A UNIFYING POINT OF VIEW IN THE PROBLEM OF PIO
Pilot In-the-loop Oscillations

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Abstract: The paper starts from the problem of PIO (Pilot-In-the-loop Oscillations), a major problem in aircraft handling and control, where the idea of the *feedback as hidden technology* is basic. The real phenomenon called PIO is modeled by a feedback structure where the pilot acts as one of the components of the loop and has to be modeled accordingly. PIO are in fact self-sustained oscillations and usually are divided into three convenient categories that are based on the nature of the pilot and vehicle dynamics behavior models and analysis needed for their explanation. Category I PIO are essentially linear while Category II PIO are quasi-linear and typically associated with rate limiting. Category III PIO are fully nonlinear and non-stationary. Since PIO II are mostly tackled *via* various robustness approaches starting from linear models, the paper strives for a unifying approach which is illustrated accordingly.

1 BASICS AND PROBLEM STATEMENT

According to the standard terminology of the field, PIO (Pilot Induced Oscillations, Pilot In-the-loop Oscillations, Pilot Involved Oscillations) are sustained or uncontrollable oscillations resulting from the effort of the pilot to control the aircraft, hence they can be considered as a closed loop destabilization of the aircraft-pilot loop (Anon., 2000),(McRuer et al., 1996). Even from this remarkably short definition it appears that the real phenomenon called PIO can be modeled by a feedback structure where the pilot acts as one of the components of the loop and has to be modeled accordingly. As mentioned in (McRuer et al., 1996) the study of the aeronautical history reveals a remarkably diverse set of severe PIOs as exemplified by the listings of "famous PIOs" (McRuer, 1994),(Klyde et al., 1995).

The feedback control character of PIOs was recognized almost from the outset because the aircraft left alone did not exhibit such oscillations. Once recognized as oscillations within a feedback system context, mathematical models were developed and used to describe the pilot's dynamic actions as a controller and active participant in PIOs.

A. Detailed analytical studies of past PIO incidents (see e.g. the references from (McRuer et al., 1996)) relied on pilot behavioral models and closed loop analysis procedures to understand and rationalize phenomena. Moreover in some cases pilot vehicle behavioral models were applied to design and assess changes to the effective vehicle to alleviate the PIO potential. Based on these results it is useful to divide PIOs into categories that reflect the analytical and pilot modeling tools. There were identified three categories of PIO as follows:

- Category I - Essentially Linear Pilot Vehicle System Oscillations: the element characteristics are essentially linear and the pilot behavior is linear (except, possibly, for simple gain shaping in series with the pilot).
- Category II - Quasi-Linear Pilot Vehicle Systems with Series Rate or Position Limiting. Rate limiting, either as a series element or as a rate limited surface actuator modifies the Category I situation by adding what is called (non-rigorously) an amplitude dependent lag and by setting the limit cycle magnitude.
- Category III - Essentially Non-Linear Pilot Vehicle System Oscillations with transitions: they fundamentally depend on nonlinear transitions in either the effective control element or in the pilot behavioral dynamics.

B. Most of the available information shows that mainly PIO I and PIO II were considered and analyzed due to the complexity of PIO III which never-

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theless have been recognized as quite rare and arising from PIO I and PIO II; consequently a consensus has been established in the PIO community that PIO III proneness may be blocked by blocking PIO I and PIO II proneness. With respect to this several PIO I and PIO II criteria have been elaborated in USA and in Europe. These criteria are viewed as sufficient conditions ensuring that the feedback system pilot-aircraft is PIO free. Most of them are obtained in the linear case i.e. for PIO I. Here several remarks are necessary. It is a trivial fact that for linear systems there exist necessary and sufficient conditions for stability what means also absence of self sustained oscillations. The PIO criteria are only sufficient conditions but they are conceived as to ensure some kind of robustness with respect to system's uncertainties. Indeed the presence of uncertainties is quite obvious. There are first the uncertainties of aircraft modeling - aerodynamic forces and coefficients depending of the flight envelope parameters - but also those of pilot modeling which depend on several modeling assumptions.

