THE LINGUISTIC GENERALIZED OWA OPERATOR AND ITS APPLICATION IN STRATEGIC DECISION MAKING

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Abstract: We introduce the linguistic generalized ordered weighted averaging (LGOWA) operator. It is a new aggregation operator that uses linguistic information and generalized means in the OWA operator. It is very useful for uncertain situations where the available information can not be assessed with numerical values but it is possible to use linguistic assessments. This aggregation operator generalizes a wide range of aggregation operators that use linguistic information such as the linguistic generalized mean (LGM), the linguistic weighted generalized mean (LWGM), the linguistic OWA (LOWA) operator, the linguistic ordered weighted geometric (LOWG) operator and the linguistic ordered weighted quadratic averaging (LOWQA) operator. We also introduce a new type of Quasi-LOWA operator by using quasi-arithmetic means in the LOWA operator. Finally, we develop an application of the new approach. We analyze a decision making problem about selection of strategies.

1 INTRODUCTION

In the literature, we find a wide range of aggregation operators for fusing the information. A very well known aggregation operator is the ordered weighted averaging (OWA) operator (Yager, 1988). The OWA operator has been studied by a lot of authors such as (Merigó, 2007; Yager and Kacprzyk, 1997).

Often, when using the OWA operator, it is considered that the available information is numerical. However, this may not be the real situation found in the decision making problem. Sometimes, the available information is vague or imprecise and it is not possible to analyze it with numerical values. Therefore, it is necessary to use another approach such as a qualitative one that uses linguistic assessments. In (Herrera et al., 1995), they introduced the first linguistic version of the OWA operator. They called it the linguistic OWA (LOWA) operator. Since then, a lot of new developments have been suggested about it such as (Herrera and Herrera-Viedma, 1997; Herrera and Martínez, 2000; Xu, 2004a; 2004b).

Another interesting extension of the OWA operator is the generalization that uses generalized means. This type of aggregation is known as the generalized OWA (GOWA) operator (Karayiannis, 2000; Yager, 2004). It generalizes a wide range of aggregation operators such as the OWA, the ordered weighted geometric (OWG) operator, etc. The GOWA operator has been further generalized (Beliakov, 2005) by using quasi-arithmetic means. The result is the Quasi-OWA operator (Fodor, 1995). For further information on the GOWA operator, see (Merigó, 2007).

The aim of this paper is to develop a generalized OWA operator for situations where the available information can not be assessed with numerical values but it is possible to use linguistic assessments. We will call it the linguistic generalized OWA (LGOWA) operator. This type of linguistic aggregation operator uses the LOWA operator and the generalized mean in the same formulation. Then, it is able to include a wide range of particular cases such as the LOWA itself, the linguistic OWG (LOWG) operator, the linguistic average (LA), the linguistic weighted average (LWA), etc. We further generalize the LGOWA operator by using quasi-arithmetic means. The result is the Quasi-LOWA operator. We should note that recently, a different linguistic Quasi-OWA operator has been studied in (Wang and Hao, 2006). We also develop an application of the new approach in a strategic decision making problem in order to see its implementation in the real life.

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This paper is organized as follows. In Section 2, we briefly comment some preliminary concepts. In Section 3, we present the LGOWA operator. Section 4 analyzes different families of LGOWA operators. In Section 5, we discuss the Quasi-LOWA operator. Section 6 develops a decision making application of the new approach. Finally, in Section 7, we summarize the main conclusions of the paper.

2 PRELIMINARIES

In this Section, we discuss the linguistic approach to be used throughout the paper, the LOWA operator and the GOWA operator.

2.1 Linguistic Approach

Usually, people are used to work in a quantitative setting, where the information is expressed by means of numerical values. However, many aspects of the real world cannot be assessed in a quantitative form. Instead, it is possible to use a qualitative one, i.e., with vague or imprecise knowledge. In this case, a better approach may be the use of linguistic assessments instead of numerical values. The linguistic approach represents qualitative aspects as linguistic values by means of linguistic variables (Zadeh, 1975).

We have to select the appropriate linguistic descriptors for the term set and their semantics. One possibility for generating the linguistic term set consists in directly supplying the term set by considering all terms distributed on a scale on which a total order is defined (Herrera and Herrera-Viedma, 1997). For example, a set of seven terms S could be given as follows:

$$S = \{s_1 = N, s_2 = VL, s_3 = L, s_4 = M, \\ s_5 = H, s_6 = VH, s_7 = P\}$$

Note that N = *None*, VL = *Very low*, L = *Low*, M = *Medium*, H = *High*, VH = *Very high*, P = *Perfect*. Usually, in these cases, it is required that in the linguistic term set there exists:

- A negation operator: Neg(s_i) = s_j such that $j = g+1-i$.
- The set is ordered: $s_i \leq s_j$ if and only if $i \leq j$.
- Max operator: Max(s_i, s_j) = s_i if $s_i \geq s_j$.
- Min operator: Min(s_i, s_j) = s_i if $s_i \leq s_j$.

