DETECTION AND CONTROL OF NON-LINEAR BEHAVIOR BY SLIDING MODES CONTROL IN A 3 D.O.F. ROBOT

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Abstract: Results from simulations of a Planar Robot Model, when it is placed in the same plane of the action of the gravity force are reviewed in this paper. The model includes several parameters usually discarded in current models, such as Driving, and Non-linear Friction, for an industrial-type robotic manipulator and its actuators. When we develop more exact representations of the dynamics of a manipulator and their actuators, chaotic behavior is detected for certain parameter values of the robotic manipulator. This chaotic behavior – without external inputs – was exactly controlled by Sliding Modes.

1 INTRODUCTION

A variable structure system (VSS) is a system whose structure is intentionally changed to achieve the desired performance. This intentional structural change is typically accomplished through discontinuous control action in accordance with a presigned algorithm and switching hyperplanes (Zohdy, M., Fadali, M.S. and Liu, J., 1992).

For nonlinear dynamical systems with uncertainties and disturbance, the sliding modes control (SMC) is a method which has some advantages. The SMC was proposed and elaborated initially in the fifties in the former Soviet Union (Park, D. and Choi, S., 1999). This control method, which can be obtained by an appropriate discontinuous law, is the principal operation mode in the variable structure control system (VSCS). This method is known for its robustness to disturbance and parameters variations (Bartoszewicz, A., 1995).

The sliding mode controllers have excellent characteristic in the sliding movement of the state on the sliding surface. During this sliding movement, the system has invariants properties, producing a robust movement regarding the unknown parameters of the system and the external interferences. The design of the VSS based sliding mode controllers can be broken down into two major phases: the first one is the determination of a stable manifold, called the sliding surface, and the second phase is to design a switching control law according to the sliding surface to satisfy the attraction manifold. When the sliding mode occurs, the system state will remain on it forever and the system behaves as an equivalent system with desired dynamics which is governed by the sliding surface equation; at the same time the system has good characteristics such as fast response, good robustness and disturbance rejection, etc. (Xu, J., Lee, T.H., Wang, M. and Yu, X., 1996).

Efficient control of industrial robots is an important issue to success of industrial automation in these years (Lu, X. and Spurgeon, S., 1999). Along with the development of robot manipulator control theory and its applications, there has been increasing demand for more efficient control schemes to achieve satisfactory results (Chen, C. and Xu, R., 1999).

The paper is organized as follows: in section 2, a friction model is given. In section 3, manipulator dynamic equations are presented. In section 4, the state-space model for a 3 link planar robot and its actuators, is developed. In section 5, the robot controller is developed. Section 6 presents some simulation results. Finally, in section 7, the conclusions are discussed.

2 FRICTION MODEL

Models representing friction effects have been widely studied in concerned literature (Canudas, C., Aström, K. and Braun, K., 1987), (Kircanski, N. and

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Goldenberg, A., 1997), (Urrea, C., 1999). In this paper, we use a model that includes both effects, whose friction curve is discontinuous and non-symmetrical.

$$F(\dot{\theta}) = (\alpha_0 + \alpha_1 \cdot e^{(\dot{\theta}/v_0)^2}) \cdot sgn(\dot{\theta}) + \alpha_2 \cdot \dot{\theta} \qquad (1)$$

where: $F(\dot{\theta})$: is the friction torque [N · m]; $\dot{\theta}$: angular velocity [rad/s]; $\alpha_0 + \alpha_1$: static friction [N·m]; v_0: Stribeck velocity [rad/s]; α_2: viscous friction [N·m·s/rad].

