# A NEW APPROXIMATE REASONING BASED ON SPMF

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Abstract: A new approximate reasoning based on standardized parametric membership functions (SPMF) is proposed. It provides an efficient mechanism for approximate reasoning within linear time complexity.

## 1 INTRODUCTION

Approximate reasoning is generally expressed in the form of the following syllogism :

$$\begin{array}{ll} \text{Rule}: & \textit{IF}\ \ \text{X is A}\ \ \textit{THEN}\ \ \text{Y is B} \\ \text{Observation}: & \text{X is A}' \\ \hline \text{Conclusion}: & \qquad\qquad \text{Y is B}' \end{array} \qquad (1)$$

where X and Y are linguistic variables, and A, B and A′ are fuzzy subsets defined in the universe of discourse U, V and U, respectively. Generally, in order to obtain the deduction of conclusions from observations and rules in a knowledge base, there are three alternative ways of doing this in fuzzy sets framework: truth value restriction (Zadeh, 1975), compositional rule of inference (CRI) (Zadeh, 1973) and an approximate analogical reasoning schema (AARS) (Turksen and Zhong, 1988). The CRI has been more widely accepted and applied in development studies. The CRI is essentially based on matrix operation. The effect of such matrix operations on membership function value propagation is not conceptually clear. Some of its undesirable consequences were pointed out in (Mizumoto, 1985, Turksen and Zhong, 1988). It has major flaws that it does not satisfy the modus ponens. That is, when A′=A, the deduced B′ is obtained as follows : $\mu_{B'} = (1+\mu_B)/2 \neq \mu_B$. This inference result indicates that the CRI does not satisfy the modus ponens, i.e., $(A \wedge (A \rightarrow B)) \rightarrow B$, which is quite reasonable demand in the approximate reasoning. In addition, 'indetermination' part of the consequence occurs because of the incompatibility between the membership functions of A in the premise of rule and A′ from observation. This incompatibility happens when the insignificant part (i.e., the zero membership range) of A includes that of A′ (Chang et al., 1991). In the meantime, the AARS uses the term similarity to express the semantics of inference. Here, the similarity of two fuzzy sets is expressed by the following equation: $SM=(1+DM)^{-1}$ where $SM \in [0,1]$. That is, the similarity measure (SM) is obtained by using the distance measure (DM). However, it did not define clearly how to obtain the DM. In addition, it did not define clearly the modification function (MF) that plays an important role in the approximate reasoning. To handle these problems, a new approximate reasoning based on SPMF is proposed.

## 2 SPMF

Let A be a fuzzy set for a linguistic term and be a subset of the universal set X, then, for $x \in X$, a triangular-type membership function can be represented by using 3 points $\mu_A(x_L, x_M, x_H)$, where $x_L < x_M < x_H$, and if the result of this membership function is normalized to [0, 1] then $\mu_A(x_L, x_M, x_H) = 0$ for every $x \in [-\infty, x_L] \cup [x_H, \infty]$ and $\mu_A(x_L, x_M, x_H) = 1$ at $x_M$. A trapezoidal-type can be represented by using 4 points $\mu_A(x_L, x_{I1}, x_{I2}, x_H)$, where $x_L < x_{I1} < x_{I2} < x_H$, and if the result of this membership function is normalized to [0, 1] then $\mu_A(x_L, x_{I1}, x_{I2}, x_H) = 0$ for every $x \in [-\infty, x_L] \cup [x_H, \infty]$ and $\mu_A(x_L, x_{I1}, x_{I2}, x_H) = 1$ at $[x_{I1}, x_{I2}]$. A more comprehensive study of SPMF can be found in (Chang et al., 1991).



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# 3 THE PROPOSED METHOD

A new approximate reasoning based on SPMF makes the DM to compare two fuzzy sets in the pattern matching phase, and then the DM is used to construct the MF. The MF will adjust the right side of the rule in the consequent deducing phase. We first consider a simple rule case where only one observation A′ and one simple rule as in Eq. (1).

## 3.1 Distance Measure (DM)

Based on their behavioral experiment (Zwick et al., 1987), they recommended the five good DM between fuzzy subset A and B of a universe of discourse U. We note that the five good DM concentrate their attention on a single value rather than performing some sort of averaging or integration. We know that the reduction of complicated membership functions to a single 'slice' may be the intuitively natural way for human beings to combine and process fuzzy concepts. Moreover, we know that the DM between two fuzzy subsets can be efficiently represented by a limited number of features. From these ideas, we define the DM based on the structure of the SPMF.