Different approaches have been developed for dealing with linguistic information such as (Herrera and Herrera-Viedma, 1997; Herrera and Martínez, 2000; Xu, 2004a; 2004b). In this paper, we will follow the ideas of (Xu, 2004a; 2004b). Then, in order to preserve all the given information, we extend the discrete linguistic term set S to a continuous set $\hat{S} = \{s_\alpha \mid s_1 < s_\alpha \leq s_t, \alpha \in [1, t]\}$, where, if $s_\alpha \in S$, we call s_α the original linguistic term, otherwise, we call s_α the virtual one.

Consider any two linguistic terms s_α, $s_\beta \in \hat{S}$, and μ, μ_1, $\mu_2 \in [0, 1]$, we define some operational laws as follows (Xu, 2004a; 2004b):

- $\mu s_\alpha = s_{\mu\alpha}$.
- $s_\alpha \oplus s_\beta = s_\beta \oplus s_\alpha = s_{\alpha+\beta}$.
- $(s_\alpha)^\mu = s_{\alpha^\mu}$.
- $s_\alpha \otimes s_\beta = s_\beta \otimes s_\alpha = s_{\alpha\beta}$.

2.2 LOWA Operator

In the literature, we find a wide range of linguistic aggregation operators (Herrera and Herrera-Viedma, 1997; Herrera et al., 1995; Herrera and Martínez, 2000; Xu, 2004a; 2004b). In this study, we will consider the LOWA operator developed by Xu (2004a; 2004b) with its particular cases that include the linguistic average (LA), among others. Then, we should point out that the LOWA operator we are going to use is also known as the extended OWA (EOWA) operator (Xu, 2004a).

Definition 1. A LOWA operator of dimension n is a mapping LOWA: $S^n \rightarrow S$, which has an associated weighting vector W such that $w_j \in [0, 1]$ and $\sum_{j=1}^{n} w_j = 1$, then:

$$LOWA(s_{\alpha_1}, s_{\alpha_2}, \ldots, s_{\alpha_n}) = \sum_{j=1}^{n} w_j s_{\beta_j} \tag{1}$$

where s_{β_j} is the jth largest of the s_{α_i}.

2.3 GOWA Operator

The GOWA operator (Karayiannis, 2000; Yager 2004) is a generalization of the OWA operator by using generalized means. It includes a wide range of means such as the OWG operator, the ordered weighted quadratic averaging operator (OWQA), etc. It can be defined as follows.

Definition 2. A GOWA operator of dimension n is a mapping $GOWA$:$R^n \rightarrow R$ that has an associated weighting vector W of dimension n such that the sum of the weights is 1 and $w_j \in [0,1]$, then:

$$GOWA(a_1, a_2, \ldots, a_n) = \left(\sum_{j=1}^{n} w_j b_j^{\lambda} \right)^{1/\lambda} \quad (2)$$

where b_j is the jth largest of the a_i, and λ is a parameter such that $\lambda \in (-\infty, \infty)$.

3 LINGUISTIC GENERALIZED OWA OPERATOR

The LGOWA operator is an extension of the OWA operator that uses linguistic information and generalized means. It provides a parameterized family of linguistic aggregation operators that includes the LOWA operator, the linguistic maximum, the linguistic minimum and the linguistic average (LA), among others. It can be defined as follows.

Definition 3. A LGOWA operator of dimension n is a mapping $LGOWA: S^n \rightarrow S$ that has an associated weighting vector W of dimension n such that the sum of the weights is 1 and $w_j \in [0,1]$, then:

$$LGOWA(s_{\alpha_1}, \ldots, s_{\alpha_n}) = \left(\sum_{j=1}^{n} w_j s_{\beta_j}^{\lambda} \right)^{1/\lambda} \quad (3)$$

where s_{β_j} is the jth largest of the s_{α_i}, and λ is a parameter such that $\lambda \in (-\infty, \infty)$.

From a generalized perspective of the reordering step, we can distinguish between the descending LGOWA (DLGOWA) operator and the ascending LGOWA (ALGOWA) operator. The weights of these operators are related by $w_j = w^*_{n-j+1}$, where w_j is the jth weight of the DLGOWA and w^*_{n-j+1} the jth weight of the ALGOWA operator.