3 MANIPULATOR DYNAMIC EQUATIONS

The dynamic equation of a robotic manipulator in the joint space can be written as follows (Leahy, M., Valavanis, K. and Saridis, G., 1989); (Mahla, I., Urrea, C., 1999):

$$D(\theta) \cdot \ddot{\theta} + B(\theta, \dot{\theta}) = \tau \qquad (2)$$

where:

$$B(\theta, \dot{\theta}) = C(\theta, \dot{\theta}) \cdot \dot{\theta} + F(\dot{\theta}) + G(\theta) \qquad (3)$$

in which θ: joint angle vector, $\theta \in R^n$; $D(\theta)$: inertial matrix, $D(\theta) \in R^{nxn}$; $C(\theta, \dot{\theta})$: Coriolis and centrifugal torque matrix, $C(\theta, \dot{\theta}) \in R^{nxn}$; $G(\theta)$: gravity torque vector, $G(\theta) \in R^n$; τ: joint torque vector, $\tau \in R^n$; n: degrees of freedom.

4 STATE-SPACE MODELS

4.1 Manipulator

If in an industrial-type robotic manipulator, the following state variables are chosen:

$$x_1 = \theta_1;\ x_2 = \dot{\theta}_1;\ x_3 = \theta_2;\ x_4 = \dot{\theta}_2;\ x_5 = \theta_3;\ x_6 = \dot{\theta}_3 \qquad (4)$$

then:

$$\dot{x}_2 = \ddot{\theta}_1;\ \dot{x}_4 = \ddot{\theta}_2;\ \dot{x}_6 = \ddot{\theta}_3 \qquad (5)$$

If some of the elements in the inertial matrix $D(\theta(t))$ are defined as constants, there it is obtained:

$$k_1 = m_1 \cdot l_{c1} + (m_2 + m_3) \cdot l_1 \qquad (6)$$

$$k_2 = m_2 \cdot l_{c2} + m_3 \cdot l_2 \qquad (7)$$

$$k_3 = m_3 \cdot l_3 \qquad (8)$$

where m_1: mass of the first link [kg]; m_2: mass of the second link [kg]; m_3: mass of the third link [kg]; l_1: length of the first link [m]; l_2: length of the second link [m]; l_3: length of the third link [m]; l_{c1}: distance between the gravity centre of the first link and its driving axis [m]; l_{c2}: distance between the gravity centre of the second link and its driving axis [m]. Replacing eq. 1 and eqs. 4 to 8 into eqs. 2 and 3 (n = 3), we have:

$$\tau_{L1}^* = (I_1 + I_2 + I_3 + m_1 \cdot l_{c1}^2 + m_2 \cdot (l_1^2 + l_{c2}^2) + m_3 \cdot (l_1^2 + l_2^2 + l_{c3}^2) + 2 \cdot k_2 \cdot l_1 \cdot cos\, x_3 + 2 \cdot k_3 \cdot (l_1 \cdot cos(x_3 + x_5) + l_2 \cdot cos(x_5))) \cdot \dot{x}_2 / N_1 + (I_2 + I_3 + m_2 \cdot l_{c2}^2 + m_3 \cdot (l_2^2 + l_{c3}^2) + k_2 \cdot l_1 \cdot cos(x_3) + k_3 \cdot (l_1 \cdot cos(x_3 + x_5) + 2 \cdot l_2 \cdot cos(x_5))) \cdot \dot{x}_4 / N_1 + (I_3 + m_3 \cdot l_{c3}^2 + k_3 \cdot (l_1 \cdot cos(x_3 + x_5) + l_2 \cdot cos(x_5))) \cdot \dot{x}_6 / N_1 + (k_2 \cdot (-l_1 \cdot sin(x_3) \cdot (2 \cdot x_2 \cdot x_4 + x_4^2)) + k_3 \cdot (-l_1 \cdot sin(x_3 + x_5) \cdot (2 \cdot x_2 \cdot (x_4 + x_6) + (x_4 + x_6)^2) - l_1 \cdot sin(x_5) \cdot (2 \cdot (x_2 + x_4) \cdot x_6 + x_6^2))) / N_1 + g \cdot (k_1 \cdot cos(x_1) + k_2 \cdot cos(x_1 + x_3) + k_3 \cdot cos(x_1 + x_3 + x_5)) / N_1 + F_1(x_2) / N_1 \qquad (9)$$