On the other hand, as already mentioned, PIO II are associated to quasi-linear models where rate and position limiters are active. The limiters are modeled as structures containing saturation nonlinear functions; the quite recent models which are based on Integral Quadratic Constraints (Megretski and Rantzer, 1997),(Megretski, 1997) take into account the simplest remark that the saturation nonlinearity (fig. 1) may be "embedded" in the larger class of the sector restricted nonlinearities (fig. 2)

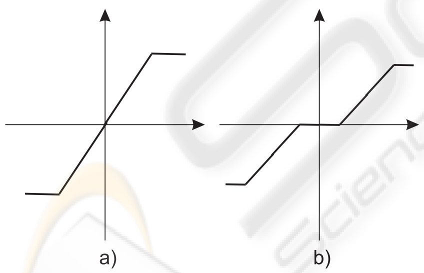

Figure 1: Saturation nonlinearities.

As known, the properties of the sector restricted nonlinearities may be expressed under the form of some quadratic constraints - see (Megretski and Rantzer, 1997) but also the pioneering paper (Yakubovich, 1967) as well as the monograph (Răsvan, 1975). This embedding of the nonlinear function in a larger class speaks about allowing some uncertainty concerning the nonlinearity; additionally, if the stability results are valid uniformly for the entire class of non linearities, some robustness is ensured.

Consequently, it is not by chance that an important class of methods associated to PIO II originate from robustness approaches. Moreover, one can find there such standard methods of the absolute stability as the Liapunov function of the form "quadratic form of the state variables plus integral of the nonlinear function" or the Popov frequency domain inequality (Anon., 2000).

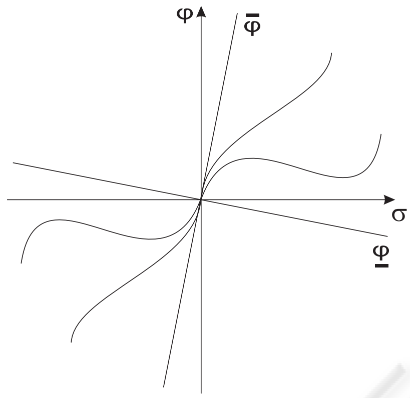

Figure 2: Sector restricted nonlinearity.

The above considerations show that it is not without interest *to discuss PIO I and PIO II within a unified context of robustness* in the sense that the robustness restrictions introduced in the totally linear case (PIO I) should be taken into account in the quasi-linear (PIO II) case. As an at-hand example, the frequency domain restrictions of the Neal-Smith criterion (Neal and Smith, 1971) should be reflected in a Popov like frequency domain inequality.

The present paper will demonstrate and motivate the above sketched approach and what remains is organized as follows. Firstly the basic feedback structure is presented in the context of fully linear models accounting for PIO I. It is then shown how rate limiters occur in the loop - the PIO II onset - and the new structure of a feedback nonlinear system with a sector restricted nonlinearity. The linear subsystem is then identified and a frequency domain inequality is then formulated. This inequality has to be valid for the frequency domain characteristic as resulting from a PIO I criterion; if this holds we may say that PIO I gives "some insurance" for PIO II. Next a specific case will be discussed to illustrate the principle and conclusions together with suggestions for future research and tests are given.

2 ROBUSTNESS VERSUS ABSOLUTE STABILITY

The analysis of the models of (Anon., 2000),(McRuer et al., 1996),(Klyde et al., 1996),(Klyde and Mitchell,

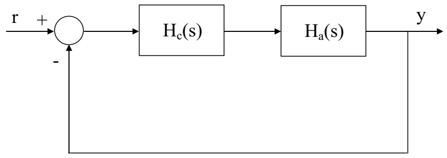

Figure 3: Basic linear feedback structure.

