ON THE USE OF SYNTACTIC POSSIBILISTIC FUSION FOR COMPUTING POSSIBILISTIC QUALITATIVE DECISION

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Abstract: This paper describes the use of syntactical data fusion to computing possibilistic qualitative decisions. More precisely qualitative possibilistic decisions can be viewed as a data fusion problem of two particular possibility distributions (or possibilistic knowledge bases): the first one representing the beliefs of an agent and the second one representing the qualitative utility. The proposed algorithm computes a pessimistic optimal decisions based on data fusion techniques. We show that the computation of optimal decisions is equivalent to computing an inconsistency degree of possibilistic bases representing the fusion of agent's beliefs and agent's preferences.

1 INTRODUCTION

This paper presents a computation of pessimistic decisions based on syntactic possibilistic fusion operations. Qualitative possibilistic decisions can be viewed as a data fusion problem of two particular possibility distributions: the first one representing the beliefs of an agent and the second one representing the qualitative utility (or agent's preferences). A possibilistic decision model (Dubois and Prade, 1995) allows a gradual expression of both agent's preferences and knowledge. The preferences and the available knowledge about the world are expressed in ordinal way. In (Dubois and Prade, 1995), the authors have proposed two qualitative criteria for ordinal decision approaches under uncertainty: the pessimistic and the optimistic decisions criteria. The first one being more cautious than the second one for computing optimal decisions.
A method for computing optimal decisions, based on ATMS, has been proposed in (Dubois et al., 1998). Using the pessimistic criteria, the procedure is translated to a problem tractable by an ATMS (Kleer, 1986a)(Kleer, 1986b). In (Berre, 2000), Le Berre has implemented the optimistic algorithm and the pessimistic one. This implementation can not deal with an important number of variables (Berre, 2000).

The rest of this paper is organized as follow. Section 2 gives a brief backgrounds on possibilistic logic, qualitative decision and data fusion in possibilistic logic. Section 3 contains an efficient and unified way of computing pessimistic qualitative decisions based on syntactic counterpart of data fusion problem. Section 4 concludes the paper.

2 BACKGROUNDS

2.1 Possibilistic Logic

This section gives a brief refresher on possibilistic logic and qualitative decision theory. See (Dubois et al., 1994b) for more details on possibilistic logic.
A possibility distribution (Dubois et al., 1994b) π is a mapping from a set of interpretations Ω into the unit interval [0,1]. $\pi(\omega)$ represents the degree of compatibility of the interpretation ω with available pieces of information.

Given a possibility distribution π, two dual measures are defined on the set of propositional formulas:

- The possibility measure of a formula ϕ, defined by:

$$\Pi(\phi) = max\{\pi(\omega) : \omega \models \phi \text{ and } \omega \in \Omega\} \quad (1)$$

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- The necessity measure of a formula ϕ, defined by:

$$N(\phi) = 1 - \Pi(\neg\phi) \quad (2)$$

A possibilistic knowledge base Σ is a set of weighted formulas:

$$\Sigma = \{(\phi_i, \alpha_i) : i = 1, ..., n\},$$

where ϕ_i is a propositional formula and $\alpha_i \in]0,1]$ represents the certainty level of ϕ_i.
The possibility distribution associated with a weighted formula (ϕ_i, α_i) is (Dubois et al., 1994b): $\forall\omega \in \Omega$,

$$\pi_{(\phi_i,\alpha_i)}(\omega) = \begin{cases} 1-\alpha_i & if\ \omega \not\models \phi_i \\ 1 & \text{otherwise} \end{cases} \quad (3)$$

More generally, the possibility distribution associated with a possibilistic knowledge base Σ is the result of combining possibility distributions associated with each weighted formula (ϕ_i, α_i) of Σ, namely $\forall\omega \in \Omega$:

$$\pi_\Sigma(\omega) = \oplus\{\pi_{(\phi_i,\alpha_i)}(\omega) : (\phi_i, \alpha_i) \in \Sigma\}. \quad (4)$$

where $\oplus$ is in general either equal to the minimum operator (in standard possibilistic logic), or to the product operator (*).

