A QUADRATIC PROGRAMMING APPROACH TO THE MINIMUM ENERGY PROBLEM OF A MOBILE ROBOT

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Abstract: As a consequence of physical constraints and of dynamical nonlinearities, optimal control problems involving mobile robots are generally difficult ones. Many algorithms have been developed to solve such problems, the more common being related to trajectory planning, minimum-time control or any specific performance index. Nevertheless optimal control problems associated to mobile robots have not been reported. Minimum energy problems subject to both equality and inequality constraints are generally intricate ones to be solved using classical methods. In this paper we present an algorithm to solve it using a Quadratic Programming approach. In order to illustrate the application of the algorithm, one practical problem was solved.

1 INTRODUCTION

1.1 Preliminaries

Mobile robotics is an important research area and for its study many researchers have dedicated a lot of time to it. There are many problems in mobile robots that are not present in industrial robots. Problems with posture maintenance, localization, equilibrium and energy consumption are common both at design and operation times. Limbed robots can be considered an important engineering conquest due to the fact that they have larger mobility, flexibility and freedom of movements than any other automatic machine (Dudek, 2000). Research in this area requires strong knowledge of mechanics, electronics, computation, and eventually biomechanics. Limbed robots are capable of walking and climbing and have been developed around the world. (Armada et al., 2003; Virk, 2005). Some of them have been used to inspect bridges (Abderrahim et al., 1999) and pipelines (Galves; Santos; Pfeiffer, 2001).

Presently the literature reports just two robots with the ability of tree climbing. The first one was developed at the Waseda University, Japan, and the second one is the RiSE robots (Robots in Scansorial Enviroments) (Saundersa et al., 2006) developed at the United States. The RiSE robot is a member of a new class of climbing robots whose design is based on animals. For all of them, the energy consumption is a big problem. Generally the battery is their heaviest part since a considerable amount of energy is necessary to drive the legs'motors.

Typically a leg has the form of a serial mechanism with a highly nonlinear dynamics. This is one of the reasons that make to find an optimal control law a difficult problem. A second reason is the presence of both equality and inequality constraints imposed on the system – e.g., the actuator of each joint is subject to saturation (Spont et al., 1989).

Many authors have worked in the optimization of robots operation in the context of trajectory planning. (Lin et al., 1983; Garg et al., 2002; Luo et al., 2004). The problems considered in general aimed to minimize the time or some quadratic performance index. Nevertheless in the majority of them both the Coriolis and Centrifugal terms were omitted.

Optimal trajectory control systems usually can be built by solving two associated sub-problems: i) the optimal trajectory planning (OTP); ii) the trajectory tracking control (TTC). Since there are many complex constraints conditions concerning robot kinematics and dynamics, the corresponding

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algorithms for solving the OTP problem have been improved in recent years (Luo et al., 2004). Even though good results have been obtained with the OTP problem for both manipulators and industrial robots, this is not the case for mobile robots.

A minimum energy formulation may be an interesting approach for mobile robots, particularly in applications where the battery weight is a critical issue. Minimum energy problems may be difficult to solve by classical methods since they involve both the nonlinear dynamics of the robot and a set of constraints.

In this paper a Quadratic Programming approach to the minimum energy problem of a mobile robot is proposed. The method is based on the discretization of the problem. Numerical tests were performed for Kamanbaré[1], a robot currently under development at the Automation and Control Laboratory (LAC), University of São Paulo.

1.2 The Robotic Platform

Kamambaré is a biomimetic robotic platform, i.e., a robotic platform inspired in nature, with the purpose of climbing trees for environmental research applications (see Fig. 1). More specifically the platform locomotion is inspired in the form lizards climb trees. The main characteristics sought in the definition of the Kamanbaré platform were: locomotion in irregular environments (unpredictability of the branch complexity that compose a tree), surmounting obstacles (nodes and small twigs), tree climbing and descending without risking stability, and keeping low structural weight (mechanics + electronics + batteries). (Bernardi et al., 2006)

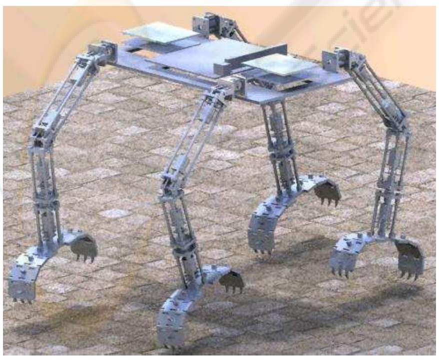

Figure 1: Robot Kamanbaré.

