrix} \ ; \ X_0 = \begin{bmatrix} v_x(t) \\ v_y(t) \\ \omega(t) \end{bmatrix}$$

$$X_R = \begin{bmatrix} cos(\theta(t)) & sin(\theta(t)) & 0 \\ -sin(\theta(t)) & cos(\theta(t)) & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot X_0 \quad (1)$$

3.1.1 Three Wheeled Robot

Wheel speeds v_0, v_1 and v_2 are related with robot's speeds v, vn and ω as described by equation 2.

$$\begin{bmatrix} v_0(t) \\ v_1(t) \\ v_2(t) \end{bmatrix} = \begin{bmatrix} -sin(\pi/3) & cos(\pi/3) & d \\ 0 & -1 & d \\ sin(\pi/3) & cos(\pi/3) & d \end{bmatrix} \cdot \begin{bmatrix} v(t) \\ vn(t) \\ \omega(t) \end{bmatrix} \quad (2)$$

Applying the inverse kinematics is possible to obtain the equations that determine the robot speeds related the wheels speed. Solving in order of v, vn and ω, the following can be found:

$$v(t) = (\sqrt{3}/3) \cdot (v_2(t) - v_0(t)) \quad (3)$$

$$vn(t) = (1/3) \cdot (v_2(t) + v_0(t)) - (2/3) \cdot v_1(t) \quad (4)$$

$$\omega(t) = (1/(3 \cdot d)) \cdot (v_0(t) + v_1(t) + v_2(t)) \quad (5)$$

3.1.2 Four Wheeled Robot

The relationship between the wheels speed v_0, v_1, v_2 and v_3, with the robot speeds v, vn and ω is described by equation 6.

$$\begin{bmatrix} v_0(t) \\ v_1(t) \\ v_2(t) \\ v_3(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 & d \\ -1 & 0 & d \\ 0 & -1 & d \\ 1 & 0 & d \end{bmatrix} \cdot \begin{bmatrix} v(t) \\ vn(t) \\ \omega(t) \end{bmatrix} \tag{6}$$

It is possible to obtain the equations that determine the robot speeds related with wheels speed but the matrix associated with equation 6 is not square. This is because the system is redundant. It can be found that:

$$v(t) = (1/2) \cdot (v_3(t) - v_1(t)) \tag{7}$$
$$vn(t) = (1/2) \cdot (v_0(t) - v_2(t)) \tag{8}$$
$$\omega(t) = (v_0(t) + v_1(t) + v_2(t) + v_3(t))/(4 \cdot d) \tag{9}$$

3.2 Dynamic

The dynamical equations relative to the accelerations can be described in the following relations:

$$M \cdot \frac{dv(t)}{dt} = \sum F_v(t) - F_{Bv}(t) - F_{Cv}(t) \tag{10}$$
$$M \cdot \frac{dvn(t)}{dt} = \sum F_{vn}(t) - F_{Bvn}(t) - F_{Cvn}(t) \tag{11}$$
$$J \cdot \frac{d\omega(t)}{dt} = \sum T(t) - T_{B\omega}(t) - T_{C\omega}(t) \tag{12}$$

where the following parameters relate to the robot as follows:

- M $[kg]$ - mass;
- J $[kg \cdot m^2]$ - inertia moment;
- F_{Bv}, F_{Bvn} $[N]$ - viscous friction forces along v and vn;
- $T_{B\omega}$ $[N \cdot m]$ - viscous friction torque with respect to the robot's rotation axis;
- F_{Cv}, F_{Cvn} $[N]$ - Coulomb frictions forces along v and vn;
- $T_{C\omega}$ $[N \cdot m]$ - Coulomb friction torque with respect to robot's rotation axis.

Viscous friction forces are proportional to robot's speed and as such $F_{Bv}(t) = B_v \cdot v(t)$, $F_{Bvn}(t) = B_{vn} \cdot vn(t)$ and $T_{B\omega}(t) = B_\omega \cdot \omega(t)$, where Bv, Bvn $[N/(m/s)]$ are the viscous friction coefficients for directions v and vn and $B\omega$ $[N \cdot m/(rad/s)]$ is the viscous friction coefficient to ω.