**(1) Triangular-type Membership Functions**

If the antecedent A of a rule is represented by A = ($x_L$, $x_M$, $x_H$) and an observation A′ is represented by A′ = ($x_L'$, $x_M'$, $x_H'$), then each DM is obtained regarding its corresponding 3 points, respectively.

$$DM_L = x_L' - x_L \\ DM_M = x_M' - x_M \qquad (2) \\ DM_H = x_H' - x_H$$

**(2) Trapezoidal-type Membership Functions**

If the antecedent A of a rule is represented by A = ($x_L$, $x_{I1}$, $x_{I2}$, $x_H$) and an observation A′ is represented by A′ = ($x_L'$, $x_{I1}'$, $x_{I2}'$, $x_H'$), then each DM is obtained regarding its corresponding 4 points, respectively.

$$DM_L = x_L' - x_L \\ DM_{I1} = x_{I1}' - x_{I1} \qquad (3) \\ DM_{I2} = x_{I2}' - x_{I2} \\ DM_H = x_H' - x_H$$

## 3.2 Pattern Matching

The pattern matching is achieved through the use of the least distance measure (LDM) between the observed fact and the antecedent of a rule.

**(1) Triangular-type Membership Functions**

$LDM_p = \min\{DM_L + DM_M + DM_H\}$ for all rules in a rule base. (4)

**(2) Trapezoidal-type Membership Functions**

$LDM_I = \min\{DM_L + DM_{I1} + DM_{I2} + DM_H\}$ for all rules in a rule base. (5)

Thus, the rule with $LDM_p$ or $LDM_I$ is selected in a rule base.

## 3.3 Modification Functions (MF)

In the proposed method, a rule $R_i : A_i \rightarrow B_i$ is to be fired with each MF that modifies the consequent $B_i$ of the rule $R_i$. We construct each MF based on its corresponding DM in Eqs. (2), (3). When deducing a consequent, the MF enables us to bypass the matrix operations of CRI. Each MF is achieved by using the following formulas :

**(1) Triangular-type Membership Functions**

Let the maximum support interval (MSI) of two fuzzy subsets (i.e., A, A′) represented by using the triangular-type membership functions be [$x_{LL}$, $x_{MH}$], where $x_{LL}$ is derived from min {$x_L$, $x_L'$} and $x_{MH}$ is derived from max {$x_H$, $x_H'$}, and let the distance of MSI of two fuzzy subsets (DMSI) be $|x_{MH} - x_{LL}|$, then each MF is obtained regarding its corresponding 3 points, respectively.

$$MF_L = (1 + (DM_L/DMSI)) \\ MF_M = (1 + (DM_M/DMSI)) \qquad (6) \\ MF_H = (1 + (DM_H/DMSI))$$

where each DM is derived from Eq. (2).

**(2) Trapezoidal-type Membership Functions**

Let the MSI of two fuzzy subsets (i.e., A, A′) represented by using the trapezoidal-type membership functions be [$x_{LL}$, $x_{MH}$], where $x_{LL}$ is derived from min {$x_L$, $x_L'$}, and $x_{MH}$ is derived from max {$x_H$, $x_H'$}, and let the DMSI of two fuzzy subsets be $|x_{MH} - x_{LL}|$, then each MF is obtained regarding its corresponding 4 points, respectively.

$$MF_L = (1 + (DM_L/DMSI)) \\ MF_{I1} = (1 + (DM_{I1}/DMSI)) \qquad (7) \\ MF_{I2} = (1 + (DM_{I2}/DMSI)) \\ MF_H = (1 + (DM_H/DMSI))$$

where each DM is derived from Eq. (3).

We can determine the overall MF(OMF) by averaging all MF in Eqs. (6) or (7), respectively.

OMF = avg{all MF(A, A′)} where the all MF(A, A′)

are derived from Eqs.(6) or (7), respectively. (8)

In Eq. (8), we consider a simple rule case where only one observation A′ and one simple rule in the form '*IF X is A THEN Y is B*'. The construction of MF is subjective in (Turksen and Zhong, 1988). To handle this problem we suggest the efficient OMF based on the structure of the SPMF.

## 3.4 Deducing a Consequent

It is assumed that we consider a simple rule case where only one observation A′ and one simple rule in the form '*IF X is A THEN Y is B*'. Let B be a fuzzy subset of the linguistic variable '*Y*' and be represented by the SPMF then, for $^{\forall}y \in Y$, the linguistic value B can be represented by ($y_L$, $y_M$ , $y_H$) or ($y_L$, $y_{I1}$, $y_{I2}$, $y_H$) in the triangular-type and trapezoidal-type membership functions, respectively. In the proposed method, we construct the deduced consequent B′ by applying the OMF to B.

