# OPTIMAL LAYOUT SELECTION USING PETRI NET IN AN AUTOMATED ASSEMBLING SHOP

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Abstract: Abstract In today's competitive manufacturing systems, it is crucial to respond quickly to the demand of customers and to decrease total cost of production. To achieve higher performance of automated assembling shop, it is needed to utilize methods to minimize production cycle time (makespan) and work-in-process (WIP) in buffers. This paper intends to focus on the selection of optimal layout based on allocation of machines to different locations as they can perform similar operations with different processing times. The time Petri net (TPN) has been used to illustrate the applications of proposed model in case study.

## 1 INTRODUCTION

Layout designing have been extensively researched in many manufacturing systems. Researches have mainly concentrated on the important class of systems called flow shops, in which components are moved linearly through the system, and manufacturing stations are totally dedicated (Adel and Baz, 2004). Now a days, automatic tools such as computer numerical control (CNC) machines and different types of robots have been used in assembly lines called automated assembling shop. Automated assembling shop consists of several types of CNC machines, robots, and automated guided vehicles designed to produce a great variety of products in multiple lines. Many products can be manufactured and assembled in automated assembling shop. The parts to be assembled are transferred by conveyors and robots. Robots transfer the parts from the conveyors to buffer. The main problem of designing an automated assembling shop is to obtain the minimum production cycle time and WIP (Hsieh et al, 2007).

In the literature, this problem is most often treated as a single objective problem and only the capacity constraints of the assembly shop are considered. For example, Boubekri and Nagaraj (1993) developed an integrated approach for the selection and design of assembly systems. A model for evolutionary implementation of efficient assembly systems was proposed by Rampersad (1994, 1995). But very little has been reported on the design of assembly systems and system layout.

Due to the discrete nature, Petri nets (PN) are widely used for modeling manufacturing systems (Park et al., 2001, Yan et al., 2003). Petri net is a graphical and mathematical modeling tool for describing and studying systems (Jehng, 2002). In the early development of Petri nets (Petri, 1962) and (Peterson, 1981), it was particularly concerned with the description of the causal relationships between events. Much of the early theory, notation, and representation of Petri nets have been developed for discrete event systems. (Ramchandani, 1974) showed how Petri nets could be applied to the modeling and analysis of systems of concurrent components. There have been reports of Petri nets applications in the representation, analysis and control of flexible assembling system/ flexible manufacturing system (Alla et al., 1984), (Cecil et al., 1992), (Muro-Medrano et al., 1992), and (Moore, 1996). Petri nets have been used to model robotic or assembly processes so that a sequence of operations is generated based on the Petri net model. On the other hand, many attempts have also been made to extend and modify conventional Petri nets to enhance their modeling power for assembly systems. This resulted in net variations such as colored Petri nets, control nets, timed Petri nets, and object Petri nets. This paper focuses on the layout

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designing in automated assembling shops. Since some machines can perform different operations in different processing times, the machines are allocated to different locations so that the total production cycle time and WIP are minimized.

# 2 PETRI NET MODELING

A timed-PN is able to describe a time dependent system. Two methods exist to model timing: either timing associated with places (the PN is said to be place-timed Petri net, or P-timed PN), or timing associated with transitions (the PN is said to be transition-timed Petri net, or T-timed PN). It also can be shown that P-timed PNs and T-timed PNs are equivalent, and it is possible to move from one model to the another (Zhang et al., 2005).

This paper addresses a production system that receives an order from customers. According to the order, an initial layout and machine allocation of production line is designed. The automated assembling shop for this model consists of two conveyor robots ($R^{'}$, $R^{''}$). There are nine machines (M1, M2,..., M9), five work pieces (A, B, C, D, E) and fourteen operations ($OP_{1,}$ $OP_2$,..., $OP_{14}$). We consider five buffers in the layout such that their amount of WIP is different. Figure 1 is a graphical representation of material flow in the manufacturing system based on the above information. The machines $M_2$, $M_3$ and $M_8$ can perform either of operations $OP_2$, $OP_3$ and $OP_{12}$ in different processing times. The problem is to find the optimum allocation of these machines for doing these operations to minimize the total production cycle time and WIP.