The Coulomb friction forces are constant in amplitude $F_{Cv}(t) = C_v \cdot sign(v(t))$, $F_{Cvn}(t) = C_{vn} \cdot sign(vn(t))$ and $T_{C\omega}(t) = C_\omega \cdot sign\ \omega(t)$, where Cv, Cvn $[N]$ are Coulomb friction coefficient for directions v e vn and $C\omega$ $[N \cdot m]$ is the Coulomb friction coefficient for ω.

3.2.1 Three Wheeled Robot

The relationship between the traction forces and rotation torque of the robot with the traction forces on the wheels is described by the following equations:

$$\sum F_v(t) = (f_2(t) - f_0(t)) \cdot \sin(\pi/3) \tag{13}$$
$$\sum F_{vn}(t) = -f_1(t) + (f_2(t) + f_0(t)) \cdot \cos(\pi/3) \tag{14}$$
$$\sum T(t) = (f_0(t) + f_1(t) + f_2(t)) \cdot d \tag{15}$$

The traction force on each wheel is estimated by traction torque, which can be determined using the motor current, as described in the following equations:

$$f_j(t) = T_j(t)/r \tag{16}$$
$$T_j(t) = l \cdot K_t \cdot i_j(t) \tag{17}$$

- l - Gearbox reduction;
- r $[m]$ - Wheel radius;
- K_t $[N \cdot m/A]$ - Motor torque constant;
- i_j $[A]$ - Motor current (j=motor number).

3.2.2 Four Wheeled Robot

The relationship between the traction forces and rotation torque of the robot with the traction forces on the wheels, is described by the following equations:

$$\sum F_v(t) = f_3(t) - f_1(t) \tag{18}$$
$$\sum F_{vn}(t) = f_0(t) - f_2(t) \tag{19}$$
$$\sum T(t) = (f_0(t) + f_1(t) + f_2(t) + f_3(t)) \cdot d \tag{20}$$

As above, the traction force in each wheel is estimated using the wheels traction torque, which is determined by the motor current, using equations 16 and 17, where j=0,1,2,3.

3.3 Motor

The prototype uses brushless motors for the locomotion of the robot. The model for brushless motors is the similar to the common DC motors, based on (Pillay and Krishnan, 1989).

$$u_j(t) = L \cdot \frac{di_j(t)}{dt} + R \cdot i_j(t) + K_v \cdot \omega_{mj}(t) \tag{21}$$
$$T_{mj}(t) = K_t \cdot i_j(t) \tag{22}$$

- L $[H]$ - Motor inductance;
- R $[\Omega]$ - Motor resistor;
- K_v $[V/(rad/s)]$ - EMF motor constant;
- u_j $[V]$ - Motor voltage (j=motor number);
- ω_{mj} $[rad/s]$ - Motor angular velocity (j=motor number);
- T_{mj} $[N \cdot m]$ - Motor torque (j=motor number).

4 PARAMETER ESTIMATION

The necessary variables to estimate the model parameters are motor current, robot position and velocity. Currents are measured by the drive electronics, position is measured by using external camera and velocities are estimated from positions.

The parameters that must be identified are the viscous friction coefficients $(Bv, Bvn, B\omega)$, the Coulomb friction coefficients $(Cv, Cvn, C\omega)$ and inertia moment J. The robot mass was measured, and it was 1.944 kg for the three wheeled robot and 2.34 kg for the four wheeled robot.

4.1 Experience 1 - Steady State Velocity

This method permits to identify the viscous friction coefficients $B\omega$ and the Coulomb friction coefficients $C\omega$. The estimation of the coefficient ω was only implemented because inertia moment is unknown, and it is necessary to have an initial estimate of these coefficients. The experimental method relies on applying different voltages to the motors in order to move the robot according his rotation axis - the tests were made for positive velocities. Once reached the steady state, the robot's speed ω and rotation torque T can be measured. The robot speed is constant, so, the acceleration is null, and as such equation 12 can be re-written as follows:

$$\sum T(t) = B_\omega \cdot \omega(t) + C_\omega \qquad (23)$$

This linear equation shows that it is possible to test different values of rotation speed and rotation torques in multiple experiences and estimate the parameters.

