DCT DOMAIN VIDEO WATERMARKING
Attack Estimation and Capacity Evaluation

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Abstract: The first difficulty when trying to evaluate with accuracy the video watermarking capacity is the lack of a reliable statistical model for the malicious attacks. The present paper brings into evidence that the attack effects in the DCT domain are stationary and computes the corresponding *pdf*s. In this respect, an in-depth statistical approach is deployed by combining Gaussian mixture estimation with the probability confidence limits. Further on, these *pdf*s are involved in capacity computation. The experimental results are obtained on a corpus of 10 video sequences (about 25 minutes each), with heterogeneous content.

1 INTRODUCTION

For property right identification purposes, the watermarking techniques insert a *mark* into some original media (*e.g.* a video). If the mark insertion does not result in visual artefacts, the method features *transparency*. If a pirate cannot eliminate the mark without damaging the marked video, the method features *robustness* (Cox & others, 2002).

In practice, the better the robustness, the worse the transparency. In order to reach a balance between these two constraints, the mark is inserted into some spectral representations of the original data, *e.g.* in the *DCT* (*D*iscrete *C*osine *T*ransform).

A crucial issue is to compute the watermarking capacity, *i.e.* the largest amount of information which can be inserted into a video, for prescribed transparency and robustness. The watermarking capacity is computed as the capacity of the noisy channel modelling the watermarking method, Figure 1. According to this model, the mark is sampled from the information source. The detection is impaired by the noise sources: the original video itself and the attacks. The side information watermarking exploits the fact that the original video is known at the insertion but unknown at the detection. As such a noise source should not decrease the channel capacity (Costa, 1983), the attacks remain the restricting factor and their intimate knowledge would grant accuracy in capacity evaluation. The present paper focuses on some real life attacks and models their effects in the DCT domain. Note that attack modelling is not a trivial task. Actually, any mathematical approach should properly answer at least the following questions:

1. *Does a general statistical model for the considered attack effects, independent with respect to the video sequence, really exist?*
2. *When considering an individual video sequence, does a reliable model exist for any (intra)frame content and any (inter)frame dependency?* Positive answers at these first questions mean a proof of stationarity concerning the attacks.
3. *In case such a model exists, which is its pdf (probability density function)?* Although the Gaussian law is generally considered, previous studies rejected this popular assumption.

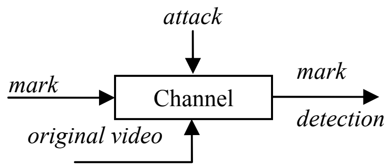

Figure 1: The watermarking model.

The paper has the following structure. After having defined a set of random variables corresponding to the attack effects, Section 2 presents the statistical investigation procedure. Section 3 describes the experimental results. The capacity evaluation is dealt with in Section 4 while Section 5 concludes the paper. The Appendix summarises some theoretical bases.

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2 INVESTIGATION PROCEDURE

2.1 Attack Effect Representation

Be there an L frame colour video. Each frame is represented in the *HSV* space. The following steps are applied to each frame (Mitrea & others, 2006):

- Compute the DCT on the original *V* component.
- Decreasingly sort the coefficients and record the largest R values in a vector n_o; record their corresponding locations in a vector l.
- Apply the DCT to the attacked *V* component and record the coefficients at the *l* locations into the new vector, denoted by n_a.
- Compute the vector: $difference = n_a - n_o$.

A set of *L* vectors of the same type as $difference$ (each of them with *R* components) is thus obtained.

Be *noise* a vector with L components, containing the values corresponding to an arbitrarily chosen rank r in the set of *difference* vectors: $noise = [n_1, n_2, ..., n_L]$. Such a vector is sampled from a random variable modelling the attack effects in the r^{th} rank of the DCT hierarchy. To model the attacks means to obtain the *pdf* for the corresponding random variable (a model for each rank).