$$\tau_{L2}^* = (I_2 + I_3 + m_2 \cdot l_{c2}^2 + m_3 \cdot (l_2^2 + l_{c3}^2) + k_2 \cdot (l_1 \cdot cos(x_3) + k_3 \cdot (l_1 \cdot cos(x_3 + x_5) + 2 \cdot l_2 \cdot cos(x_5))) \cdot \dot{x}_2 / N_2 + (I_2 + I_3 + m_2 \cdot l_{c2}^2 + m_3 \cdot (l_2^2 + l_{c3}^2) + k_3 \cdot 2 \cdot l_2 \cdot cos(x_5)) \cdot \dot{x}_4 / N_2 + (I_3 + m_3 \cdot l_{c3}^2 + k_3 \cdot l_2 \cdot cos(x_5)) \cdot \dot{x}_6 / N_2 + (k_2 \cdot (l_1 \cdot sin(x_3 + x_5) \cdot x_2^2) + k_3 \cdot (l_1 \cdot sin(x_3 + x_5) \cdot x_2^2 - l_2 \cdot sin(x_6) \cdot (2 \cdot (x_2 + x_4) \cdot x_6 + x_6^2))) / N_2 + g \cdot (k_2 \cdot cos(x_1 + x_3) + k_3 \cdot cos(x_1 + x_3 + x_5)) / N_2 + F_2(x_4) / N_2 \qquad (10)$$

$$\tau_{L3}^* = (I_3 + m_3 \cdot l_{c3}^2 + k_3 \cdot (l_1 \cdot cos(x_3 + x_5) + l_2 \cdot cos(x_5))) \cdot \dot{x}_2 / N_3 + (I_3 + m_3 \cdot l_{c3}^2 + k_3 \cdot l_2 \cdot cos(x_5)) \cdot \dot{x}_4 / N_3 + (I_3 + k_3^2) \cdot \dot{x}_6 / N_3 + k_3 \cdot (l_1 \cdot sin(x_3 + x_5) \cdot x_2^2 + l_2 \cdot sin(x_5) \cdot (x_2 + x_4)^2) / N_3 + g \cdot k_3 \cdot cos(x_1 + x_3 + x_5) / N_3 + F_3(x_6) / N_3 \qquad (11)$$

where τ_{L1}^*: torque applied in the first link, referred to the first motor axis [N·m]; τ_{L2}^*: torque applied in the second link, referred to the second motor axis [N·m]; τ_{L3}^*: torque applied in the third link, referred to the third motor axis [N·m]; N_1: reduction factor of the first gear train; N_2: reduction factor of the second gear train; N_3: reduction factor of the third gear train; I_1: moment of inertia of the first link [Kg·m]; I_2: moment of inertia of the second link [Kg·m]; I_3: moment of inertia of the third link [Kg·m]; F_1: is the friction torque in the first link axis, [N·m]; F_2: is the friction torque in the second link axis, [N·m]; F_3: is the friction torque in the third link axis, [N·m]; g: is the gravity force [N·m].

Defining the following functions:

$$f_1^* = (I_1 + I_2 + I_3 + m_1 \cdot l_{c1}^2 + m_2 \cdot (l_1^2 + l_2^2) + m_3 \cdot (l_1^2 + l_2^2 + l_{c3}^2) + 2 \cdot k_2 \cdot l_1 \cdot \cos x_3 + 2 \cdot k_3 \cdot (l_1 \cdot \cos(x_3 + x_5) + l_2 \cdot \cos(x_5)))/N_1 \tag{12}$$

$$f_2^* = (I_2 + I_3 + m_2 \cdot l_{c2}^2 + m_3 \cdot (l_2^2 + l_{c3}^2) + k_2 \cdot l_1 \cdot \cos(x_3) + k_3 \cdot (l_1 \cdot \cos(x_3 + x_5) + 2 \cdot l_2 \cdot \cos(x_5)))/N_1 \tag{13}$$