4.2 Experience 2 - Null Traction Forces

This method allows for the estimation of the viscous friction coefficients (Bv, Bvn), the Coulomb friction coefficients (Cv, Cvn) and the inertia moment J. The experimental method consists in measuring the robot acceleration and speed when the traction forces were null. The motor connectors were disconnected and with a manual movement starting from a stable position, the robot was pushed through the directions v, vn and rotated according to his rotation axis. During the subsequent deceleration, velocity and acceleration were measured. Because the traction forces were null during the deceleration equations 10, 11, and 12 can be re-written as follows:

$$\frac{dv(t)}{dt} = -\frac{B_v}{M} \cdot v(t) - \frac{C_v}{M} \qquad (24)$$

$$\frac{dvn(t)}{dt} = -\frac{B_{vn}}{M} \cdot vn(t) - \frac{C_{vn}}{M} \qquad (25)$$

$$\frac{d\omega(t)}{dt} = -\frac{B_\omega}{J} \cdot \omega(t) - \frac{C_\omega}{J} \qquad (26)$$

These equations are also a linear relation and estimation of all parameters is possible.

The inertia moment J is estimated using the values obtained previously in section 4.1. To do this, equation 26 must be solved in order of J:

$$J = -\frac{\omega(t)}{(d\omega(t)/dt)} \cdot B_\omega - \frac{1}{(d\omega(t)/dt)} \cdot C_\omega \qquad (27)$$

4.3 DC Motor Parameters

The previous electrical motor model (equation 21) includes an electrical pole and a much slower, dominant mechanical pole - thus making inductance L value negligible. To determinate the relevant parameters K_v and R, a constant voltage is applied to the motor. Under steady state condition, the motor's current and the robot's angular velocity are measured. The tests are repeated several times for the same voltage, changing the operation point of the motor, by changing the friction on the motor axis.

In steady state, the inductance L disappears of the equation 21, being rewritten as follows:

$$u_j(t) = R \cdot i_j(t) + K_v \cdot \omega_{mj}(t) \qquad (28)$$

As seen in equation 29, by dividing (28) by $i_j(t)$, a linear relation is obtained and thus estimation is possible.

$$\frac{u_j(t)}{i_j(t)} = K_v \cdot \frac{\omega_{mj}(t)}{i_j(t)} + R \qquad (29)$$

5 RESULTS

5.1 Robot Model

By combining previously mentioned equations, it is possible to show that model equations can be rearranged into a variation of the state space that can be described as:

$$(dx(t)/dt) = A \cdot x(t) + B \cdot u(t) + K \cdot sign(x) \qquad (30)$$

$$x(t) = [v(t) \;\; vn(t) \;\; w(t)]^T \qquad (31)$$

This formulation is interesting because it shows exactly which part of the system is non non-linear.

5.1.1 Three Wheeled

Using equations on section 3.2, 13 to 17 and 28, the equations for the three wheeled robot model are:

$$A = \begin{bmatrix} A_{11} & 0 & 0 \\ 0 & A_{22} & 0 \\ 0 & 0 & A_{33} \end{bmatrix} \qquad (32)$$

$$A_{11} = -\frac{3 \cdot K_t^2 \cdot l^2}{2 \cdot r^2 \cdot R \cdot M} - \frac{B_v}{M}$$

$$A_{22} = -\frac{3 \cdot K_t^2 \cdot l^2}{2 \cdot r^2 \cdot R \cdot M} - \frac{B_{vn}}{M}$$

$$A_{33} = -\frac{3 \cdot d^2 \cdot K_t^2 \cdot l^2}{r^2 \cdot R \cdot J} - \frac{B_w}{J}$$

$$B = \frac{l \cdot K_t}{r \cdot R} \cdot \begin{bmatrix} -\sqrt{3}/(2 \cdot M) & 0 & \sqrt{3}/(2 \cdot M) \\ 1/(2 \cdot M) & 1/M & 1/(2 \cdot M) \\ d/J & d/J & d/J \end{bmatrix} \quad (33)$$

$$K = \begin{bmatrix} -C_v/M & 0 & 0 \\ 0 & -C_{vn}/M & 0 \\ 0 & 0 & -C_w/J \end{bmatrix} \quad (34)$$