2.2 Qualitative Decision

Let $D = \{l_i\}$ be a set of decision variables, where l_i are distinguished variables of the language L. Let $d \subseteq D$, then the decision $d^\wedge$ is the logical conjunction of literals in the chosen subset. Each set of decision d induces a possibility distribution π_{K_d} in the following way (Dubois et al., 1994a):

$$\pi_{K_d}(\omega) = \begin{cases} 1 \text{ if } \forall(\phi_i,\alpha_i) \in K, \omega \models \phi_i \text{ and } \omega \models d^\wedge \\ min_{(\phi_i,\alpha_i)\in K/\omega\models\neg\phi_i}(1-\alpha_i) \text{ if } \omega \models d^\wedge \\ 0 \text{ if } \omega \not\models d^\wedge \end{cases}$$

Where α_i represents the degrees of necessity of the formulas in the corresponding layers of $K \cup \{(d,1)\}$.
The utility function μ is built over Ω in a similar way:

$$\mu(\omega) = \begin{cases} 1 \text{ if } \forall(\psi_j,\beta_j) \in P, \omega \models \psi_j \\ min_{(\psi_j,\beta_j)\in P/\omega\models\neg\psi_j}(1-\beta_j) \end{cases}$$

where β_j represents a degree of priority of a formulas in P.
Making a decision amounts to choosing a subset d of the decision set D. The objective is to rank-order decisions on the basis of K and P.
The pessimistic utility function is expressed in terms of the possibility distribution π_{K_d} and the utility function μ (Dubois et al., 1999):

$$u_*(d) = min_{\omega\in\Omega} max(1 - \pi_{K_d}(\omega), \mu(\omega)) \quad (5)$$

In the pessimistic case, the decision d must satisfy (Dubois et al., 1997):

$$K^\wedge_\alpha \wedge d^\wedge \vdash P^\wedge_{>(1-\alpha)} \quad (6)$$

The decision d associated with the most certain part of K entails the satisfaction of the goals, even those with low priority. The pessimistic utility u_* of decision d, defined at the syntactic level, takes the form (Dubois et al., 1997):

$$u_*(d) = \begin{cases} max_{K^\wedge_\alpha \wedge d^\wedge \vdash P^\wedge_{>(1-\alpha)}, K^\wedge_\alpha \wedge d^\wedge \neq \perp}\alpha \\ 0 \text{ if } \{K^\wedge_\alpha \wedge d^\wedge \vdash P^\wedge_{>(1-\alpha)}, K^\wedge_\alpha \wedge d^\wedge \neq \perp\} = \emptyset \end{cases}$$

2.3 Fusion in Possibilistic Logic

Let Σ_1, Σ_2 be two possibilistic bases and π_1, π_2 be their associated possibility distributions. Let $\oplus$ be a two-place function whose domain is [0,1]×[0,1], to be used for aggregating π_1 and π_2. The only requirements for $\oplus$ are the following properties (Benferhat et al., 1997):

- $1 \oplus 1 = 1$,
- if $\forall\omega, \omega'$ if $\pi_1(\omega) \geq \pi_1(\omega')$ and $\pi_2(\omega) \geq \pi_2(\omega')$, then $\pi_1(\omega) \oplus \pi_2(\omega) \geq \pi_1(\omega') \oplus \pi_2(\omega')$.