$$f_3^* = (I_3 + m_3 \cdot l_{c3}^2 + k_3 \cdot (l_1 \cdot \cos(x_3 + x_5) + l_2 \cdot \cos(x_5)))/N_1 \tag{14}$$

$$f_4^* = (k_2 \cdot (l_1 \cdot \sin(x_3) \cdot (2 \cdot x_2 \cdot x_4 + x_4^2)) + k_3 \cdot (l_1 \cdot \sin(x_3 + x_5) \cdot (2 \cdot x_2 \cdot (x_4 + x_6) + (x_4 + x_6)^2) + l_1 \cdot \sin(x_5) \cdot (2 \cdot (x_2 + x_4) \cdot x_6 + x_6^2)))/N_1 - g \cdot (k_1 \cdot \cos(x_1) + k_2 \cdot \cos(x_1 + x_3) + k_3 \cdot \cos(x_1 + x_3 + x_5))/N_1 - F_1(x_2)/N_1 \tag{15}$$

$$f_5^* = (I_2 + I_3 + m_2 \cdot l_{c2}^2 + m_3 \cdot (l_2^2 + l_{c3}^2) + k_2 \cdot (l_1 \cdot \cos(x_3) + k_3 \cdot (l_1 \cdot \cos(x_3 + x_5) + 2 \cdot l_2 \cdot \cos(x_5)))/N_2 \tag{16}$$

$$f_6^* = (I_2 + I_3 + m_2 \cdot l_{c2}^2 + m_3 \cdot (l_2^2 + l_{c3}^2) + k_3 \cdot 2 \cdot l_2 \cdot \cos(x_5))/N_2 \tag{17}$$

$$f_7^* = (I_3 + m_3 \cdot l_{c3}^2 + k_3 \cdot l_2 \cdot \cos(x_5))/N_2 \tag{18}$$

$$f_8^* = -(k_2 \cdot (l_1 \cdot \sin(x_3 + x_5) \cdot x_2^2) + k_3 \cdot (l_1 \cdot \sin(x_3 + x_5) \cdot x_2^2) - l_2 \cdot \sin(x_6) \cdot (2 \cdot (x_2 + x_4) \cdot x_6 + x_6^2)))/N_2 - g \cdot (k_2 \cdot \cos(x_1 + x_3) + k_3 \cdot \cos(x_1 + x_3 + x_5)/N_2 - F_2(x_4)/N_2 \tag{19}$$

$$f_9^* = (I_3 + m_3 \cdot l_{c3}^2 + k_3 \cdot (l_1 \cdot \cos(x_3 + x_5) + l_2 \cdot \cos(x_5)))/N_3 \tag{20}$$

$$f_{10}^* = (I_3 + m_3 \cdot l_{c3}^2 + k_3 \cdot l_2 \cdot \cos(x_5))/N_3 \tag{21}$$

$$f_{11}^* = (I_3 + k_3^2)/N_3 \tag{22}$$

$$f_{12}^* = -k_3 \cdot (l_1 \cdot \sin(x_3 + x_5) \cdot x_2^2 + l_2 \cdot \sin(x_5) \cdot (x_2 + x_4)^2)/N_3 - g \cdot k_3 \cdot \cos(x_1 + x_3 + x_5)/N_3 - F_3(x_6)/N_3 \tag{23}$$

then:

$$\tau_{L1}^* + f_4^* = f_1^* \cdot \dot{x}_2 + f_2^* \cdot \dot{x}_4 + f_3^* \cdot \dot{x}_6 \tag{24}$$

$$\tau_{L2}^* + f_8^* = f_5^* \cdot \dot{x}_2 + f_6^* \cdot \dot{x}_4 + f_7^* \cdot \dot{x}_6 \tag{25}$$

$$\tau_{L3}^* + f_{12}^* = f_9^* \cdot \dot{x}_2 + f_{10}^* \cdot \dot{x}_4 + f_{11}^* \cdot \dot{x}_6 \tag{26}$$

