REDUCED ORDER H∞ SYNTHESIS USING A PARTICLE SWARM OPTIMIZATION METHOD

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Abstract: H∞ controller synthesis is a well known design method for which efficient dedicated methods have been developed. However, such methods compute a full order controller which has often to be reduced to be implemented. Indeed, the reduced order H∞ synthesis is a non convex optimization problem due to rank constraints. In this paper, a particle swarm optimization method is used to solve such a problem. Numerical results show that the computed controller has a lower H∞ norm than the controller computed from a classical Hankel reduction of the full order H∞ controller.

1 INTRODUCTION

H∞ synthesis is an efficient tool, which aims to compute controllers in a closed loop framework, achieving high and various performances. Two principal solution methods have been developed for this purpose, based on Linear Matrix Inequalities (Gahinet and Apkarian, 1994), or on Riccatti equation solutions (Glover and Doyle, 1988). The main drawback of such approaches is the controller order: H∞ synthesis provides a controller whose order is the same as the synthesis model. A classical way to get low order controllers is to reduce the full controller, for example with a Hankel decomposition method. However, this approach may lead to a high H∞ norm of the closed loop system and a high sensitivity to high frequency noises. To avoid high order controllers, the H∞ optimization problem can be solved, adding some order constraints. However, this kind of constraints is expressed with rank constraints and the reduced-order synthesis problem appears to be a non convex optimization problem, and classical algorithms may fail in the solution.

In this paper, a new approach is proposed, using Particle Swarm Optimization (PSO). With such a method, the optimality of the computed solution can never be guaranteed, but the structure of costs and constraints is not an essential point. The mathematical descriptions of the full and reduced order H∞ synthesis are called up in section 2. PSO is presented in section 3. The proposed algorithm is used for the multivariable control of a pendulum in the cart. Results are given in section 4. Finally conclusions are drawn in section 5.

2 REDUCED-ORDER H∞ SYNTHESIS

2.1 Full-Order H∞ Synthesis

Consider the closed loop of figure 1, with s the Laplace variable. The transfer matrix is:

$$\begin{bmatrix} \varepsilon(s) \\ u(s) \end{bmatrix} = \begin{bmatrix} S & -SG \\ KS & -KSG \end{bmatrix} \begin{bmatrix} r(s) \\ d(s) \end{bmatrix} = T(s) \begin{bmatrix} r(s) \\ d(s) \end{bmatrix} \quad (1)$$

with $S(s) = (I + G(s)K(s))^{-1}$

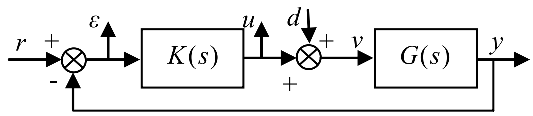

Figure 1: Classical closed loop system.

The H∞ synthesis problem is defined as follows. Find a stabilizing controller $K(s)$ such that:

$$\gamma = \min_{K(s)} \|T(s)\|_\infty \quad (2)$$

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It can be reformulated into a convex problem and solved with Riccati equations or LMI formulations. This solution is called "full order" synthesis, as the solution of problem (2) is a controller $K(s)$ whose order is equal to the order of $G(s)$. Some design filters are added to the synthesis model to tune the performances (figure 2). The new system is:

$$\begin{bmatrix} e_1(s) \\ e_2(s) \end{bmatrix} = \begin{bmatrix} W_1 S & -W_1 SGW_3 \\ W_2 KS & -W_2 KSGW_3 \end{bmatrix} \begin{bmatrix} r(s) \\ d(s) \end{bmatrix} \quad (3)$$

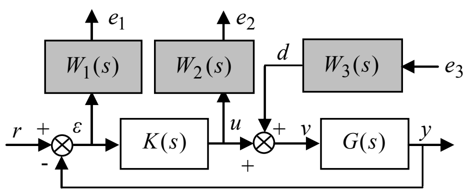

Figure 2: Synthesis model.

Finally, solving the H∞ problem for this system induces frequency dependent constraints for each transfer of matrix (1).

2.2 Reduced-Order H∞ Synthesis

The reduced-order H∞ problem refers to the solution of the following optimization problem:

$$\gamma = \min_{K(s)} \|T(s)\|_\infty \text{ s.t. } \partial^\circ K(s) = n_r \quad (4)$$

where $\partial^\circ K$ denotes the order of $K(s)$, and n_r is strictly less than the order of the synthesis model. It can be reformulated into LMI equations by adding rank constraints on matrices, loosing the property of convexity (El Ghaoui et al., 1997).