2.2 *Pdf* Estimation for Attacks

There are many *pdf* estimation tools based on *iid* (*i*ndependent and *i*dentically *d*istributed) data, but thiey do not apply here. The *noise* vector is computed on successive (dependent) frames and the *a priori* lack of support for attack stationarity can raise suspicions about the data identical distribution. Hence, a general estimation procedure should be considered (Mitrea & others, 2007):

- *Eliminate the data dependency*. Sample the $noise = [n_1, n_2, ..., n_L]$ vector with a D period. Shift the sampling origin and get D *iid* data sets $[x^i{}_1, x^i{}_2, ..., x^i{}_N] = [n_i, n_{D+i}, ..., n_{(N-1)D+i}]$, where $i = 1,2,...,D$ and $N = L / D$.
- *Extract partial information from each iid data set.* Obtain $\hat{p}^i(x)$ ($i = 1,2,...,D$) by Gaussian mixture estimation (Appendix).
- *Extract global information*. Apply the Gaussian mixture estimation to the *noise* vector and obtain $\hat{p}_{av}(x)$ (an *average* model).
- *Define the model*. First, define a similarity measure between two *pdfs*, eq. (1):

$$m(u(x), v(x)) = \frac{\sum_{s=1}^{S}\left(\int_{I_s}(u(x) - v(x))dx\right)^2}{\sum_{s=1}^{S}\left(\int_{I_s} v(x)dx\right)^2}, \quad (1)$$

where I_s, $s = 1,2,...S$ is a subdivision of the $[x_{\min}; x_{\max}]$ interval on which the u and v *pdf*s take non-zero values. Secondly, define the attack model as $\hat{p}(.)$ which is the *iid* estimate closest to the $\hat{p}_{av}(.)$ in the $m(.)$ sense:

$$\hat{p}(x) = \arg\min_i m(\hat{p}_{av}(x), \hat{p}^i(x)). \quad (2)$$

- *Evaluate the model accuracy*. Calculate the average similarity measure between each of the *D pdfs* and the $\hat{p}(.)$ model, eq. (3):

$$Error = \frac{1}{D}\sum_{i=1}^{D} m(\hat{p}^i(x), \hat{p}(x)). \quad (3)$$

If this procedure is successful when applied to an individual video (Section 3.1), then positive answers to the last two questions in the Introduction are obtained. A positive answer to the first question is obtained iff. the same model is obtained for different video sequences (Section 3.2).

3 EXPERIMENTAL RESULTS

The corpus contains 10 video sequences (64 Kbit/s), each of them of $L = 35000$ frames (about 25 minutes each). The content is heterogeneous, combining film, news, and home video excerpts.

The frame size is 192×80 pixels. The *V* component is normalised to the $[0, 1]$ interval. The DCT is individually applied to whole frames, and the largest $R = 360$ coefficients are investigated.

3.1 Model Computation

The model is computed for an arbitrarily chosen video sequence. The following parameters are considered: $D = 250$ frames (*i.e.* 10s); $K = 10$ *pdfs* in the mixture; $N_{iter} = 200$ iterations in the EM algorithm; $S = 20$ evenly distributed intervals.

Table 1 presents the models for three ranks (1, 150, 300) and three attacks (Gaussian filtering, sharpening, and StirMark). In each case, the $\hat{p}(x)$ model is computed according to (2), its the parameters ($P(k)$, μ_k, σ_k) according to (A3) and the corresponding errors to (3). Notice that each and every time, the *Error* values are lower than 0.04.

In order to illustrate the results in Table 1, Figure 2 depicts in continuous line the models for one rank ($r = 300$) and the three attacks. For comparison, Figure 2 also represents (in dashed line) the Gaussian *pdf* with the same mean values and variances as the computed models.

The same results were obtained for each of the 10 video sequences in the corpus and for each investigated attack: the parameters were slightly different but the errors were lower than 0.05.