[1] *Kamanbaré* is the word in the Tupi indian language for chameleon.

The prototype of the Kamanbaré platform presented in this work was developed considering certain capabilities (abilities), such as: locomotion in irregular environments (unpredictability of the branch complexity that compose a tree), surmounting obstacles (nodes and small twigs), tree climbing and descent without risking stability, and keeping low structural weight (mechanics + electronics + batteries). (Bernardi et al., 2006)

Each leg has three rigid links connected by two rotational joints with one degree-of-freedom (d.o.f.) each. The first link is connected to the platform by a two d.o.f. rotational joint (Fig. 2).

Figure 2: Limb of the Kamanbaré Platform.

All joins are controlled by DC motors. The control problem for mobile robot is the problem of determining the time history of join required to cause the end-effectors (the gripper) executed a commanded motion.

There are many control techniques and methodologies that can be applied to the control of limbed robots. The particular control method chosen as well as the manner in which it is implemented can have a significant impact on the performance of the robot and consequently on the range of its possible applications. Optimal control of energy can be an interesting policy when the robot must operate autonomously for a long time. The control problem for the tree climbing robot considered here is to determine the time history of each limb joint required to cause the leg to move from an initial angle to a final one in such a way that the energy loss in the motors is minimal.

2 FORMULATION AND SOLUTION OF THE OPTIMAL CONTROL PROBLEM

2.1 The State Space Model

This work consider the control strategy named independent joint control. In this type of control each axis of the limb is controlled as a single input /single output system. Any coupling effects due to the motion of the others links is either ignored or treated as a disturbance. (Spont et al., 1989)

The space state model that describes the dynamics of the DC motor located at the *i*-th joint ($1 \le i \le n$) of the leg can be expressed by the equations:

$$\dot{x} = Ax + Bu \tag{1}$$

$$y = Cx + Du + E\tau \tag{2}$$

where

$$A = \begin{bmatrix} 0 & 1 \\ 0 & -b/J \end{bmatrix}, \tag{3}$$

$$B = \begin{bmatrix} 0 \\ K_m / J \end{bmatrix}, \tag{4}$$

$$C = \begin{bmatrix} 0 & -K_b / R_a \end{bmatrix}, \tag{5}$$

$$D = 1/R_a, \tag{6}$$

$$E = r/(R_a K_m), \tag{7}$$

and

$$x = \begin{bmatrix} \theta_m \\ \dot{\theta}_m \end{bmatrix}, \tag{8}$$

$$u = V - \frac{r}{K_m}\tau \cdot \tag{9}$$

J is the moment of inertia of the rotor, b is the viscous damping coefficient of the mechanical system, V is the armature voltage (control variable), R_a is the armature resistance, $Y = i_a$ is the armature current, K_m is the torque coefficient of the motor, K_b is the counter-electromotive coefficient, τ is the load torque, $r < 1$ is the gear reduction factor, θ_m is the angular position of the rotor and $\dot{\theta}_m$ is the angular speed of the rotor. Notice that the input variable u depends on both the control variable V and the load torque τ, which also depends on V through θ_m and $\dot{\theta}_m$.

The load torque τ, which depends on the robot dynamics, is given by : (Spong, 1989)

$$\tau = M(q)\ddot{q} + B(q)[\dot{q}_i\dot{q}_z] + C(q)[\dot{q}^2] + G(q) \tag{10}$$

where $M(q)$ is the leg inertia matrix, $B(q)$ is the Coriolis torque matrix, $C(q)$ is the centrifugal torque matrix, $G(q)$ is the gravitational torque vector,

$$[\dot{q}_i\dot{q}_z] = [\dot{q}_1\dot{q}_2 \quad \dot{q}_1\dot{q}_3 \quad \dots \quad \dot{q}_{n-1}\dot{q}_n]^T,$$
$$\forall i \ne z : 1 < i < n, 1 < z < n$$

$$[\dot{q}^2] = [\dot{q}_1^{\,2} \quad \dot{q}_2^{\,2} \quad \dots \quad \dot{q}_n^{\,2}]^T$$

and

$$q = [q_1 \quad q_2 \quad \dots \quad q_n]^T,$$

where q_i, $1 \le i \le n$, is the generalized coordinate of joint *i* and *n* is the number of joints of the leg.

Some dynamical effects like friction were not included in (10) although they may be significant for some limbs. In addition, a more detailed model of the leg dynamics could include various sources of flexibility, defection of the links under load and vibrations (Borrow et al., 2004). Nevertheless, this model is sufficiently accurate for our purposes since these effects are not significant for the leg under consideration.