The LGOWA operator is a mean or averaging operator. This is a reflection of the fact that the operator is commutative, monotonic, bounded and idempotent. It is commutative because any permutation of the arguments has the same evaluation. It is monotonic because if $s_{\alpha_i} \geq s_{\delta_i}$, for all α_i, then, $LGOWA(s_{\alpha_1}, \ldots, s_{\alpha_n}) \geq LGOWA(s_{\delta_1}, \ldots, s_{\delta_n})$. It is bounded because the LGOWA aggregation is delimitated by the minimum and the maximum: $\text{Min}\{s_{\alpha_i}\} \leq LGOWA(s_{\alpha_1}, \ldots, s_{\alpha_n}) \leq \text{Max}\{s_{\alpha_i}\}$. It is idempotent because if $s_{\alpha_i} = s_{\alpha}$, for all s_{α_i}, then, $LGOWA(s_{\alpha_1}, \ldots, s_{\alpha_n}) = s_{\alpha}$.

Another interesting issue to consider is the attitudinal character of the LGOWA operator. Using a similar methodology as it was used by (Yager, 2004) for the GOWA operator we can define the following measure:

$$\alpha(W) = \left(\sum_{j=1}^{n} w_j \left(\frac{n-j}{n-1} \right)^{\lambda} \right)^{1/\lambda} \quad (4)$$

Note that other measures could be discussed such as the entropy of dispersion, the divergence of W and the balance operator (Merigó, 2007).

4 FAMILIES OF LGOWA OPERATORS

Different families of linguistic aggregation operators are found in the LGOWA operator. Basically, we can classify them in two big groups.

4.1 Analysing the Weighting Vector *W*

By choosing a different manifestation of the weighting vector in the LGOWA operator, we are able to obtain different types of aggregation operators. For example, we can obtain the linguistic maximum, the linguistic minimum, the linguistic generalized mean (LGM) and the linguistic weighted generalized mean (LWGM).

The linguistic maximum is obtained if $w_1 = 1$ and $w_j = 0$, for all $j \neq 1$. The linguistic minimum is obtained if $w_n = 1$ and $w_j = 0$, for all $j \neq n$. More generally, if $w_k = 1$ and $w_j = 0$, for all $j \neq k$, we get for any λ, $LGOWA(s_{\alpha_1}, \ldots, s_{\alpha_n}) = b_k$, where b_k is the kth largest argument a_i. The LGM is found when $w_j = 1/n$, for all a_i. The LWGM is obtained when the ordered position of i is the same than j.

Following a similar methodology as it has been developed in (Merigó, 2007; Yager, 1993), we could study other particular cases of the LGOWA operator such as the step-LGOWA, the window-LGOWA, the olympic-LGOWA, the centered-LGOWA operator, the S-LGOWA operator, the median-LGOWA, the E-Z LGOWA, the maximal entropy LGOWA weights, the Gaussian LOWA weights, the minimal variability OWA weights, the nonmonotonic LGOWA operator, etc.

For example, if $w_1 = w_n = 0$, and for all others $w_{j^*} = 1/(n - 2)$, we are using the olympic-LGOWA that it is based on the olympic average (Yager, 1996). Note that if $n = 3$ or $n = 4$, the olympic-LGOWA is transformed in the median-LGOWA and if $m = n - 2$ and $k = 2$, the window-LGOWA is transformed in the olympic-LGOWA.

When $w_{j^*} = 1/m$ for $k \leq j^* \leq k + m - 1$ and $w_{j^*} = 0$ for $j^* > k + m$ and $j^* < k$, we are using the window-LGOWA operator. Note that k and m must be positive integers such that $k + m - 1 \leq n$.

Another interesting family is the S-LGOWA operator based on the S-OWA operator (Yager, 1993; Yager and Filev, 1994). It can be subdivided in three classes, the "orlike", the "andlike" and the generalized S-LGOWA operator. The "orlike" S-LGOWA operator is found when $w_1 = (1/n)(1 - \alpha) + \alpha$, and $w_j = (1/n)(1 - \alpha)$ for $j = 2$ to n with $\alpha \in [0, 1]$. The "andlike" S-LGOWA operator is found when $w_n = (1/n)(1 - \beta) + \beta$ and $w_j = (1/n)(1 - \beta)$ for $j = 1$ to $n - 1$ with $\beta \in [0, 1]$. Finally, the generalized S-LGOWA operator is obtained when $w_1 = (1/n)(1 - (\alpha + \beta)) + \alpha$, $w_n = (1/n)(1 - (\alpha + \beta)) + \beta$, and $w_j = (1/n)(1 - (\alpha + \beta))$ for $j = 2$ to $n - 1$ where $\alpha, \beta \in [0, 1]$ and $\alpha + \beta \leq 1$. Note that if $\alpha = 0$, the generalized S-LGOWA operator becomes the "andlike" S-LGOWA operator and if $\beta = 0$, it becomes the "orlike" S-LGOWA operator.