Defining:

$$f_{13}^* = 1/[f_3^* \cdot f_5^* \cdot f_{10}^* - f_3^* \cdot f_9^* \cdot f_6^* - f_2^* \cdot f_5^* \cdot f_{11}^* - f_2^* \cdot f_9^* \cdot f_7^* \cdot f_6^* \cdot f_{11}^* - f_1^* \cdot f_{10}^* \cdot f_7^*] \tag{27}$$

$$f_{14}^* = f_{13}^* \cdot [f_6^* \cdot f_{11}^* - f_{10}^* \cdot f_7^*] \tag{28}$$

$$f_{15}^* = f_{13}^* \cdot [f_9^* \cdot f_7^* - f_5^* \cdot f_{11}^*] \tag{29}$$

$$f_{16}^* = f_{13}^* \cdot [f_5^* \cdot f_{10}^* - f_9^* \cdot f_6^*] \tag{30}$$

$$f_{17}^* = f_{13}^* \cdot [f_3^* \cdot f_{10}^* - f_2^* \cdot f_{11}^*] \tag{31}$$

$$f_{18}^* = f_{13}^* \cdot [f_1^* \cdot f_{11}^* - f_3^* \cdot f_9^*] \tag{32}$$

$$f_{19}^* = f_{13}^* \cdot [f_2^* \cdot f_9^* - f_1^* \cdot f_{10}^*] \tag{33}$$

$$f_{20}^* = f_{13}^* \cdot [f_2^* \cdot f_7^* - f_3^* \cdot f_6^*] \tag{34}$$

$$f_{21}^* = f_{13}^* \cdot [f_3^* \cdot f_5^* - f_1^* \cdot f_7^*] \tag{35}$$

$$f_{22}^* = f_{13}^* \cdot [f_1^* \cdot f_6^* - f_2^* \cdot f_5^*] \tag{36}$$

Redefining functions,

$$f_{23}^* = f_{14}^* \cdot f_4^* \tag{37}$$

$$f_{24}^* = f_{15}^* \cdot f_8^* \tag{38}$$

$$f_{25}^* = f_{16}^* \cdot f_{12}^* \tag{39}$$

$$f_{26}^* = f_{17}^* \cdot f_4^* \tag{40}$$

$$f_{27}^* = f_{18}^* \cdot f_8^* \tag{41}$$

$$f_{28}^* = f_{19}^* \cdot f_{12}^* \tag{42}$$

$$f_{29}^* = f_{20}^* \cdot f_4^* \tag{43}$$

$$f_{30}^* = f_{21}^* \cdot f_8^* \tag{44}$$

$$f_{31}^* = f_{22}^* \cdot f_{12}^* \tag{45}$$

The state equation model for the three-link planar RRR arm can be written as:

$$\dot{x}_1 = x_2 \tag{46}$$

$$\dot{x}_2 = f_{14}^* \cdot \tau_{L1}^* + f_{23}^* + f_{15}^* \cdot \tau_{L2}^* + f_{24}^* + f_{16}^* \cdot \tau_{L3}^* + f_{25}^* \tag{47}$$

$$\dot{x}_3 = x_4 \tag{48}$$

$$\dot{x}_4 = f_{17}^* \cdot \tau_{L1}^* + f_{26}^* + f_{18}^* \cdot \tau_{L2}^* + f_{27}^* + f_{19}^* \cdot \tau_{L3}^* + f_{28}^* \tag{49}$$

$$\dot{x}_5 = x_6 \tag{50}$$

$$\dot{x}_6 = f_{20}^* \cdot \tau_{L1}^* + f_{29}^* + f_{21}^* \cdot \tau_{L2}^* + f_{30}^* + f_{22}^* \cdot \tau_{L3}^* + f_{31}^* \tag{51}$$