One of the characteristics of the independent joint model is that τ is multiplied by the gear reduction r. Thus effect of the gear ratio is to reducing the coupling nonlinearities presents in dynamics of the limbs.

The solution to equation (1) is given by:

$$x(t) = e^{At}x(0) + \int_0^t e^{A(t-\varsigma)}Bu(\varsigma)d\varsigma \tag{11}$$

2.2 The Optimal Control Problem

In this section it is assumed that $\tau(t)$ is known for all t in the interval $[0 \; t_f]$. The performance index adopted is the Joule loss in the armature resistance of each motor during the motion:

$$\min \varepsilon = \int_0^{t_f} P\,dt \qquad (12)$$

where t_f is the time required to move the joint from the initial to the final position, and P is the power dissipated:

$$P = R_a i_a^2 = R_a Y^2 = R_a\left[x'C' + u'D' + \tau'E'\right]\left[Cx + Du + E\tau\right] \qquad (13)$$

where Y was defined in (2).

The optimal control problem is subject to the following constraints:

$$-\frac{K_b}{R_a}x_2(t) + \frac{1}{R_a}u(t) + \frac{r}{R_aK_m}\tau(t) \le I_m, \qquad (14)$$

$$-I_m \le -\frac{K_b}{R_a}x_2(t) + \frac{1}{R_a}u(t) + \frac{r}{R_aK_m}\tau(t), \qquad (15)$$

$$-V_m \le u(t) \le V_m, \qquad (16)$$

$$x_1(t_f) = q_f / r, \qquad (17)$$

$$x_2(t_f) = 0, \qquad (18)$$

where I_m and V_m are, respectively, the maximum armature current and maximum armature voltage of the motor and $t \in \left[0 \quad t_f\right]$. With no loss of generality we take $x_1(0) = 0$. Considering the motor initially at rest, $x_2(0) = 0$.

This type of problem is hard to solve and generally involve a great computational effort (Kirk, 1998). Mobile robots require a quick solution and to solve it in real-time is practically impossible. These are among the reasons for which we decided to look for another kind of solution.

2.3 Discretization

Let us to define the time-step as:

$$\Delta t = \frac{t_f}{N} = t_k - t_{k-1} \qquad (1 < k \le N) \qquad (19)$$

where N can be chosen sufficiently large to discretize t_f. Then it is assumed that $u(t)$ is a stepwise constant function and u_k is used to denote the value of $u(t)$ for all t in the k -th time interval $[t_{k-1}, t_k)$.

Taking into account equations (11), (12), (13) and (19), the functional ε can be rewritten as:

$$\min \varepsilon = \sum_{k=1}^{N} P_k \Delta t = \sum_{k=1}^{N} R_a i_a^2 \Delta t \qquad (20)$$

and the solution of system (1) as:

$$x(t_k) = e^{At_k}\Gamma I_k U \qquad (21)$$

where,

$$\Gamma = \int_0^{t_1} e^{-A\varsigma}Bd\varsigma + \int_0^{t_2} e^{-A\varsigma}Bd\varsigma + \ldots + \int_{t_{N-1}}^{t_N} e^{-A\varsigma}Bd\varsigma, \qquad (22)$$

$$U = \left[u_1 \quad u_2 \quad \ldots \quad u_N\right]^T \qquad (23)$$

and

$$I_k = \begin{bmatrix} e_1 & 0 & 0 & 0 \\ 0 & e_2 & 0 & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & e_n \end{bmatrix} \qquad (24)$$

Where $e_i = 1$ for $1 \le i \le k$ and $e_i = 0$ for $k < i \le N$.

Matrix Γ can be calculated offline since it depends only on the motor parameters.

From equations (20) and (21) it is possible rewrite (20) as:

$$\min \varepsilon(U) = \frac{1}{2}U^TQU + \tau K^TU + \sigma \qquad (25)$$

where matrices Q and K depend only on the motor parameters and σ is a constant. The constraints for the problem are:

$$-I_m \le -\frac{K_b}{R_a}x_{2_k} + \frac{1}{R_a}u_k + \frac{r}{R_aK_m}\tau_k \le I_m \qquad (26)$$

$$-V_m \le u_k \le V_m \qquad (27)$$

$$x_{1_N} = q_f / r \qquad (28)$$

$$x_{2_N} = 0 \qquad (29)$$

where constraints (26) and (27) apply for all k, $1 \le k \le N$.

Since σ is a constant, it is not relevant for the optimization.