5.1.2 Four Wheeled

Using equations on section 3.2 and equations 16 to 20 and 28 we get the following equations to the four wheeled robot model.

$$A = \begin{bmatrix} A_{11} & 0 & 0 \\ 0 & A_{22} & 0 \\ 0 & 0 & A_{33} \end{bmatrix} \quad (35)$$

$$A_{11} = -\frac{2 \cdot K_t^2 \cdot l^2}{r^2 \cdot R \cdot M} - \frac{B_v}{M}$$

$$A_{22} = -\frac{2 \cdot K_t^2 \cdot l^2}{r^2 \cdot R \cdot M} - \frac{B_{vn}}{M}$$

$$A_{33} = -\frac{4 \cdot d^2 \cdot K_t^2 \cdot l^2}{r^2 \cdot R \cdot J} - \frac{B_w}{J}$$

$$B = \frac{l \cdot K_t}{r \cdot R} \cdot \begin{bmatrix} 0 & -1/M & 0 & 1/M \\ 1/M & 0 & -1/M & 0 \\ d/J & d/J & d/J & d/J \end{bmatrix} \quad (36)$$

$$K = \begin{bmatrix} -C_v/M & 0 & 0 \\ 0 & -C_{vn}/M & 0 \\ 0 & 0 & -C_w/J \end{bmatrix} \quad (37)$$

5.2 Experimental Data for Robot Model

Experience 1 was conducted using an input signal corresponding to a ramped up step. This way wheel sleeping was avoided, that is, wheel - traction problems don't exist.

Shown in Figure 4 are the experimental plots regarding the 4 wheeled system. Due to space constraints only $T vs. \omega$ and results of the experiment 2 along the v direction are shown.

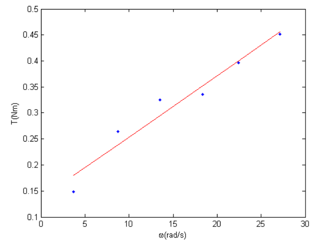
The motor model was presented earlier in equation 21.

Experimental tests to the four motors were made to estimate the value of resistor R and the constant K_v. The numerical value of the torque constant K_t is identical to the EMF motor constant K_v.

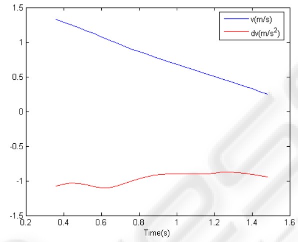
Figure 5 plots experimental runs regarding motor 0. Other motors follow similar behavior.

5.3 Numerical Results

Table 1 presents the experimental results to the friction coefficients and inertial moment. From the experimental runs from all 4 motors, the parameters found are $K_v = 0.0259\ V/(rad/s)$ and $R = 3.7007\ \Omega$.



(a) Experience 1 - T and ω.



(b) Experience 2 - Direction v.

Figure 4: Experimental Results for the four wheeled robot.

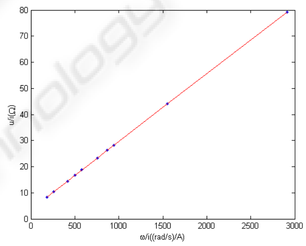


Figure 5: Experimental tests for motor 0.

Table 1: Friction coefficients and inertia moment.

Parameters	3 wheels	4 wheels
$J(kg \cdot m^2)$	0.015	0.016
$B_v(N/(m/s))$	0.503	0.477
$B_{vn}(N/(m/s))$	0.516	0.600
$B_\omega N \cdot m/(rad/s)$	0.011	0.011
$C_v(N)$	1.906	1.873
$C_{vn}(N)$	2.042	2.219
$C_\omega(N \cdot m)$	0.113	0.135

5.4 Sensitivity Analysis

To understand which model parameters have more influence on the robot's dynamics, a comparison was made between the matrices of the models.