4.2 Actuators

By employing state equations models for three DC motors, and from (Craig, J., 1996), we have equations (52) to (54):

$$\dot{x}_7 = [k_{a1} \cdot v_{a1}(t) - r_{a1} \cdot x_7 - k_{a1} \cdot k_{b1} \cdot x_2 \cdot N_1]/L_{a1} \tag{52}$$

$$\dot{x}_8 = [k_{a2} \cdot v_{a2}(t) - r_{a2} \cdot x_8 - k_{a2} \cdot k_{b2} \cdot x_4 \cdot N_2]/L_{a2} \tag{53}$$

$$\dot{x}_9 = [k_{a3} \cdot v_{a3}(t) - r_{a3} \cdot x_9 - k_{a3} \cdot k_{b3} \cdot x_6 \cdot N_3]/L_{a3} \tag{54}$$

where k_{aj} : proportional j-motor-torque constant [N · m / A]; v_{aj} : armature voltage [V]; r_{aj} : j-motor armature resistance [Ω]; $x_{j+6} = \tau_j$: torque generated by the j-motor axis [N · m]; k_{bj} : j-motor proportionality constant [V · rad / s]; L_{aj} : j-motor armature inductance [H]; with $j = 1, 2, 3$.

4.3 State Equations Model

The torque generated in the *j*-motor axis is equal to the sum of the *j*-motor and its load, *i.e.*:

$$\tau_j(t) = J_{mj}(t)\cdot\ddot{\theta}_{mj} + \tau_{Lj}^{*}(t) + T_{fmj}(\ddot{\theta}_{mj}) \quad (55)$$

with J_{mj} : *j*-motor inertia moment reflected to *j*-motor axis $[N\cdot m\cdot s^2/rad]$; $\ddot{\theta}_{mj}$: *j*-motor angular acceleration referred to *j*-motor axis $[rad/s^2]$; T_{fmj} : friction torque generated in the *j*-motor axis referred to *j*-motor axis [N·m]; $\dot{\theta}_{mj}$: *j*-motor angular velocity referred to *j*-motor axis [rad/s], with j = 1, 2, 3. From eq. 55:

$$\tau_{L1}^{*} = x_7 - J_{m1}\cdot\dot{x}_2\cdot N_1 - T_{fm1}(x_2\cdot N_1) \quad (56)$$

$$\tau_{L2}^{*} = x_8 - J_{m2}\cdot\dot{x}_4\cdot N_2 - T_{fm2}(x_4\cdot N_2) \quad (57)$$

$$\tau_{L3}^{*} = x_9 - J_{m3}\cdot\dot{x}_6\cdot N_3 - T_{fm3}(x_6\cdot N_3) \quad (58)$$

From eq. 52 to 54, and replacing eq. 56 to 58 in eqs. 46 to 51, the following state equation are obtained:

$$\dot{x}_1 = x_2 \quad (59)$$

$$\dot{x}_2 = f_{14}^{*}\cdot(x_7 - J_{m1}\cdot\dot{x}_2\cdot N_1 - T_{fm1}(x_2\cdot N_1)) + f_{23}^{*} + f_{15}^{*}\cdot(x_8 - J_{m2}\cdot\dot{x}_4\cdot N_2 - T_{fm2}(x_4\cdot N_2)) + f_{24}^{*} + f_{16}^{*}\cdot(x_9 - J_{m3}\cdot\dot{x}_6\cdot N_3 - T_{fm3}(x_6\cdot N_3)) + f_{25}^{*} \quad (60)$$

$$\dot{x}_3 = x_4 \quad (61)$$

$$\dot{x}_4 = f_{17}^{*}\cdot(x_7 - J_{m1}\cdot\dot{x}_2\cdot N_1 - T_{fm1}(x_2\cdot N_1)) + f_{26}^{*} + f_{18}^{*}\cdot(x_8 - J_{m2}\cdot\dot{x}_4\cdot N_2 - T_{fm2}(x_4\cdot N_2)) + f_{27}^{*} + f_{19}^{*}\cdot(x_9 - J_{m3}\cdot\dot{x}_6\cdot N_3 - T_{fm3}(x_6\cdot N_3)) + f_{28}^{*} \quad (62)$$