The problem given by equations (25)-(29) has thus the form of a Quadratic Programming problem which is certainly the kind of nonlinear programming problem closest to linear programming from analytical and computational point of view. Solution for this kind of problem can be efficiently found by numerical methods. (Avriel, 1976) (Winston, 1995).

2.4 The Algorithm

An iterative algorithm to solve the minimum energy problem based in equations (13) and (20) is proposed in this section.

To start the algorithm it is assumed that $\tau_k =: \tau_k^0 \equiv 0$ for all k, $1 \le k \le N$. Then Quadratic Programming problem of minimizing the function (20) subject to constraints (26) to (29) is solved. Denote by U_k^{0*} the optimal solution for this problem. Using equation (17) and recalling equation (1), both the motor angular position $\theta_m^0(k)$ and angular speed $\dot{\theta}_m^0(k)$ can be evaluated for all k, $1 \le k \le N$.

The second step of the algorithm begins by using the leg dynamical equations (10) to evaluate a new torque time history τ_k^1 for all k, $1 \le k \le N$. The new Quadratic Programming problem is then solved and U_k^{1*} is obtained. $\theta_m^1(k)$ and $\dot{\theta}_m^1(k)$ are evaluated for all k, $1 \le k \le N$.

The process is repeated until $\left\| \tau_k^{j+1} - \tau_k^j \right\| < \varepsilon_\tau$ for a given accuracy ε_τ and two consecutive steps *j* and *j*+1. When convergence is attained the optimal vector of armature voltages V^{j+1*} can be evaluated using equation (9).

The algorithm is expected to converge since the gear reduction ratio *r* is generally small and the effect of the torque on the motor dynamics is correspondingly small too.

The algorithm may thus be summarized as:

Algorithm

$\tau_k^0 = 0$

$j = -1$

`repeat`

$j = j + 1$

$$\begin{cases} \min \varepsilon(U) = \frac{1}{2} U^T Q U + \tau_k^j K^T U \\ \textit{subject to constrains: equations } (26) \text{ to } (29) \end{cases}$$

$$\dot{x}_k^j = \begin{bmatrix} 0 & 1 \\ 0 & -\frac{b}{J} \end{bmatrix} x_k^j + \begin{bmatrix} 0 \\ \frac{K_m}{J} \end{bmatrix} U_k^{j*}$$

$$\begin{bmatrix} q & \dot{q} & \ddot{q} \end{bmatrix}_k^{j^T} = r \begin{bmatrix} x & \dot{x} & \ddot{x} \end{bmatrix}_k^{j^T}$$

$$\tau_k^{j+1} = M(q)\ddot{q} + B(q)[\dot{q}\dot{q}] + C(q)\left[\dot{q}^2\right] + G(q)$$

`until` $\left(\max\left(\tau_k^{j+1} - \tau_k^j \right) \le \varepsilon_\tau \right)$

3 SIMULATION RESULTS

This section presents the results obtained from the application of the algorithm above to a leg similar to that of the Kamanbaré platform.

The algorithm code was written in MatLab. The following data were used: $t_f = 2s$, $N = 10$, $\varepsilon_\tau = 10^{-11}$ and

$$\begin{bmatrix} \theta_{m_{0N}} & \theta_{m_{1N}} & \theta_{m_{2N}} \end{bmatrix}^T = \begin{bmatrix} \pi & 0 & -\frac{\pi}{4} \end{bmatrix}^T$$

Algorithm convergence occurred in 15 iterations. Table 1 shows the algorithm steps until the optimal solution is reached. The overall processing time was quite small.

Figure 3 show the optimal solution $U_k^{15^*}$ for each motor.

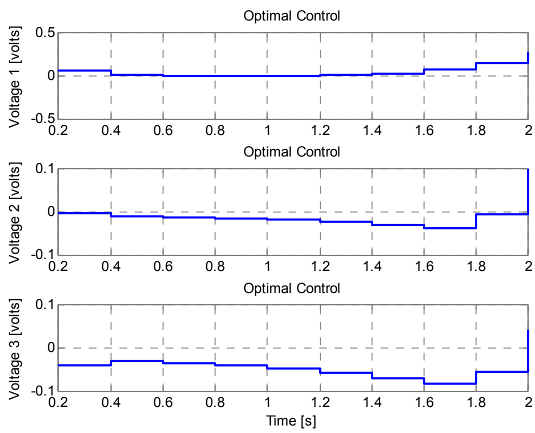

Figure 3: Optimal Solution.

Table 1: Minimum Energy Consumption.