The model equation 30 is a sum of fractions. Analyzing the contribution of each parcel and of the variable portion within each fraction, a sensitivity analysis is performed, one estimated parameter at a time.

1. Matrix A, robot moving along v direction;
   - Three wheeled robot:

$$\left(\frac{3 \cdot K_t^2 \cdot l^2}{2 \cdot r^2 \cdot R \cdot M}\right) = \frac{K_{a1}}{R} = 3.3110$$
$$(B_v/M) = K_{a2} \cdot B_v = 0.3245$$

   - Four wheeled robot:

$$\left(\frac{2 \cdot K_t^2 \cdot l^2}{r^2 \cdot R \cdot M}\right) = \frac{K_{a1}}{R} = 3.6676$$
$$(B_v/M) = K_{a2} \cdot B_v = 0.2041$$

2. Matrices B and K, robot moving along v direction with constant voltage motor equal to $6V$;
   - Three wheeled robot:

$$\left(\frac{\sqrt{3} \cdot l \cdot K_t}{2 \cdot r \cdot R \cdot M}\right) \cdot 12 = \frac{K_b}{R} \cdot 12 = 5.7570$$
$$(C_v/M) = K_k \cdot C_v = 0.8728$$

   - Four wheeled robot:

$$\left(\frac{l \cdot K_t}{r \cdot R \cdot M}\right) \cdot 12 = \frac{K_b}{R} \cdot 12 = 5.5227$$
$$(C_v/M) = K_k \cdot C_v = 0.7879$$

The same kind of analysis could be taken further by analyzing other velocities (vn and ω). Conclusions reaffirm that motor parameters have more influence in the dynamics than friction coefficients. This means that it is very important to have an accurate estimation of the motor parameters. Some additional experiences were designed to improve accuracy. The method used previously does not offer sufficient accuracy to the estimation of R. This parameter R is not a physical parameter and includes a portion of the non-linearity of the H bridge powering the circuit that, in turn, feeds 3 rapidly switching phases of the brushless motors used. In conclusion, additional accuracy in estimating R is needed.

5.5 Experience 3 - Parameter Estimation Improvement

The parameter improving experience was made using a step voltage with an initial acceleration ramp.

As seen in 5.1 the model was defined by the equation 30 and we can improve the quality of the estimation by using the Least Squares method. The system model equation can be rewritten as:

$$y = \theta_1 \cdot x_1 + \theta_2 \cdot x_2 + \theta_3 \cdot x_3 \tag{38}$$

Where $x_1 = x(t)$, $x_2 = u(t)$, $x_3 = 1$ and $y = dx(t)/dt$. The parameters θ are estimated using:

$$\theta = \left(x^T \cdot x\right)^{-1} \cdot x^T \cdot y \tag{39}$$

$$x = [x_1(1) \dots x_1(n)\ x_2(1) \dots x_2(n)\ x_3(1) \dots x_3(n)]^T \tag{40}$$

Estimated parameters can be skewed and for this reason instrumental variables are used to minimize the error, with vector of states defined as

$$z = [\overline{x_1}(1) \dots \overline{x_1}(n)\ x_2(1) \dots x_2(n)\ x_3(1) \dots x_3(n)]^T \tag{41}$$

The parameters θ are now calculated by:

$$\theta = \left(z^T \cdot x\right)^{-1} \cdot z^T \cdot y \tag{42}$$

Three experiments were made for each configuration of 3 and 4 wheels, along v, vn and ω. For the v and vn experiments values C_v and C_{vn} are kept from previous analysis. For the ω experiment, the value of the R parameter used is the already improved version from previous v and vn experimental runs of the current section.

The numerical value of R for each motor was estimated for each motor and then averaged to find R=4.3111 Ω. The results are present on followings tables. Table 2 shows values estimated by the experiment mentioned in this section.

Table 2: Parameters estimated using the method 3.

Parameters	3 wheels	4 wheels
$J(kg \cdot m^2)$	0.0187	0.0288
$B_v(N/(m/s))$	0.5134	0.5181
$B_{vn}(N/(m/s))$	0.4571	0.7518
$B_\omega N \cdot m/(rad/s)$	0.0150	0.0165
$C_\omega(N \cdot m)$	0.0812	0.1411

The final values for friction and inertial coefficients are averaged with results from all 3 experimental methods and the numerical values found are presented in Table 3.