4.2 Analysing the Parameter λ

If we analyze different values of the parameter λ, we obtain another group of particular cases such as the usual LOWA operator, the LOWG operator, the LOWHA operator and the LOWQA operator. Note that it is possible to distinguish between descending and ascending orders in all the cases.

When $\lambda = 1$, we get the LOWA operator.

$$LGOWA(s_{\alpha_1}, \ldots, s_{\alpha_n}) = \sum_{j=1}^{n} w_j s_{\beta_j} \tag{5}$$

Note that if $w_j = 1/n$, for all a_i, we get the LA and if the ordered position of $i = j$, the LWA.

When $\lambda = 0$, we get the LOWG operator.

$$LGOWA(s_{\alpha_1}, \ldots, s_{\alpha_n}) = \prod_{j=1}^{n} s_{\beta_j}{}^{w_j} \tag{6}$$

If $w_j = 1/n$, for all a_i, we get the linguistic geometric average (LGA) and if $i = j$, for all a_i, the linguistic weighted geometric average (LWGA).

When $\lambda = -1$, we get the LOWHA operator.

$$LGOWA(s_{\alpha_1}, \ldots, s_{\alpha_n}) = \frac{1}{\sum_{j=1}^{n} \frac{w_j}{s_{\beta_j}}} \tag{7}$$

Note that if $w_j = 1/n$, for all a_i, we get the linguistic harmonic mean (LHM) and if $i = j$, for all a_i, the linguistic weighted harmonic mean (LWHM).

When $\lambda = 2$, we get the LOWQA operator.

$$LGOWA(s_{\alpha_1}, \ldots, s_{\alpha_n}) = \left(\sum_{j=1}^{n} w_j s_{\beta_j}^2 \right)^{1/2} \tag{8}$$

If $w_j = 1/n$, for all a_i, we get the linguistic LQA and if $i = j$, for all a_i, the linguistic weighted quadratic mean (LWQM).

Note that we could analyze other families by using different values in the parameter λ and study these families individually.

5 QUASI-ARITHMETIC MEANS IN THE LOWA OPERATOR

As it is explained in (Beliakov, 2005), a further generalization of the GOWA operator is possible by using quasi-arithmetic means. Following the same methodology than (Fodor et al., 1995), we can suggest a similar generalization of the LGOWA operator by using quasi-arithmetic means. We will call this generalization the Quasi-LOWA operator. Note that this generalization is different than (Wang and Hao, 2006) because it uses a different linguistic approach. The Quasi-LOWA operator can be defined as follows.

Definition 4. A Quasi-LOWA operator of dimension n is a mapping $QLOWA$: $S^n \rightarrow S$ that has an associated weighting vector W of dimension n such that the sum of the weights is 1 and $w_j \in [0,1]$, then:

$$QLOWA(s_{\alpha_1}, \ldots, s_{\alpha_n}) = g^{-1}\left(\sum_{j=1}^{n} w_j g\left(s_{\beta_j}\right) \right) \tag{9}$$

where s_{β_j} is the jth largest of the s_{α_i}.

As we can see, we replace s_β^λ with a general continuous strictly monotone function $g(s_\beta)$. In this case, the weights of the ascending and descending versions are also related by $w_j = w^*_{n-j+1}$, where w_j is the *j*th weight of the Quasi-DLOWA and w^*_{n-j+1} the *j*th weight of the Quasi-ALOWA operator.

Note that all the properties and particular cases commented in the LGOWA operator are also included in this generalization. For example, we could study different families of Quasi-LOWA operators such as the Quasi-LA, the Quasi-LWA, the Quasi-step-LOWA, the Quasi-window-LOWA, the Quasi-olympic-LOWA, etc.

6 APPLICATION IN STRATEGIC DECISION MAKING

In the following, we are going to develop a numerical example about the use of the LGOWA operator in a business decision making problem. We will analyze a strategic decision making problem where an enterprise is analysing which is the most appropriate global strategy for them. We will assume that they consider five alternatives for the next period. As the environment is very uncertain, the group of experts of the enterprise is not able to use numerical information in the analysis. Instead, they will use linguistic information. Note that other decision making applications could be developed with the LGOWA operator such as financial decision making (Merigó, 2007), human resource selection (Merigó, 2007), etc.

Assume an enterprise is analyzing its general policy for the next year and they consider five possible strategies to follow.