2005) which deal with longitudinal PIO will suggest the general feedback structure of fig. 3.

We denoted there by $H_a(s)$ the transfer function of the "uncontrolled plant" which might be some longitudinal or lateral motion of the aircraft; by $H_c(s)$ we denoted the "controller" which in this man/machine system might be some pilot model (for instance the so-called *synchronous pilot* is just a gain - see (Klyde et al., 1996)).

Some remarks are necessary from this very beginning. Since various assumptions on pilot behavior may require *pole/zero cancelation*, only LHP (left hand plane) i.e. stable poles and zeros may be canceled, otherwise uncontrollable unstable modes will appear. This is particularly true for the so-called *crossover model* where we have

$$H_c(s)H_a(s) \equiv \frac{K}{s}\mathrm{e}^{-\tau s} \qquad (1)$$

and this clearly implies pole/zero cancelation. Since it is well known that modern fighters become unstable for high speed points of the flight envelope, they are made stable by additional stabilizing feedback - the SAS (Stability Augmentation System). Equality (1) avoids unstable pole/zero cancelation only if the SAS is active[1].

If the limiters are to be considered, the system of fig. 3 will become a standard feedback control structure with a nonlinear (saturated) actuator (fig. 4)

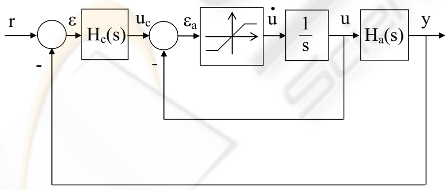

Figure 4: Feedback structure with rate limiter.

In order to obtain the standard structure of the absolute stability problem, we consider the state realizations of the two proper rational transfer functions $H_a(s)$ and $H_c(s)$ embedded in the structure of fig. 4

[1]This explains also in some way the X-15 landing flare PIO evoked in (Klyde et al., 1996) since it is mentioned there that the "pitch damper was off", the pitch damper being the SAS of the channel

$$\begin{aligned}
&\dot{x}_a = Ax_a + bu\ ,\ y = c^T x_a + h_0 u\\
&\dot{u} = \varphi(\varepsilon_a)\ ,\ \varepsilon_a = u_c - u\\
&\dot{x}_c = A_c x_c + b_c\varepsilon\ ,\ \varepsilon = r(t) - y\\
&u_c = f_c^T x_c + h_c\varepsilon
\end{aligned} \qquad (2)$$

which becomes

$$\begin{aligned}
&\dot{x}_a = Ax_a + bu\\
&\dot{x}_c = -b_c c^T x_a + A_c x_c - h_0 b_c u + b_c r(t)\\
&\dot{u} = \varphi(\sigma)\\
&\sigma = -h_c c^T x_a + f_c^T x_c - (1 + h_c h_0)u + h_c r(t)
\end{aligned} \qquad (3)$$

For $r(t) \equiv 0$ what means the system is considered in deviations with respect to some steady state (equilibrium) the feedback structure of fig. 5 is obtained

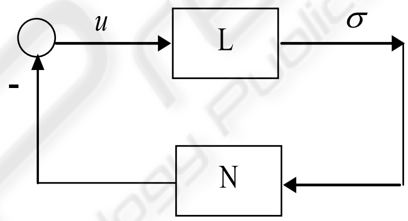

Figure 5: Absolute stability feedback structure.