The syntactic counterpart of the fusion of π_1 and π_2 is the following possibilistic base, denoted by $\Sigma_\oplus = \Sigma_1 \oplus \Sigma_2$, which is made of the union of :

- the initial bases, however with new necessity degrees defined by :
  $\{(\phi_i, 1-(1-\alpha_i) \oplus 1) : (\phi_i, \alpha_i) \in \Sigma_1\} \cup$
  $\{(\psi_j, 1-(1-\beta_j) \oplus 1) : (\psi_j, \beta_j) \in \Sigma_2\}$
- and the knowledge common to Σ_1 and Σ_2 defined by :
  $\{(\phi_i \vee \psi_j, 1-(1-\alpha_i) \oplus (1-\beta_j)) : (\phi_i, \alpha_i) \in \Sigma_1$ and $(\psi_j, \beta_j) \in \Sigma_2\}$

The conjunctive operators exploit the symbolic complementarities between sources.
$\oplus$ is said to be a conjunctive operator if $\forall a \in [0,1], \oplus(a,1) = \oplus(1,a) = a$.

The operator minimum (min) is an idempotent conjunctive one. At the syntactic level, the base associated to π_{min}, such that $\pi_{min}(\omega) = min(\pi_1(\omega), \pi_2(\omega))$, is $\Sigma_{min} = \Sigma_1 \cup \Sigma_2$ (Benferhat et al., 1997);

3 COMPUTATION OF QUALITATIVE PESSIMISTIC OPTIMAL DECISIONS BASED ON DATA FUSION TECHNICAL

A good pessimistic decision d maximizing $u_*(d)$ is such that:

$$u_*(d) = min_{\omega\in\Omega} max(1-\pi_{K_d}(\omega), \mu(\omega)) \quad (7)$$

which is equivalent to:

$$u_*(d) = 1 - max_{\omega\in\Omega} min(\pi_{K_d}(\omega), 1-\mu(\omega)) \quad (8)$$

Besides, the syntactic counterpart of $min(\pi_1(\omega), \pi_2(\omega))$ is the possibilistic base $\Sigma_{min} = \Sigma_1 \cup \Sigma_2$. Thus, combining these results, the corresponding base Σ_{min} associated to $min(\pi_{K_d}(\omega), 1-\mu(\omega))$ is the possibilistic base $K \cup n_P \cup \{(d,1)\}$, such that n_P is the possibilistic base corresponding to the utility function $1-\mu(\omega)$.

3.1 Transformations Steps

In this subsection, we define the possibilistic base n_P corresponding to the utility function $1-\mu(\omega)$, from the preferences base P.

Let $P = \{(\phi_i, \alpha_i) : i = 1,..n\}$ be a preferences base. We assume that: $\alpha_0 = 0 < \alpha_1 < ... < \alpha_n$. The following definition gives the possibilistic knowledge base associated with the negation of P.

Definition 1. The negated base of $P = \{(\phi_i, \alpha_i) : i = 1,..n\}$ is a possibilistic base, denoted by n_p, and defined by:

$$n_P = \{(d_i, 1-\alpha_i) : i = 1,...,n\} \cup \{(\perp, 1-\alpha_n)\}$$

where $d_i = \neg\phi_i \vee \neg\phi_{i+1} \vee ... \vee \neg\phi_n$.
The following proposition shows that n_P is indeed encodes the negation of P:

Proposition 1. Let $P = \{(\phi_i, \alpha_i) : i = 1,..n\}$ be a preference base, and n_p its negated base obtained using definition 7. Let μ_P and μ_{n_P} be the utility distributions associated with P and n_P respectively. Then:

$$\forall\omega \in \Omega, \mu_P(\omega) = 1 - \mu_{n_P}(\omega)$$

The obtained base n_P must be put in clausal form. So, we get C_{n_P}.
If α_n is different to 1, then the utility function $1-\mu(\omega)$ is not normalized. In this case, it will be necessary to add contradiction to the possibilistic base C_{n_P} with priority degree $1-\alpha_n$. Let C'_{n_P} be this base. Then C_{n_P} and C'_{n_P} are equivalents.