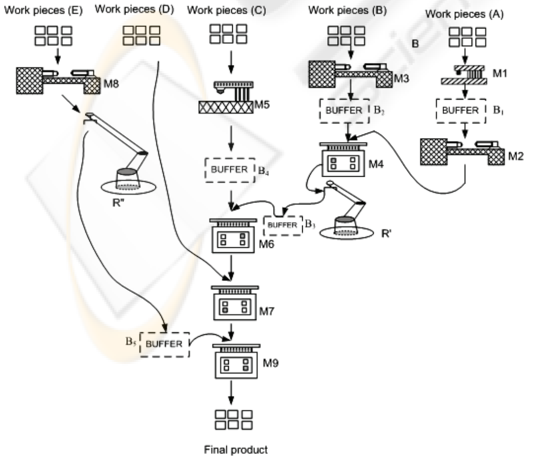

Figure 1: System configuration of automated assembeling shop.

Figure 2 shows a specific allocation of these machines in which operations $OP_2$, $OP_3$ and $OP_{12}$ are performed by machines $M_2$, $M_3$ and $M_8$, respectively.

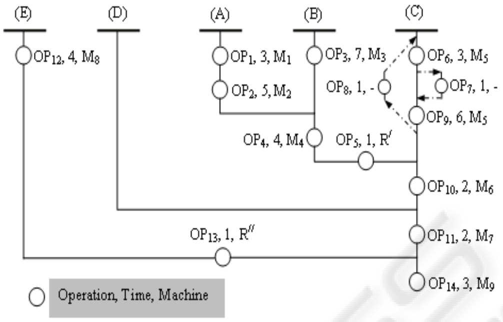

Figure 2: The OPC of automated assembling shop.

In Figure 2, the Operation Process Chart (OPC) of the manufacturing system is shown. The OPC is used for showing the procedure through which work pieces are assembled and all operations in the process of manufacturing system. In OPC, purchased work piece (work piece D) is connected to the basic line (work piece C) by a horizontal line. The operations that are connected to the basic line by dash line ($OP_{7,}$ $OP_8$) represent tool changing in machine $M_5$.

## 2.1 P-timed PN Model of Automated Assembling Shop

For the PN model of automated assembling shop shown in Figure 3. Place-timed Petri nets (P-timed PN) are used to model the system, in which transitions represent events and the places represent states, or conditions.

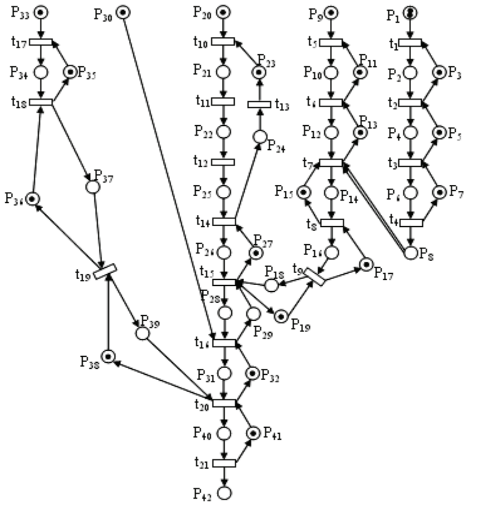

Figure 3: The PN model of automated assembling shop.

The role of transitions and places in the proposed PN model are shown in Tables 1 and 2, respectivly.

Table 1: Role of transitions in the proposed PN model.

| Transitions |
|---|
| $t_1$: Operation OP$_1$ starts |
| $t_2$: Operation OP$_1$ finishes |
| $t_3$: Operation OP$_2$ starts |
| $t_4$: Operation OP$_2$ finishes |
| $t_5$: Operation OP$_3$ starts |
| $t_6$: Operation OP$_3$ finishes |
| $t_7$: Operation OP$_4$ starts |
| $t_8$: Operation OP$_4$ finishes& Operation OP$_5$ starts |
| $t_9$: Operation OP$_5$ finishes |
| $t_{10}$: Operation OP$_6$ starts |
| $t_{11}$: Operation OP$_6$ finishes& Operation OP$_7$ starts |
| $t_{12}$: Operation OP$_7$ finishes& Operation OP$_9$ starts |
| $t_{13}$: Operation OP$_8$ finishes |
| $t_{14}$: Operation OP$_9$ finishes& Operation OP$_8$ starts |
| $t_{15}$: Operation OP$_{10}$ starts |
| $t_{16}$: Operation OP$_{10}$ finishes & Operation OP$_{11}$ starts |
| $t_{17}$: Operation OP$_{12}$ starts |
| $t_{18}$: Operation OP$_{12}$ finishes& Operation OP$_{13}$ starts |
| $t_{19}$: Operation OP$_{13}$ finishes |
| $t_{20}$: Operation OP$_{11}$ finishes& Operation OP$_{14}$ starts |
| $t_{21}$: Operation OP$_{14}$ finishes |

Table 2: Role of places in the proposed PN model.