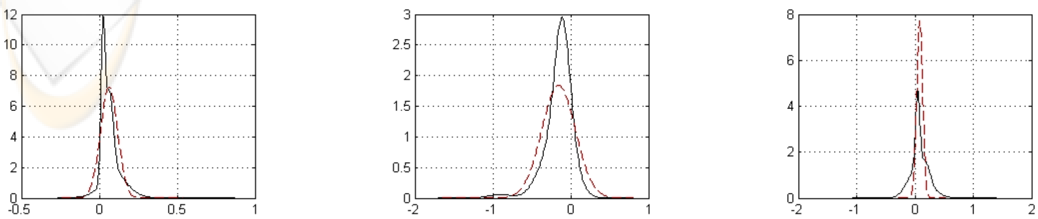
The models, for all 360 ranks and for other attacks (Frequency Model Laplacian Removal, median filtering, small rotations, JPEG compression) can be obtained by contacting the authors.

3.2 Model Validation

Up to now, the experimental results point to the existence of a model for the attack effects on a particular video sequence and estimate this model (*i.e.* elucidates the second & third questions in the Introduction).

Table 1: Statistical model for the watermarking attacks in the DCT hierarchy.

Attack	Rank	Model parameters											Error
Gaussian filtering	r = 1	P(k)	0.076	0.072	0.228	0.095	0.021	0.096	0.071	0.133	0.120	0.083	0.035
		μ(k)	0.274	0.319	0.214	0.305	0.788	0.283	0.330	0.052	0.440	0.374	
		σ(k)	0.110	0.118	0.031	0.116	0.037	0.112	0.108	0.055	0.091	0.112	
	r=150	P(k)	0.019	0.025	0.190	0.213	0.091	0.075	0.271	0.025	0.031	0.056	0.013
		μ(k)	0.224	0.195	0.031	0.009	0.072	0.121	0.063	0.101	0.201	0.131	
		σ(k)	0.130	0.135	0.011	0.021	0.083	0.010	0.010	0.110	0.134	0.046	
	r=300	P(k)	0.080	0.044	0.048	0.146	0.056	0.038	0.041	0.284	0.093	0.164	0.013
		μ(k)	0.131	0.096	0.094	0.066	0.083	0.091	0.090	0.023	0.062	0.055	
		σ(k)	0.053	0.112	0.111	0.025	0.069	0.110	0.110	0.012	0.026	0.029	
Sharpening	r = 1	P(k)	0.319	0.057	0.083	0.066	0.095	0.056	0.101	0.073	0.074	0.071	0.020
		μ(k)	-1.478	-1.185	-2.325	-2.151	-2.341	-0.971	-2.424	-0.536	-2.347	-2.141	
		σ(k)	0.284	0.789	0.557	0.604	0.551	0.747	0.441	0.648	0.549	0.606	
	r=150	P(k)	0.102	0.029	0.091	0.033	0.171	0.033	0.140	0.081	0.155	0.159	0.021
		μ(k)	-0.324	0.213	-0.124	-1.104	-0.080	-0.716	-0.063	-0.279	-0.078	-0.035	
		σ(k)	0.137	0.028	0.102	0.316	0.103	0.168	0.103	0.150	0.104	0.098	
	r=300	P(k)	0.054	0.151	0.283	0.105	0.078	0.067	0.073	0.098	0.023	0.063	0.021
		μ(k)	-0.211	-0.105	-0.106	-0.114	-0.369	-0.030	-0.115	-0.120	-0.905	-0.203	
		σ(k)	0.180	0.096	0.095	0.156	0.133	0.133	0.156	0.158	0.155	0.178	
StirMark	r = 1	P(k)	0.070	0.087	0.105	0.116	0.097	0.100	0.059	0.131	0.069	0.161	0.037
		μ(k)	-0.480	0.075	-0.354	-0.289	-0.115	-0.389	-0.401	-0.348	-0.619	-0.150	
		σ(k)	0.368	0.397	0.271	0.263	0.436	0.270	0.404	0.269	0.357	0.215	
	r=150	P(k)	0.235	0.083	0.043	0.096	0.070	0.149	0.087	0.048	0.075	0.109	0.019
		μ(k)	0.036	-0.084	0.206	0.103	0.422	0.006	0.017	-0.096	0.092	0.124	
		σ(k)	0.084	0.204	0.027	0.063	0.073	0.100	0.136	0.203	0.155	0.146	
	r=300	P(k)	0.085	0.075	0.046	0.078	0.137	0.036	0.010	0.046	0.300	0.093	0.018
		μ(k)	0.004	0.191	0.197	-0.003	0.192	0.165	-0.005	0.212	0.047	0.015	
		σ(k)	0.144	0.193	0.192	0.141	0.074	0.195	0.140	0.189	0.036	0.148	