- A_1 = Strategy 1.
- A_2 = Strategy 2.
- A_3 = Strategy 3.
- A_4 = Strategy 4.
- A_5 = Strategy 5.

In order to evaluate these strategies, the group of experts considers that the key factor is the economic situation of the company for the next year. After careful analysis, the experts have considered five possible situations that could happen in the future: N_1 = Very bad, N_2 = Bad, N_3 = Regular, N_4 = Good, N_5 = Very good. The linguistic expected results depending on the situation N_i and the alternative A_k are shown in Table 1.

Table 1: Linguistic payoff matrix.

	N_1	N_2	N_3	N_4	N_5
A_1	S_3	S_6	S_2	S_4	S_5
A_2	S_7	S_3	S_1	S_2	S_6
A_3	S_5	S_4	S_4	S_3	S_4
A_4	S_2	S_3	S_6	S_5	S_4
A_5	S_4	S_2	S_7	S_5	S_2

In this example, we assume that the group of experts assumes the following weighting vector for all the cases: W = (0.1, 0.2, 0.2, 0.2, 0.3). Note that this weighting vector will be used as a weighted average in the LWA, but for the LOWA, ALOWA, LOWG and LOWQA, it will be used as the attitudinal character of the enterprise.

With this information, we can aggregate it in order to take a decision. First, we consider some basic linguistic aggregation operators. The results are shown in Table 2.

Table 2: Linguistic aggregated results 1.

	Max	Min	LA	LGA	LQA
A_1	S_3	S_6	S_2	S_4	S_5
A_2	S_7	S_3	S_1	S_2	S_6
A_3	S_5	S_4	S_4	S_3	S_4
A_4	S_2	S_3	S_6	S_5	S_4
A_5	S_4	S_2	S_7	S_5	S_2

As we can see, the decision is different depending on the aggregation operator used.

Now, we are going to consider the results obtained by using other particular cases of LGOWA operators such as the LWA, the LOWA, the ALOWA, the LOWG and the LOWQA operator. The results are shown in Table 3.

Table 3: Linguistic aggregated results 2.

	LWA	LOWA	ALOWA	LOWG	LOWQ
A_1	S_3	S_6	S_2	S_4	S_5
A_2	S_7	S_3	S_1	S_2	S_6
A_3	S_5	S_4	S_4	S_3	S_4
A_4	S_2	S_3	S_6	S_5	S_4
A_5	S_4	S_2	S_7	S_5	S_2

As we can see, in this case we also get different results depending on the aggregation operator used. Note that more particular cases of the LGOWA operator could be considered in the analysis such the ones explained in the previous sections.

Another interesting issue is to establish an ordering of the strategies. Note that this is useful when we want to consider more than one strategy in the analysis. The results are shown in Table 4.

Table 4: Ordering of the strategies.

	Ordering
Max	$A_2=A_5 \succ A_1=A_4 \succ A_3$
Min	$A_3 \succ A_1=A_4=A_5 \succ A_2$
LA	$A_1=A_3=A_4=A_5 \succ A_2$
LGA	$A_3 \succ A_1=A_4 \succ A_5 \succ A_1$
LQA	$A_2 \succ A_5 \succ A_1 \succ A_4 \succ A_3$
LWA	$A_1 \succ A_4 \succ A_3 \succ A_5 \succ A_2$
LOWA	$A_3 \succ A_1=A_4 \succ A_5 \succ A_2$
ALOWA	$A_5 \succ A_1=A_4 \succ A_3 \succ A_2$
LOWG	$A_3 \succ A_1=A_4 \succ A_5 \succ A_2$
LOWQA	$A_5 \succ A_2 \succ A_1=A_3=A_4$

As we can see, depending on the linguistic aggregation used, the ordering is different.

7 CONCLUSIONS

We have presented the LGOWA operator. It is an aggregation operator that uses linguistic information and generalized means in the OWA operator. We have seen that this operator is very useful for situations where the available information can not be assessed with numerical values but it is possible to use linguistic ones. We have studied some of its main properties and we have found a wide range of particular cases. We have seen that it is possible to further generalize it by using quasi-arithmetic means obtaining the Quasi-LOWA operator.

We have applied the new approach in a business decision making problem. We have analyzed the selection of strategies. We have seen that the results and decisions are different depending on the particular LGOWA operator used.

In future research, we expect to develop more extensions of the LGOWA operator by introducing more characteristics in the problem and applying it in different business problems. For example, we could mention the possibility of using different linguistic approaches and the use of different extensions of the OWA operator such as the induced LGOWA operator or the hybrid LGOWA operator.

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