| Places |
|---|
| $p_1$: Work piece A available |
| $p_2$: Operation OP$_1$ |
| $p_3$: Machine M1 available |
| $p_4$: Work piece A ready for the operation OP$_2$ |
| $p_5$: Buffer of work piece A available |
| $p_6$: Operation OP$_2$ |
| $p_7$: Machine M2 available |
| $p_8$: Work pieces A available to assemble |
| $p_9$: Work piece B available |
| $p_{10}$: Operation OP$_3$ |
| $p_{11}$: Machine M3 available |
| $p_{12}$: Work piece A ready for the operation OP$_4$ |
| $p_{13}$: Buffer of work piece B available |
| $p_{14}$: Operation OP$_4$ |
| $p_{15}$: Machine M4 available |
| $p_{16}$: Operation OP$_5$ |
| $p_{17}$: Robot $R^{/}$ available |
| $p_{18}$: Work piece B ready for the assemble |
| $p_{19}$: Buffer of work piece B available |
| $p_{20}$: Work piece C available |
| $p_{21}$: Operation OP$_6$ |
| $p_{22}$: Operation OP$_7$ |
| $p_{23}$: Machine M5 available |
| $p_{24}$: Operation OP$_8$ |
| $p_{25}$: Operation OP$_9$ |
| $p_{26}$: Work piece C ready for the operation OP$_{10}$ |
| $p_{27}$: Buffer of work piece C available |
| $p_{28}$: Operation OP$_{10}$ |

Table 2: Role of places in the proposed PN model(cont).

| Places |
|---|
| $p_{29}$: Machine M6 available |
| $p_{30}$: Work piece D available |
| $p_{31}$: Operation OP$_{11}$ |
| $p_{32}$: Machine M7 available |
| $p_{33}$: Work piece E available |
| $p_{34}$: Operation OP$_{12}$ |
| $p_{35}$: Machine M8 available |
| $p_{36}$: Robot $R^{//}$available |
| $p_{37}$: Operation OP$_{13}$ |
| $p_{38}$: Buffer of work piece E available |
| $p_{39}$: Work piece E ready for to assemble |
| $p_{40}$: Operation OP$_{14}$ |
| $p_{41}$: Machine M9 available |
| $p_{42}$: Final product available |

# 3 PROPOSED METHOD TO SELECT OPTIMAL LAYOUT

The production cycle time is obtained by MATLAB Petri net toolbox. The maximum WIP (WIP$_{max}$) is calculated according to the maximum number of tokens in buffer places ($p_4$, $p_{12}$, $p_{18}$, $p_{26}$, $p_{39}$). The average work-in-process (WIP$_{average}$) for each buffer can be obtained as discussed below.

We define the following notation:

$i$ : is the number of work pieces $\left(i = 1, 2, \ldots., N\right)$

$j$ : is the number of buffers $\left(j = 1, 2, \ldots., M\right)$

$t$ : is the discrete unit time $\left(t = 1, 2\ldots., T\right)$

$k$ : is the number of allocations $\left(k = 1, 2, \ldots, L\right)$

**Decision Variable**:

$$W_{ijt} = \begin{cases} 1 & \text{If work piece } i \text{ is in buffer } j \text{ at time } t \\ 0 & \text{Otherwise} \end{cases}$$

According to the notations, we obtain the WIP$_{average}$ of $j^{th}$ buffer for each state as given in equation (1).

$$\left(WIP_{average}\right)_j = \left(\overline{WIP}\right)_j = \frac{\sum_{i=1}^{N}\sum_{t=1}^{T} W_{ijt}}{T} \tag{1}$$

We calculate the average WIP of buffer $j$ among all the allocations as given in equation (2).

$$\text{Average WIP within all allocations} = \left(\overline{\overline{WIP}}\right)_j = \left(\frac{\sum_{k=1}^{L}\left(\sum_{i=1}^{N}\sum_{t=1}^{T} W_{ijt}\right)_k}{TL}\right) \tag{2}$$

For allocation *k*, we calculate the value $\left(F_z\right)_k$ as a decision criterion for selection of optimum allocation as given in equation (3).

$$\left(F_z\right)_k = \left(F_x\right)_k + \left(F_y\right)_k \ : \tag{3}$$

where

$$\left(F_x\right)_k = \sum_{j=1}^{M} C_j \left( \left(\overline{WIP}\right)_j - \left(\overline{\overline{WIP}}\right)_j \right)^2 =$$

$$\sum_{j=1}^{M} C_j \left( \left( \frac{\sum_{i=1}^{N}\sum_{t=1}^{T} W_{ijt}}{T} \right) - \frac{\sum_{k=1}^{L} \left( \sum_{i=1}^{N}\sum_{t=1}^{T} W_{ijt} \right)_k}{TL} \right)^2 .$$

$$\left(F_y\right)_k = C_T \left(T_k - T_{\min}\right).$$

$C_j$ : is the cost coefficient of buffer *j*.
$C_T$ : is the cost coefficient of production cycle time.

$$\sum_{j=1}^{M} C_j + C_T = 1 \cdot$$

$T_k$ : is the makespan of allocation *k*.
$T_{\min}$ : is the minimum makespan among all the allocations.
Finally, the allocation with minimum $F_z$ is selected.