Gaussian filtering Sharpening StirMark

Figure 2: The attack models (continuous line) and the corresponding Gaussian laws (dashed line) for the rank r=300.

The model independence w.r.t. the video sequence and the estimation procedure is now to be investigated.

First, it should be précised whether the model computed on a particular video sequence can be representative for the whole corpus or not. In this respect, the investigation algorithm is resumed on the rest of 9 video sequences from the corpus and the corresponding models are computed. The errors between the reference model and these new models are evaluated according to three criteria: the similarity measure in eq. (1), the Kullback-Leibler divergence and the Hellinger distance (Appendix). For each criterion, and for three ranks, the minimal, maximal, and average errors are reported in Table 2. The numerical values obtained for the distance in eq. (1) ascertain a quite good accuracy (and generality) for the model provided in Table 1: the average errors are acceptably low, with one exception, namely the Gaussian filtering. The Kullback-Leibler divergence and the Hellinger distance lead to acceptably small values for all the attacks. In order to compute these three measures, the following instantiations were made in eqs. (1), (A4) and (A5): $v(x)$ is the reference *pdf* while $u(x)$ is, successively, each of the other 9 individual models computed on the corpus. The interval I is $I = [\mu_{mix} - 3 * \sigma_{mix}, \mu_{mix} + 3 * \sigma_{mix}]$, where μ_{mix} and σ_{mix} are the mixture mean and variance.

Secondly, the concordance between the maximum likelihood estimation (which is the basis for the EM Gaussian mixture algorithm) and the popular confidence limit estimation is checked. Note that the EM Gaussian mixture estimation results in a continuous *pdf* while the confidence limit estimation provides values for the probability that a random variable takes values in a given interval, but not the *pdf* itself. Consequently, the interval where the Gaussian mixture *model* takes non-zero values is evenly divided into 10 sub-intervals. On the one hand, confidence limits for the probability that the noise effects would take values in these sub-intervals are derived. On the other hand, the integral of the Gaussian mixture model on the same sub-intervals are computed. The experiments bring into evidence that each and every time (*i.e.* for each type of investigated attack and for each rank) the integral on the EM Gaussian model belongs to the corresponding confidence limits.

This sub-section shows that an individual model (Table 1), computed on a particular video sequence, is valuable for all the video sequences involved in the experiments and, moreover, that it does not depend on the estimation procedure. This means a positive answer to the first question in Introduction.

4 CAPACITY COMPUTATION

As discussed in Introduction, any side-information watermarking technique can be modelled by a noisy channel, where the mark is a sample from the information source and the noise is represented by the attacks. In order to evaluate the capacity of such a channel, the eqs. (A7) and (A8) are considered.

For the capacity limits in eq. (A7), the noise power N is the variance of the *noise* vector, Section 2.2. The signal power *P* was derived from transparency constraints, so as to ensure a mark 30dB lower than the original (unmarked) coefficients.

Table 2: The errors (minimal, maximal, average) between the reference model and the 9 models obtained on different video sequences, for three ranks: r = 1, r=150, and r = 300.