The linear subsystem is described by the controlled system of ordinary differential equations with linear output

$$\begin{aligned}
&\dot{x}_a = Ax_a + bu\\
&\dot{x}_c = -b_c c^T x_a + A_c x_c - h_0 b_c u\\
&\dot{u} = -\mu(t)\\
&\sigma = -h_c c^T x_a + f_c^T x_c - (1 + h_c h_0)u
\end{aligned} \qquad (4)$$

in feedback connection with the nonlinear static block

$$\mu = -\varphi(\sigma) \qquad (5)$$

The transfer function of (4) is

$$H(s) = \frac{\tilde{\sigma}(s)}{\tilde{\mu}(s)} = \frac{1}{s} + \frac{1}{s}H_c(s)H_a(s) \qquad (6)$$

and the characteristic equation clearly has a zero root. This might be the simplest critical case of the absolute stability, but if (1) holds the case corresponds to the non-simple zero root - the most special critical case, that was studied separately of the other ones, due to its specific problems; moreover the presence of the delay in (1) will complicate the approach (Răsvan, 1975); we give here an adaptation of Theorems 6.1 and 7.2 of (Răsvan, 1975)

Theorem 1. *Consider the system of fig. 5 where the linear subsystem has its transfer function of the form (6) with $H_c(s)H_a(s)$ being a meromorphic function - ratio of quasi-polynomials; the denominator has a simple zero root and all other roots with negative real parts. The nonlinear function φ is subject to the following conditions*

$$\begin{aligned} &\sigma\varphi(\sigma) > 0 \ (\sigma \neq 0) \ , \ \varphi(0) = 0 \ , \\ &\lim_{\sigma\to\pm\infty} \int_0^{\sigma} \varphi(\lambda)\mathrm{d}\lambda = \infty \end{aligned} \tag{7}$$

Assume that

$$1 - \lim_{s\to 0_+} H_c(s)H_a(s) > 0 \tag{8}$$

and also that the frequency domain inequality

$$1 + \mathrm{Re}\, H_c(\imath\omega)H_a(\imath\omega) > 0 \tag{9}$$

holds for all $\omega > 0$. Then the system has the absolute stability property.

3 A SIMPLE APPLICATION: ROBUSTNESS OF THE NEAL - SMITH CRITERION

We shall consider here one of the cases of (Klyde and Mitchell, 2005), the so-called crossover PVS (Pilot Vehicle System) model with $H_c(s)H_a(s)$ as in (1), the parameters being chosen to satisfy

- crossover frequency $\omega_c = 1.4$ rad/sec,
- neutral stability frequency $\omega_0 = 1.73$ rad/sec,
- phase margin $\Phi_c = 20^o$,
- gain margin $M_c = 4.45$ dB,
- peak magnification ratio 3.39 at 1.48 rad/sec

Remark that all these performance indicators are stated in the frequency domain and three of them deal with open loop characteristics (ω_c,Φ_c,M_c) while the other are concerned with the closed loop properties. Worth mentioning that there are only two free parameters of the model (1) while five conditions are imposed. We may check them as follows. Since ω_c corresponds to 0 dB in the *gain/log* characteristic, it follows that $K/\omega_c = 1$, hence $K = \omega_c$. On the other hand, since the phase margin is 20^o, the phase should be -160^o at the crossover frequency what will give $\tau\omega_c = 70^o$ hence $\tau = 7\pi/(18\omega_c)$. With this (1) is completely determined and we have to check the other properties. In order to verify the gain margin we need the frequency of phase reversal (when the phase equals -180^o. It follows easily that $\tau\omega_\pi = \pi/2$, therefore

$$\omega_\pi = \frac{9}{7}\omega_c \ , \ A(\omega_\pi) = \frac{K}{\omega_\pi} = \frac{\omega_c}{\omega_\pi} = \frac{7}{9} \tag{10}$$

hence $M_c = 20\lg(9/7) = 2.18$ dB. The condition on the neutral stability frequency ω_0 has to indicate a tolerable increase of the gain provided the time lag is kept constant or, conversely, a tolerable increase of the time lag provided the gain is kept constant. The closed loop characteristic equation is

$$s + K\mathrm{e}^{-\tau s} = 0 \tag{11}$$

and the neutral stability will require a pair of zeros of (11) on $\imath\mathbb{R}$ hence the conditions