$$\dot{x}_5 = x_6 \quad (63)$$

$$\dot{x}_6 = f_{20}^{*}\cdot(x_7 - J_{m1}\cdot\dot{x}_2\cdot N_1 - T_{fm1}(x_2\cdot N_1)) + f_{29}^{*} + f_{21}^{*}\cdot(x_8 - J_{m2}\cdot\dot{x}_4\cdot N_2 - T_{fm2}(x_4\cdot N_2)) + f_{30}^{*} + f_{22}^{*}\cdot(x_9 - J_{m3}\cdot\dot{x}_6\cdot N_3 - T_{fm3}(x_6\cdot N_3)) + f_{31}^{*} \quad (64)$$

$$\dot{x}_7 = [k_{a1}\cdot v_{a1}(t) - r_{a1}\cdot x_7 - k_{a1}\cdot k_{b1}(t)\cdot x_2\cdot N_1]/L_{a1} \quad (65)$$

$$\dot{x}_8 = [k_{a2}\cdot v_{a2}(t) - r_{a2}\cdot x_8 - k_{a2}\cdot k_{b2}(t)\cdot x_4\cdot N_2]/L_{a2} \quad (66)$$

$$\dot{x}_9 = [k_{a3}\cdot v_{a3}(t) - r_{a3}\cdot x_9 - k_{a3}\cdot k_{b3}(t)\cdot x_6\cdot N_3]/L_{a3} \quad (67)$$

5 CONTROLLER MODEL

The controller is modelled as:

$$V_{a1}(t) = -k_1\cdot sgn(s_1) \quad (68)$$

$$V_{a2}(t) = -k_2\cdot sgn(s_2) \quad (69)$$

$$V_{a3}(t) = -k_3\cdot sgn(s_3) \quad (70)$$

in which:

$$s_1 = w_1\cdot(x_1 - x_{1d}) + \dot{x}_1 \quad (71)$$

$$s_2 = w_2\cdot(x_2 - x_{3d}) + \dot{x}_2 \quad (72)$$

$$s_3 = w_3\cdot(x_3 - x_{3d}) + \dot{x}_3 \quad (73)$$

where k_j : *j*-discontinuity gain [V]; s_j : *j*-sliding surface [rad/s] ; w_j : *j*-position gain [1/s]; with j = 1, 2, 3.

6 SIMULATIONS RESULTS

In these simulations the specified changes in revolution joint angles are sinusoidal signals:

Table 1: References trajectories.

Joint	Amplitude [rad]	Frequency [rad/s]
x_1	$\pi/3$	$3\pi/5$
x_3	$\pi/4$	π
x_5	1	0

The given initial conditions were $[x_1(0)\ x_2(0)\ x_3(0)\ x_4(0)\ x_5(0)\ x_6(0)\ x_7(0)\ x_8(0)\ x_9(0)]^T = [0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0]^T$; the required torque to be delivered by the actuators was determined.

When usually neglected nonlinearities are considered, for certain system parameters (see appendix), chaotic behavior was detected in the end-of-arm, just as it is presented in the figures 1 and 2.

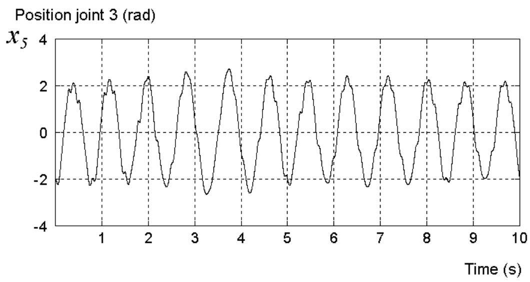

Figure 1: Last link position.