Type	Attack	r = 1			r = 150			r = 300		
		Min	Max	Average	Min	Max	Average	Min	Max	Average
Error	Gaussian filtering	0.662	0.758	0.710	0.058	0.216	0.137	0.071	0.153	0.112
	Sharpening	0.109	0.148	0.128	0.028	0.065	0.046	0.055	0.087	0.071
	StirMark	0.077	0.093	0.085	0.065	0.109	0.087	0.084	0.131	0.108
D_{KL}.	Gaussian filtering	0.131	0.204	0.167	0.011	0.016	0.014	0.029	0.030	0.029
	Sharpening	0.035	0.712	0.374	0.066	0.110	0.088	0.081	0.098	0.089
	StirMark	0.106	0.110	0.108	0.015	0.018	0.016	0.024	0.026	0.025
D_{HL}.	Gaussian filtering	0.054	0.075	0.064	0.006	0.014	0.010	0.006	0.010	0.008
	Sharpening	0.213	0.255	0.234	0.009	0.019	0.014	0.015	0.015	0.015
	StirMark	0.025	0.029	0.027	0.002	0.003	0.002	0.004	0.004	0.004

The N_1 entropic power was also estimated on the n_o original coefficient vector. The bandwidth W was computed as half the frame rate.

When considering the capacity value in eq. (A8), the model provided by the present study (*i.e.* the *pdf* in Table 1) is considered as the noise *pdf* $p_N(n)$. The $p_X(x)$ function giving the capacity value is searched for by means of a numerical strategy. Actually, it is considered that $p_X(x)$ itself can be represented as a mixture of 5 Gaussian laws, thus restricting the searching to a space with 15 dimensions (5 weights, 5 means values and 5 variances). These 15 dimensions are not independent. First, the sum of weights should equal 1. Secondly, the mean of the mark (*i.e.* the mixture mean) is set to 0 (a generally accepted assumption in watermarking). Thirdly, the mixture variance was set so as to ensure a good transparency (*i.e.* 30dB lower than the host video).

The capacity values computed with the general formula and with Shannon limits are shown in Table 3. A general agreement between the two types of capacity estimation can e noticed, with some exceptions (for $r = 1$ of Gaussian filtering, StirMark). At the same time, the capacity estimation starting from the attack models is compulsory when a certain degree of precision is required: that capacity evaluation by limits can lead at relative errors of about 100% and larger!

Table 3: Capacity value and limits (lower and upper) for rank r = 1, r = 150 and r=300.

Rank	Attack / Capacity	Gaussian filtering	Sharpening	StirMark
$r = 1$	value	3.567	1.332	2.307
	limits	(3.632 ; 3.651)	(1.268 ; 1.725)	(2.394 ; 2.415)
$r = 150$	value	0.339	0.251	0.259
	limits	(0.037 ; 0.949)	(0.004 ; 0.569)	(0.005 ; 0.273)
$r = 300$	value	0.055	0.006	0.009
	limits	(0.018 ; 0.935)	(0.002 ; 0.621)	(0.002 ; 0.273)

5 CONCLUSIONS

The present paper brings into evidence that some real life watermarking attack effects are stationary in the DCT hierarchy and accurately estimates the corresponding probability density functions. Then, these models are involved in capacity evaluation.

From the applicative point of view, beyond watermarking itself (*i.e.* reaching the capacity limit in a practical application), these results are the starting point for a large variety of applications in the multimedia content processing, as smart indexing or in-band content enrichment, for instance.

Further work will be also devoted to considering the Blahut approach for watermarking capacity evaluation.