3 PSO ALGORITHM

PSO was introduced by Russel and Eberhart (Eberhart and Kennedy, 1995). P particles are moving in the search space. x_p^k (v_p^k) is the position (velocity) of particle p at iteration k, b_p^k is the best position found by particle p until iteration k, $V(x_p^k) \subset \{1,2,\ldots,P\}$ is the set of "friend particles" of particle p at iteration k, g_p^k best position found by the friend particles of particle p until iteration k, and $\otimes$ element wise multiplication of vectors. The particles move in the search space according to the following transition rule:

$$\begin{aligned} v_p^{k+1} &= w.v_p^k + c_1 \otimes (b_p^k - x_p^k) \\ &\quad + c_2 \otimes (g_p^k - x_p^k) \\ x_p^{k+1} &= x_p^k + v_p^{k+1} \end{aligned} \quad (5)$$

In this equation, w is the inertia factor and c_1, c_2 are random vectors in the range $[0,\bar{c}]$. The choice of parameters is very important to ensure the satisfying convergence of the algorithm, see (Eberhart and Shi, 2000). However, it is not in the scope of this study to look for fine strategies of tuning. Thus, standard values, given in (Kennedy and Clerc, 2006) will be used P: $P = 10 + \sqrt{n}$ (n is the number of optimization variables), $w = 1/(2\ln(2))$, $\bar{c} = 0{,}5 + \ln(2)$, $\dim(V(x_p^k)) \le 3$.

4 NUMERICAL RESULTS

4.1 Case Study

The proposed method has been tested for a pendulum in the cart (figure 3).

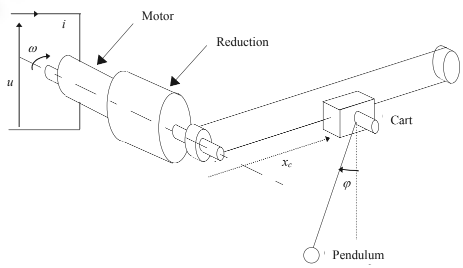

Figure 3: Pendulum in the cart.

The system can be modelled by:

$$\begin{aligned} &L\frac{d\,i(t)}{d\,t} + R\,i(t) + K_e\,\omega(t) = u(t) \\ &J\frac{d\,\omega(t)}{d\,t} + f\,\omega(t) + d(t) = K_e i(t) \\ &\frac{d\,x_c(t)}{d\,t} = \frac{r}{N}\omega(t) \\ &\cos(\varphi)\frac{d^2 x_c}{d\,t^2} + l\frac{d^2\varphi}{d\,t^2} + \alpha\frac{d\,\varphi}{d\,t} + g\sin(\varphi) = 0 \end{aligned} \quad (6)$$

Variables are i and u (current and input voltage of the motor), ω (rotation speed), x_c (position of the cart), φ (angle of the pendulum), d (disturbance moment). Constants are L, R, J (motor inductor, resistance, inertia), K_e (electromagnetic constant), f (friction coefficient), r (pulley radius), N (gear reduction), l (pendulum length), α (pendulum friction coefficient) and g (weight acceleration).

Specifications are: tracking of the reference of figure 4, no steady state error, time response $\leq 6\text{s}$, rejection of disturbance d and $|\varphi(t)| \leq 0.05\text{rad}$;

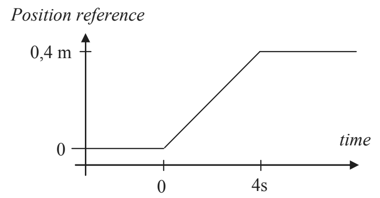

Figure 4: Position reference.

4.2 Three Outputs H∞ Synthesis

To show the versatility of the method, a three measurement controller is designed (synthesis model of figure 5). The filters are defined as:

$$W_1 = \frac{1}{2} \cdot \frac{s+1.7}{s+0.0009}, \; W_2 = 100 \cdot \frac{s+2}{s+2000} \qquad (8)$$
$$W_3 = 0.01, \; W_4 = 2, \; W_5 = 1, \; W_6 = 0.1$$

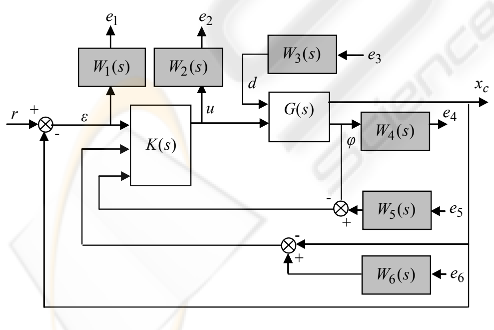

Figure 5: Synthesis model for the “3 output” case.

The solution of the full order synthesis leads to a H∞ norm $\gamma = 1.06$. The full-order controller is of order 6. The Hankel reduction leads to a very large H∞ norm $\gamma = 56.7$ for the order 2 controller. A controller is computed by the PSO algorithm, with the filters of the full order synthesis. Results are given in table 1 for 100 tests. Computation times are 30s (Pentium IV, 2GHz; Matlab 6.5).

Table 1: Optimisation results for the three output case.

Worst	Best	Mean
$\| \|_\infty = 4.53$	$\| \|_\infty = 2.60$	$\| \|_\infty = 3.50$

Figure 6 gives the Bode diagram of the transfers of matrix (1) (full order, Hankel reduction controller, and PSO). Figure 7 represents the corresponding time responses. As can be seen, results of the Hankel reduction controller are quite similar as for the full order controller, except at high frequencies. Figure 8 and 9 give the same results obtained with the mean controller of the PSO method. Note first that the response of $\varphi(t)$ is quite similar as the previous ones and remains therefore satisfying. A slight overshoot is observed on the reference tracking.