Iteration (j)	Energy Consumption $\varepsilon = \sum_{k=1}^{N} P_k \Delta t$ [watts]		
	Joint 1 (Motor 1)	**Joint 2 (Motor 2)**	**Joint 3 (Motor 3)**
1	0.22583525809518	0	0.01411470363095
2	0.54691612008194	0.00249201214070	0.02563789026998
3	0.60424685648279	0.00151916407557	0.02084265484899
4	0.60913169232854	0.00162074115964	0.02201351589222
5	0.60878562782495	0.00159940182719	0.02201765824694
6	0.60878328339302	0.00160200730314	0.02202687314770
7	0.60877614492969	0.00160176202487	0.02202629388348
8	0.60877639908587	0.00160179288124	0.02202637161877
9	0.60877635993567	0.00160178879636	0.02202635576755
10	0.60877636906069	0.00160178928402	0.02202635725193
11	0.60877636847905	0.00160178921712	0.02202635705179
12	0.60877636859543	0.00160178922523	0.02202635707836
13	0.60877636858174	0.00160178922422	0.02202635707548
14	0.60877636858322	0.00160178922435	0.02202635707588
15	**0.60877636858300**	**0.00160178922433**	**0.02202635707583**

Figures 4 and 5 show kinematics of each joint; the angular position and the speed and acceleration achieved for the optimal control $U_k^{15^*}$.

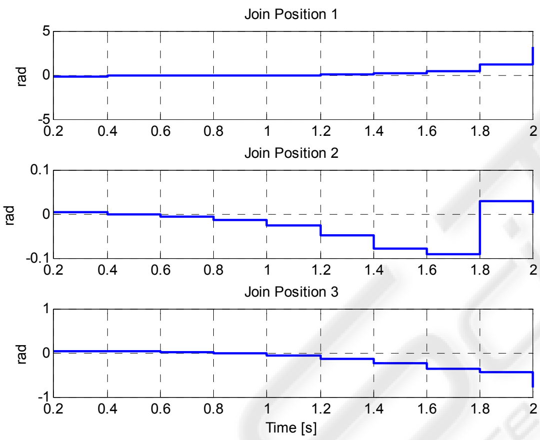

Figure 4: Joints Positions.

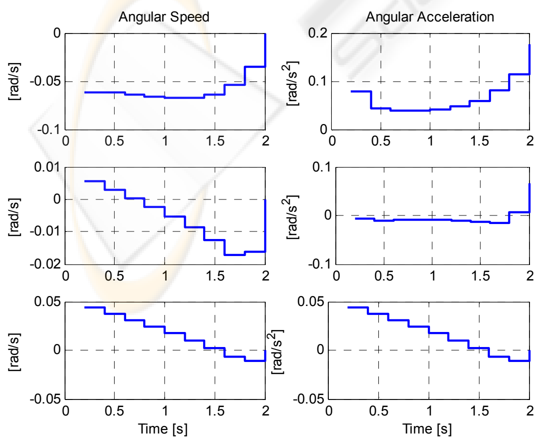

Figure 5: Speed's Joints and Acceleration's Joints.

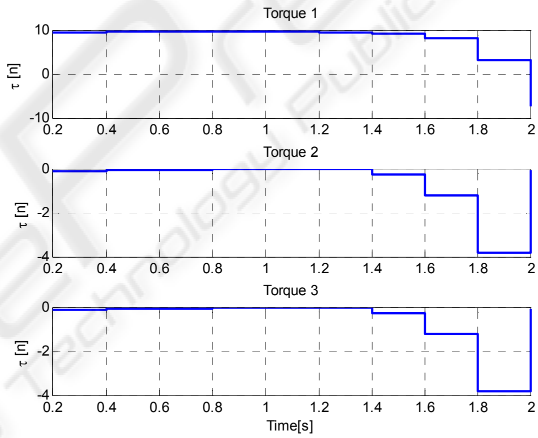

Figure 6: Torques for each motor.

In this particularly case the motor's 2 and 3 have the same torque.

4 CONCLUSIONS

This paper discussed the formulation and solution of an important problem related to mobile robotics: the minimum energy loss problem.

An optimal control problem was formulated to represent this case. After discretization in time the optimal control problem was rewritten in the form of a Quadratic Programming problem whose solution could be obtained efficiently. The algorithm was tested for a leg similar to that of the Kamanbaré platform.

Although the whole problem is nonlinear and quite complex the algorithm converged quickly in all the tests performed by now. It is thus expected that the it can be used to operate in real-time.

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