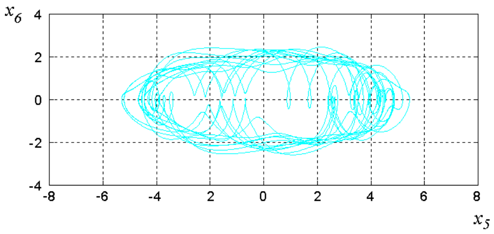

Figure 2: Phase plane (x_5 , $\dot{x}_5$).

For every robot joint, a prescribed path is considered. In $t = 1$ [s] step response for the end-of-arm is imposed; for the first and second link, in $t = 0$ [s] sinusoidal signals are imposed (see figure 3).

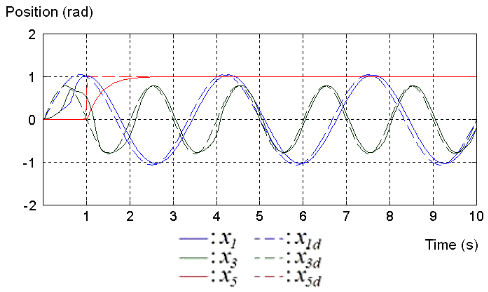

Figure 3: Angular position of the links.

From figure 3, it is possible to appreciate that the chaotic behavior was controlled and the desired paths were tracked.

7 CONCLUSIONS

In this article, models are developed for the actuator and manipulator that address some of the nonlinearities usually neglected in current models.

The manipulator is placed in the same plane of the action of the gravity force and effects such as viscous, static and Coulomb friction in DC motors; viscous, static and Coulomb friction in manipulator joints; actuators and gear trains, are considered in this dynamic model.

The controller design has allowed controlling the detected chaotic behavior.

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APPENDIX

Simulation Parameters. The following parameter values were taken from (Hu, J. and Dawson, D., 1996), (Van Willigenburg, L. and Loop, R., 1991), (Vukobratovic´, M., 1997).

Motors and their Reduction Gears

Motor M1		Motor M2		Motor M3		
L_{a1}	0.0048	L_{a2}	0.0048	L_{a2}	0.0048	[H]
R_{a1}	1.6	R_{a2}	1.6	R_{a2}	1.6	[Ω]
K_{a1}	0.35	K_{a2}	0.35	K_{a2}	0.35	[N · m / A]
K_{b1}	0.04	K_{b2}	0.04	K_{b2}	0.04	Volts·s / rad]
α_0	260	α_0	260	α_0	260	[N · m]
α_1	1.64	α_1	1.64	α_1	1.64	[N · m]
α_2	0.018	α_2	0.018	α_2	0.018	[N· m· s / rad]
v_0	0.01	v_0	0.01	v_0	0.01	[rad / s]
N_1	62.55	N_2	62.55	N_3	62.55	

Manipulator

Link 1		Link 2		Link 3			
m_1	9.86	m_2	6.38	m_3	3.21	3*	[Kg]
l_1	0.45	l_2	0.5	l_3	0.3		[m]
l_{c1}	0.3	l_{c2}	0.3243	l_{c3}	0.2	0.25*	[m]
I_1	1.1835	I_2	0.1371	I_3	0.0268		[Kg · m]
α_0	100	α_0	100	α_0	100		[N ·m]
α_1	1.01	α_1	1.01	α_1	1.01		[N ·m]
α_2	0.018	α_2	0.018	α_2	0.018		[N·m·s /rad]
v_0	0.01	v_0	0.01	v_0	0.01		[N·m·s /rad]

The manipulator parameter values that generated chaotic behavior were denoted with *. This chaotic behavior was eliminated by the following parameter values that we have proposed for the controllers.

Controllers

1. First Actuator (M1) Controller

k_1	260	[Volts]
w_1	10	[1 / s]

2. Second Actuator (M2) Controller

k_2	100	[Volts]
w_2	20	[1 / s]

3. Third Actuator (M3) Controller

k_3	200	[Volts]
w_3	3	[1 / s]