However, consider figure 10, where a measurement noise d_m has been added on the cart position. The control input u is represented both for Hankel reduction and PSO controllers. As can be seen from figure 6, Hankel reduction leads to a modification of the closed loop transfers for high frequencies. As a result, high gains for high frequencies lead to an amplification of measurement noises and thus to chattering control inputs. On the contrary, the reduced order synthesis leads to closed loop systems with smaller H∞ norm. The system is more robust against measurement disturbances.

5 CONCLUSIONS

In this paper, a metaheuristic method based on Particle Swarm Optimization has been presented. PSO is a stochastic optimization method which does not require any particular structure for costs and constraints. As a result, the method can be used to optimize many kinds of criterions and solve non convex, non linear or non analytic problems. In this paper, the method is used to solve a well known problem of modern Automatic Control, namely the reduced order H∞ synthesis. The problem is known to be a non convex problem, for which the traditional approach is an a posteriori reduction of the full order synthesis. Results, computed for a pendulum in the cart have shown the viability of the approach. Computed controllers lead to a slight decrease of nominal performances but to a more robust controller with an important decrease of the closed loop H_∞ norm.

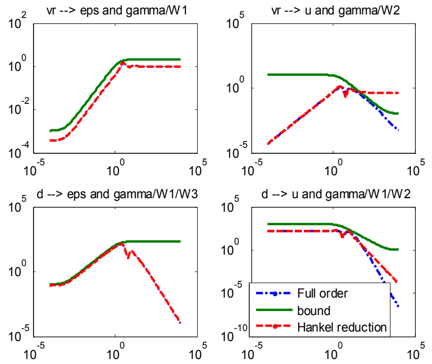

Figure 6: Bode transfer of full order and Hankel reduction.

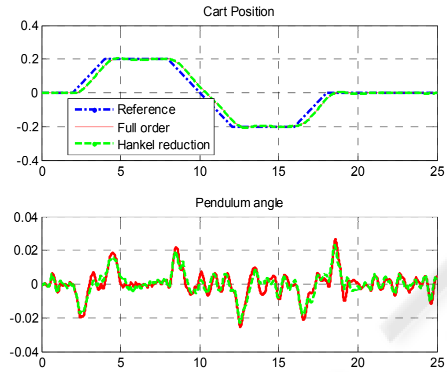

Figure 7: Time response - full order and Hankel reduction.

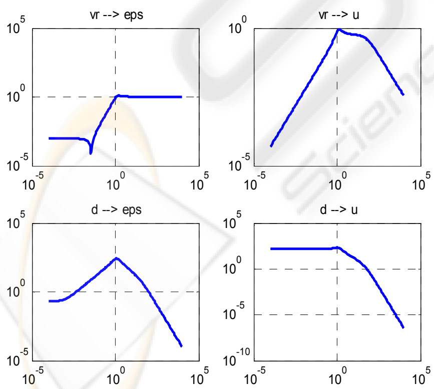

Figure 8: Bode transfer for PSO controller.

Figure 9: Time response for PSO controller.

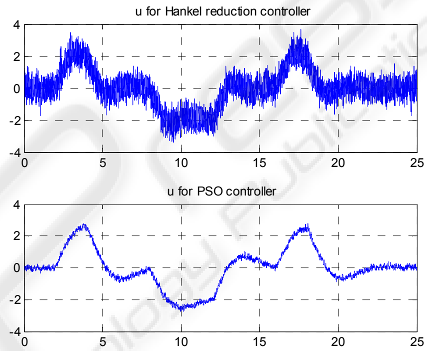

Figure 10: Control input for Hankel reduction and PSO controllers.

REFERENCES

Eberhart, R. C, Kennedy, J., 1995. A new optimizer using particle swarm theory. In: *Proceedings of the Sixth International Symposium on Micromachine and Human Science*, Nagoya, Japan. pp. 39-43.

Eberhart, R. C., Shi, Y., 2000. Comparing inertia weigths and constriction factors in particle swarm optimization. Proceedings of the IEEE Congress on Evolutionary Computation (CEC 2000), San Diego, CA, USA, p 84-88.

El Ghaoui, L., Oustry, F., AitRami, M., 1997. A cone complementary linearization algorithm for static output feedback and related problems. In: *IEEE Transactions on Automatic Control*, Vol. 42(8), pp. 1171-1176.

Gahinet, P., Apkarian P., 1994. A linear matrix inequality approach to H_∞ control. In *International Journal of Robust and Nonlinear Control*, Vol. 4, pp. 421-448.

Glover, K., Doyle, J.C., 1988. State-state formulae for all stabilizing controllers that satisfy an H_∞-norm bound and relations to risk sensitivity. In *Systems and Control Letters*, Vol.11, pp. 167-172.

Kennedy, J. and M. Clerc, 2006. Standard PSO. http://www.particleswarm.info/Standard_PSO_2006.c