Table 3: Parameters of dynamical models.

Parameters	3 wheels	4 wheels
$d(m)$	0.089	
$r\ (m)$	0.0325	
l	5	
$K_v\ (V/(rad/s))$	0.0259	
$R\ (\Omega)$	4.3111	
$M(kg)$	1.944	2.34
$J\ (kg \cdot m^2)$	0.0169	0.0228
$B_v\ (N/(m/s))$	0.5082	0.4978
$B_{vn}\ (N/(m/s))$	0.4870	0.6763
$B_\omega\ (N \cdot m/(rad/s))$	0.0130	0.0141
$C_v\ (N)$	1.9068	1.8738
$C_{vn}\ (N)$	2.0423	2.2198
$C_\omega\ (N \cdot m)$	0.0971	0.1385

5.6 Model Validation Experiences

The models were validated with experimental tests on using a step voltage with an initial acceleration ramp.

Due to space constraints in this document, figures 6 to 9 show plots for some of the runs only. Other runs confirm the global validity of the model as simulation follows reality closely.

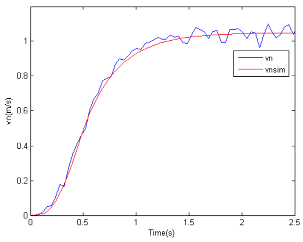


Figure 6: 3 wheel model validation - velocity along *vn*.

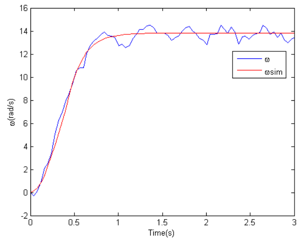


Figure 7: 3 wheel model validation - angular velocity ω.

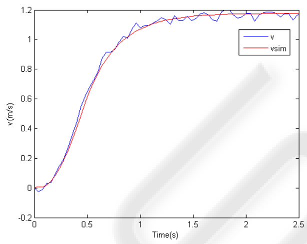


Figure 8: 4 wheel model validation - velocity along *v*.

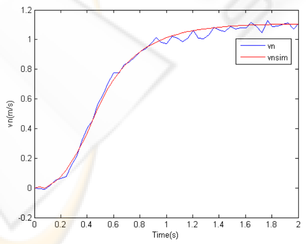


Figure 9: 4 wheel model validation - velocity along *vn*.

6 CONCLUSIONS

This paper presents models for mobile omni-directional robots with 3 and 4 wheels. The derived model is non-linear but maintains some similarities with linear state space equations. Friction coefficients are most likely dependent on robot and wheels construction and also on the weight of the robot. The model is derived assuming no wheel slip as in most standard robotic applications.

A prototype that can have either 3 or 4 omni-directional wheels was used to validate the presented model. The test ground is smooth and carpeted. Experience data was gathered by overhead camera capable of determining position and orientation of the robot with good accuracy.

Experiences were made to estimate the parameters of the model for the prototypes. The accuracy of the presented model is discussed and the need for additional experiences is proved. The initial estimation method used two experiences to find all parameters but a third experience is needed to improve the accuracy of the most important model parameters. Sensitivity analysis shows that the most important model parameters concern motor constants.

Observing estimated model parameters, the four wheel robot has higher friction coefficients in the *vn* direction when compared to the *v* direction. This means of course higher maximum speed for movement along *v* axis and higher power consumption for movements along the *vn* direction. This difference in performance points to the need of mechanical suspension to even wheel pressure on the ground.

The found model was shown to be adequate for the prototypes in the several shown experimental runs.

7 FUTURE WORK

The work presented is part of a larger study. Future work will include further tests with different prototypes including prototypes with suspension. The model can also be enlarged to include the limits for slippage and movement with controlled slip for the purpose of studying traction problems. Dynamical models estimated in this work can be used to study the limitations of the mechanical configuration and allow for future enhancements both at controller and mechanical configuration level. This study will enable effective full comparison of 3 and 4 wheeled systems.

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