$$K\cos\,\omega_0\tau = 0 \ , \ \omega_0 - K\sin\,\omega_0\tau = 0 \tag{12}$$

The first equality gives $\omega_0\tau$ which is substituted in the second to obtain the admissible value of K since τ follows by fixing ω_0. We may continue in this way by checking the other conditions. Our aim however is to check the usefulness of the proposed approach by applying Theorem 1. Considering the transfer function of (1) we check the frequency domain inequality (9)

$$1 + \mathrm{Re}\,\frac{K\mathrm{e}^{-\imath\omega\tau}}{\imath\omega} = 1 - K\tau\frac{\sin\,\omega\tau}{\omega\tau} > 0 \ , \ \forall\omega$$

and for it fulfilment it is necessary and sufficient to have $1 - K\tau > 0$ which is exactly (8). Taking into account the computations of the linear case we find that $1 - K\tau$=$1 - \omega_c\tau$=$1 - 7\pi/18 < 0$!. This is quite unpleasant and it deserves some comment. Condition (8) accounts for the so-called *limit stability property* (Răsvan, 1975) - a necessary condition for absolute stability within the sector $(0,\overline{\varphi})$ - exponential stability for linear characteristics within arbitrarily small sector $(0,\varepsilon)$. A more general and less restrictive necessary condition might be the so called *minimal stability* (Popov, 1973) which requires stability for a single linear characteristic within the sector; nevertheless, if limit stability fails this will require a linear characteristic within a sector $(\underline{\varphi},\overline{\varphi})$ with $\underline{\varphi} > 0$ and this is unacceptable since the saturation nonlinearity belongs to the sector $(0,\overline{\varphi})$.

Coming back to the condition $1 - \omega_c\tau > 0$ which does not hold, it follows that robustness assumed in the linear case is not enough to ensure it in the PIO II (system with rate limiter) case. If we require from the beginning $\omega_c\tau < 1$, the phase at the crossover frequency will be larger than $-(1 + \pi/2)$ rad hence the phase margin has to be larger than $\pi/2 - 1$ rad i.e. $\approx 33^o$ - a result that was at some extent expected.

4 CONCLUSIONS

This paper is demonstrating a point of view that seemed very natural when absolute stability i.e. robust global asymptotic stability for systems with sector restricted nonlinear functions was investigated. This point of view is that robust stability of linear systems should imply the same property for nonlinear systems also, at least for those with sector restricted nonlinearities. Such a point of view is transparent throughout all research concerning the so called Aizerman and Kalman problems (Popov, 1973),(Răsvan, 1975) and geometric similarities of the Nyquist and Popov frequency domain criteria strengthened it. Stating it, obviously is not enough; this *position paper* is pointing to critical analysis and further research, mainly application oriented. We have chosen the field of aircraft oscillations to illustrate this point of view for its practical importance (proved by the intense research activities around PIO problem) as well as for its feedback-based modeling of the dynamics: control appears here as a genuine *hidden technology* and hidden paradigm.

Since the field of aircraft dynamics and handling qualities has very strict requirements and procedures, the amount of the necessary research appears to be high and with a certain degree of complexity. The point of view stated here is to be applied possibly to all cases of PIO I i.e. corresponding to fully linearized systems; for the entire set of criteria, see (Anon., 2000),(Klyde et al., 1995). But for each criterion one may wish to consider several cases of pilot models. For *all these cases* we have to consider the PIO II i.e. the nonlinear, rate limited counterpart. But, besides the comparison of the PIO I criteria and of their nonlinear counterpart, a comparison with the other PIO II criteria, obtained independently of the approach presented in this paper is also necessary.

All this analysis and various comparison of the criteria contain the necessary amount of critical assessment of the present position paper proposal. To this we add the specific PIO approach in aircraft studies: conversion in a checkable form and application on "real data" stored in the aviation databases. Nevertheless it is hoped to follow the approach described here in the next research on other PIO criteria.

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