**(1) Triangular-type Membership Functions**

$$y_L' = \text{OMF} \times y_L$$
$$y_M' = \text{OMF} \times y_M \qquad (9)$$
$$y_H' = \text{OMF} \times y_H$$ where the OMF is derived from Eq. (8).

**(2) Trapezoidal-type Membership Functions**

$$y_L' = \text{OMF} \times y_L$$
$$y_{I1}' = \text{OMF} \times y_{I1} \qquad (10)$$
$$y_{I2}' = \text{OMF} \times y_{I2}$$
$$y_H' = \text{OMF} \times y_H$$ where the OMF is derived from Eq. (8).

The OMF obtained in the pattern matching phase is applied to the points such as $y_L$, $y_H$, etc, in the consequent deducing phase as in Eqs. (9), (10).

***Definition 1.*** According to Eqs. (2), (3), (6)-(8), in case of a positive dependency (e.g., '*good* → *big*', see Example 1) between *A* and *B* in a rule, the directionality of modification in the consequent deducing phase is determined.

Case 1 : If OMF < 1, then the left shift with OMF occurs regarding all points such as $y_L$, $y_H$, etc.

Case 2 : If OMF = 1, then no shift occurs. As a special case, for a pair (A, A′), if all DM in Eqs. (2) or (3) is zero, then the exact matching occurs between the observed fact A′ and the antecedent A of a rule.

Case 3 : If OMF > 1, then the right shift with OMF occurs regarding all points such as $y_L$, $y_H$, etc.

On the contrary, in case of a negative dependency (e.g., '*high weight* → *low speed*') between *A* and *B* in a rule, the directionality of modification in the consequent deducing phase is determined reversely.

We note that when the special case of Case 2 of the Definition 1 occurs (i.e., A = A′), the reasoning result of the proposed method becomes B′ = B. This is one of the advantages of the proposed method over CRI. In other words, the proposed method satisfies the modus ponens but the CRI does not satisfy the modus ponens.

***Example 1.*** We consider a simple rule case where only one observation A′ and one simple rule in the form '*IF X is A THEN Y is B*'. It is assumed that the selected rule is '*IF economic conditions were good THEN the earning was big*', and one observation is '*economic conditions are good′*'. We assume that the stockholder defines fuzzy subsets regarding the goodness of the linguistic variable *economic conditions* in the interval [0, 100] by using the trapezoidal-type.

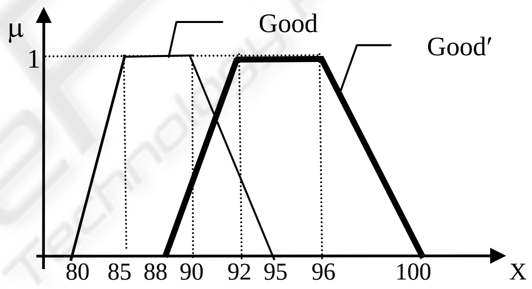


Figure 1: An example of fuzzy subsets regarding the goodness of economic conditions (X).

In Figure 1, the antecedent A of the selected rule is assumed to be '*good*', whereas the observation A′ is assumed to be '*good′*'. In this case, each MF is computed by using Eq. (7).

$\text{MF}_L$ = (1+ ((88-80)/(100-80)) = (1 + (8/20)) = 1.4.
$\text{MF}_{I1}$ = (1+ ((92-85)/(100-80)) = (1 + (7/20)) = 1.35.
$\text{MF}_{I2}$ = (1+ ((96-90)/(100-80)) = (1 + (6/20)) = 1.3.
$\text{MF}_H$ = (1+ ((100-95)/(100-80)) = (1 + (5/20)) = 1.25.

Thus, we obtain the OMF = (1.4+1.35+1.3+1.25)/4 = 1.33 by using Eq. (8). In the meantime, we assume that the stockholder defines the fuzzy subset '*big earning*' as in Figure 2. Using Eq. (10), we construct the deduced consequent B′ by applying the OMF to B as in Figure 2.

$$y_L' = \text{OMF} \times y_L = 1.33 \times \$10 = \$13.3.$$
$$y_{I1}' = \text{OMF} \times y_{I1} = 1.33 \times \$12 = \$15.96.$$
$$y_{I2}' = \text{OMF} \times y_{I2} = 1.33 \times \$13 = \$17.29.$$
$$y_H' = \text{OMF} \times y_H = 1.33 \times \$15 = \$19.95.$$

Thus, we obtain the deduced consequent B′, i.e., the deduced earning = ($y_L'$, $y_{I1}'$, $y_{I2}'$, $y_H'$) = (13.3, 15.96, 17.29, 19.95).