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APPENDIX

A1 *Pdf* Estimation Tools

Be there $[x_1, x_2, ..., x_N]$ a set of N experimental data complying with the *iid* model. Suppose that these data are sampled from a random variable X whose *pdf* $p(x)$ is unknown and should be estimated. A $\hat{p}(x)$ *Gaussian mixture* is a linear combination of Gaussian laws and can approximate any continuous $p(x)$ *pdf* (Archambeau & other, 2003):

$$\hat{p}(x) = \sum_{k=1}^{K} P(k) p_k(x), \qquad \text{(A1)}$$

$$\text{where } p_k(x) = \frac{1}{\sqrt{2\pi\sigma_k^2}} \exp(-\frac{(x-\mu_k)^2}{2\sigma_k^2}).$$

The number of mixtures K is pre-established by the experimenter and $P(k), \mu_k, \sigma_k$ are *3K* parameters to be estimated by the EM (expectation maximisation) algorithm (Dempster & other, 1977), based on a maximum likelihood criterion:

- the E step:

$$p^{(i)}(k/x_n) = \frac{p_k^{(i)}(x_n) P^{(i)}(k)}{\hat{p}^{(i)}(x_n)}, \qquad \text{(A2)}$$

- the M step:

$$\mu_k^{(i+1)} = \frac{\sum_{n=1}^{N} p^{(i)}(k/x_n) x_n}{\sum_{n=1}^{N} p^{(i)}(k/x_n)},$$

$$(\sigma_k^2)^{(i+1)} = \frac{\sum_{n=1}^{N} p^{(i)}(k/x_n)\left(x_n - \mu_k^{(i+1)}\right)^2}{\sum_{n=1}^{N} p^{(i)}(k/x_n)} \qquad \text{(A3)}$$

$$P^{(i+1)}(k) = \frac{1}{N} \cdot \sum_{n=1}^{N} p^{(i)}(k/x_n),$$

where the (i) upper index denotes the current iteration; the total number of iterations is also subject to the experimenters choice.

The relationship among the parameters of the individual Gaussian laws and the mixture parameters is given in (Trailovic, Pao, 2002).

Alongside with the similarity measure defined in eq. (1), two popular methods for *pdf* comparison are involved in the experiments (Basseville, 1996):

- Kullback-Leibler divergence:

$$D_{KL}(u,v) = \int_I u(x) \log_2 \frac{u(x)}{v(x)} dx; \qquad \text{(A4)}$$

- Hellinger distance:

$$D_{HL}(u,v) = \frac{1}{2} \int_I \left(\sqrt{u(x)} - \sqrt{v(x)}\right)^2 dx. \qquad \text{(A5)}$$

A2 Capacity Evaluation Basis

The capacity of a continuous channel, whose input and output information sources are denoted by X and Y is given by the Shannon's formula (A6):

$$C = \max_{p_X(x)} \int_{x_1}^{x_2} \int_{y_1}^{y_2} f_{XY}(x,y) dx dy, \qquad \text{(A6)}$$

$$f_{XY}(x,y) = p_{XY}(x,y) \log_2 \frac{p_{XY}(x,y)}{p_X(x) p_Y(y)},$$

where $p_X(x)$ and $p_Y(y)$ stand for the input and output *pdf*s, while $p_{XY}(x,y)$ is the joint *pdf* of X and Y. The x_1, x_2 and y_1, y_2 are the limits of the intervals on which the input and output *pdf*s have non-zero values. In the case of a non-Gaussian noise, Shannon derives some upper and lower limits, eq. (A7):

$$W \log_2 \frac{P + N_1}{N_1} \le C \le W \log_2 \frac{P+N}{N_1}, \qquad \text{(A7)}$$

where W is the channel bandwidth, P is the signal power, N is the noise power, and N_1 is the noise entropy power (*i.e.* the power of a white-type noise which has the same bandwidth and entropy as the considered noise).

When assuming the noise is additive and independent, eq. (A6) becomes:

$$C = \max_{p_X(x)} \int_{x_1}^{x_2} \int_{x_1+n_1}^{x_2+n_2} g_{XY}(x,y) dy dx, \qquad \text{(A8)}$$

$$g_{XY}(x,y) = p_X(x) p_N(y-x) \log_2 \frac{p_N(y-x)}{\int_{n_1}^{n_2} p_N(n) p_X(y-n) dn},$$

where the noise limits are $n_1 = y_1 - x_1$ and $n_2 = y_2 - x_2$ (Dumitru, & others, 2007).