## 4 COMPUTATIONAL RESULTS

Assume that ten products are to be produced in the manufacturing system discussed above. Table 3 shows six possible allocations of machines $M_2$, $M_3$ and $M_8$ for doing operations $OP_2$, $OP_3$ and $OP_{12}$.

Table 3: Possible allocations of machines $M_2$, $M_3$ and $M_8$ for operations $OP_2$, $OP_3$ and $OP_{12}$.

| Operation<br>Allocation | $OP_2$ | $OP_3$ | $OP_{12}$ |
|---|---|---|---|
| 1 | $M_3$ | $M_2$ | $M_8$ |
| 2 | $M_3$ | $M_8$ | $M_2$ |
| 3 | $M_2$ | $M_8$ | $M_3$ |
| 4 | $M_2$ | $M_3$ | $M_8$ |
| 5 | $M_8$ | $M_2$ | $M_3$ |
| 6 | $M_8$ | $M_3$ | $M_2$ |

The simulation results of WIP in each buffer and the production cycle time of each allocation have been given in Tables 4 and 5, respectively.

As a managerial consideration, let us assume that the cost coefficient value of production cycle time is 0.4, i.e. $C_T$= 0.4, and the cost coefficient value of each buffer is as shown in Table 6.

Table 4: The average and maximum WIP of each buffer in different allocations.

| Allocation<br>Buffer | | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|---|
| 1 | $WIP_{ave}$ | 4.8 | 4.8 | 3.3 | 3.3 | 1.6 | 1.6 |
| | $WIP_{max}$ | 11 | 11 | 8 | 8 | 5 | 5 |
| 2 | $WIP_{ave}$ | 3.4 | 3.7 | 3.1 | 1.7 | 1.7 | 0.7 |
| | $WIP_{max}$ | 7 | 8 | 7 | 4 | 5 | 2 |
| 3 | $WIP_{ave}$ | 0 | 0 | 0.01 | 0.01 | 0.7 | 0.7 |
| | $WIP_{max}$ | 0 | 0 | 1 | 1 | 2 | 2 |
| 4 | $WIP_{ave}$ | 1.6 | 1.6 | 0.3 | 0.3 | 0.1 | 0.8 |
| | $WIP_{max}$ | 3 | 3 | 1 | 1 | 1 | 1 |
| 5 | $WIP_{ave}$ | 4.3 | 3.8 | 2.4 | 3.8 | 2.1 | 3.1 |
| | $WIP_{max}$ | 8 | 8 | 5 | 8 | 4 | 6 |

Table 5: The production cycle time of each allocation.

| Allocation | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| Cycle time | 155 | 155 | 116 | 116 | 116 | 116 |

Table 6: Cost coefficient value of each buffer.

| Buffer | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| $C_j$ | 0.08 | 0.1 | 0.16 | 0.21 | 0.05 |

Based on the computational results, the values of functions $F_x$, $F_y$ and $F_z$ of each allocation are shown in Table 7.

Table 7: The values of functions $F_x$, $F_y$ and $F_z$ of each allocation.

| Allocation | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| $(F_x)_k$ | 0.64 | 0.67 | 0.12 | 0.14 | 0.29 | 0.41 |
| $(F_y)_k$ | 15.6 | 15.6 | 0 | 0 | 0 | 0 |
| $(F_z)_k$ | 16.2 | 16.3 | 0.12 | 0.1 | 0.29 | 0.41 |

As seen in Table 7, the allocation 3 has resulted in minimum $F_z$ and therefore this layout is selected as the optimum allocation of machines.

## 5 CONCLUSIONS

In this paper, the allocation of machines for doing different operations in an automated assembling shop has been discussed. The system features identical multi-functional machines with different processing times. A P-timed PN is applied for modeling of the manufacturing system. The proposed model is able to determine the average and maximum WIP in different buffers as well as the production cycle time associated with each allocation pattern. The optimal layout is obtained based on minimum $WIP_{average}$ and production cycle time.

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