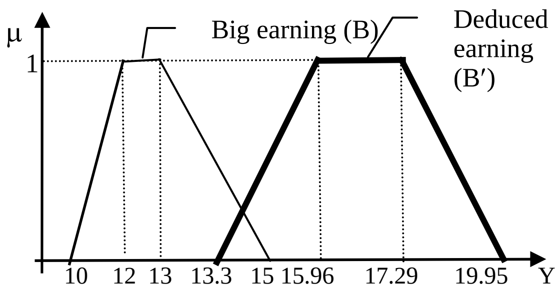


Figure 2: An example of fuzzy subsets.

Now, we consider the composite rules with '*OR*' and '*AND*' connectives.

**(1) '*OR*' Composition**

Given a rule with the following format : [$A_{i1}$ *OR* $A_{i2}$ *OR* …*OR* $A_{iK}$] → $B_i$, it can be decomposed into simple rules as $A_{i1} \rightarrow B_i$ , $A_{i2} \rightarrow B_i$ , … , $A_{iK} \rightarrow B_i$, and can be treated as individual simple rules, respectively (Turksen and Zhong, 1988).

**(2) '*AND*' Composition**

Given a rule with the following format : [$A_{i1}$ *AND* $A_{i2}$ *AND* … *AND* $A_{iK}$] → $B_i$, we can determine the overall MF($OMF_i$) based on Eqs. (6) or (7) by averaging $MF_{ij}$ regarding all corresponding pairs of ($A_{ij}$, $A_{ij}'$), where i denotes the i[th] rule and j = 1,2,…,k. In this case, Eq. (8) is changed into as follows :

$OMF_i$=avg {avg $MF_{ij}(A_{ij}, A_{ij}')$} where each $MF_{ij}(A_{ij}, A_{ij}')$ is derived from Eq.(6) or (7), respectively, and a group of observations has the same form [$A_{i1}'$ *AND* $A_{i2}'$ *AND*… *AND* $A_{iK}'$]. (11)

***Example 2.*** We consider the i[th] rule with '*AND*' connectives in the form '*IF $X_1$ is $A_{i1}$ AND $X_2$ is $A_{i2}$ AND $X_3$ is $A_{i3}$ THEN Y is B*'. For simplicity, let $A_{i1}$ =(1,2,3), $A_{i2}$=(3,4,5,6), $A_{i3}$=(6,7,8), and $A_{i1}'$=(2,3,4), $A_{i2}'$=(4,5,6,7), $A_{i3}'$=(7,8,9), respectively, (i.e., k = 3) then the $OMF_i$ is obtained by using Eqs.(6),(7), (11).

$$OMF_i = \left(\sum_{j=1}^{3} \text{avg}\,(MF_{ij})\right)/3$$

$$=\{[(1+((2-1)/(4-1)))+(1+((3-2)/(4-1)))+(1+((4-3)/(4-1)))]/3+[(1+((4-3)/(7-3)))+(1+((5-4)/(7-3)))+(1+((6-5)/(7-3)))+(1+((7-6)/(7-3)))]/4+[(1+((7-6)/(9-6)))+(1+((8-7)/(9-6)))+(1+((9-8)/(9-6)))]/3\}/3$$

$$= \{(1+(1/3))+(1+(1/4))+(1+(1/3))\}/3 = 1.3.$$

The $OMF_i$ (i.e., 1.3) will be used in the consequent deducing phase as follows :

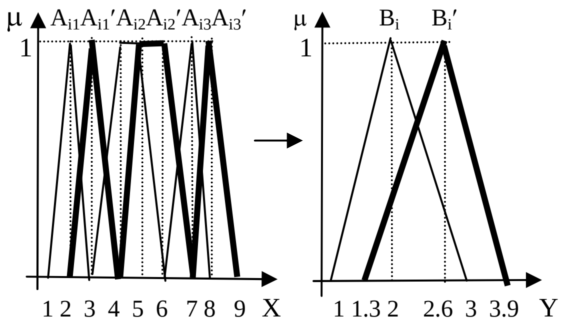


Figure 3: An example of '*AND*' connectives.

## 4 COMPARISONS

Some comparisons are in Table 1.

Table 1: Comparisons.

| Attributes | Existing methods | Proposed method |
|---|---|---|
| Mem. fn. | Ad-hoc | SPMF |
| Method | Genreally, CRI | Based on SPMF |
| Formula | Complex | Simple |
| Operation | Generally, matrix | Linear |

## 5 CONCLUSIONS

The proposed method provides an efficient mechanism for approximate reasoning within linear time complexity.

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