Lemma 1. Let $\Sigma = \{(\phi_i, \alpha_i), i = 1,n\}$ be a preferences base and let $\alpha_1, \ldots, \alpha_n$ be the distinct valuations appearing in Σ, ranked increasingly : $0 \le \alpha_1, \ldots, \alpha_n \le 1$ and let $\mu(\omega)$ be the utility function associated to the preferences base Σ. Let $n_\Sigma = \{(\psi_i, \beta_i), i = 1,n\}$ be the preferences base associated to the utility function $\mu_{n_\Sigma}(\omega) = 1-\mu(\omega)$. The base $n'_\Sigma = n_\Sigma \cup \{(\perp, 1-\alpha_n)\}$ is equivalent to n_Σ. We have:
$\forall\omega \in \Omega, \mu_{n_\Sigma}(\omega) = \mu_{n'_\Sigma}(\omega)$.

3.2 Computation of Pessimistic Decisions

We recall that:

$$u_*(d) = 1 - max_{\omega\in\Omega}\{\pi_{min}(\omega)\} \quad (9)$$

where $\pi_{min}(\omega) = min(\pi_{K_d(\omega)}, 1-\mu(\omega))$ and $\Sigma_{min} = K \cup C_{n_P} \cup \{(d,1)\}$.
On the other hand, the inconsistency degree of a possibilistic base K, $Inc(K)$ is defined as follow (Dubois et al., 1994b):

$$Inc(K) = 1 - max\{\pi_{K_d}(\omega)\} \quad (10)$$

Proposition 2. Then clearly, the pessimistic utility function associated to decision d is :

$$u_*(d) = Inc(\Sigma_{min})$$

Where $Inc(\Sigma_{min})$ represents the inconsistency degree of the base $K \cup C_{n_P} \cup \{(d,1)\}$.
Then, the computation of optimal pessimistic decisions is obtained using the following algorithm.

Algorithm: Computation of Optimal Pessimistic Decisions
Input : *K:knowledge base,*
n_P:revers preferences base,
N:number of decision variables,
D:set of decisions,
Output : *Decision: optimal decisions,*
Begin
$i := 1$;
$max := 0$;
$Inc := 1$;
For *i=1* **to** *N* **do**
Begin
$Inconsi(K \cup C_{n_P} \cup \{(d_{i\in[1,n]}, 1)\}, Inc, bool)$;
/ $d_i \in D$*/*
if *(bool=true)* **then**
if*(Inc > max)***then**
$max := Inc$;
$Decision := \{d_i\}$;
else

if*(Inc=max)***then**
$Decision := Decision \cup \{d_i\}$;
endif;
endif
endif
end
return $< Decision >$*;*
end

The computation of inconsistency degree is performed by a call to the function $Inconsi(B \cup \{(\neg\phi, 1)\}, Inc, bool)$. This function has three parameters: a stratified knowledge base, an integer representing current inconsistency degree and a boolean variable. More precisely, the function Inconsi is defined as follows:

Function $Inconsi(B \cup \{(\neg\phi, 1)\}, Inc, bool)$ **Input :**

B:stratified base,
ϕ:weighted formula,
n: number of strate in base B,
Output : Inc: inconsistency degree,
bool:boolean,
Begin
$l := 0$; /*initially pointed on the last strate of the base*/
$u := n$; /*initially pointed on first strate of the base*/
$bool := true$;
while $(l < u)$ **do**
Begin
$r := [(l+u)/2]$;
/*pointer uses for dichotomy*/
if($B^*_{\geq\alpha_r} \wedge \neg\phi$ consistent)
/*$B^*_{\geq\alpha_r} = \{\phi_i/\alpha_i \geq \alpha_r\}$*/
then
$u := r - 1$;
/*check inconsistency in most big base*/
else
$l := r$;
/*check the inconsistency delimited by u,l*/
endif
end
if($\alpha_r < inc$) **then** $bool := false$;
else $Inc := \alpha_r$; /*$Inc = N(\phi)$*/
endif
return $< Inc, bool >$
end

4 CONCLUSIONS

The main contribution of this paper is a proposition of a new approach to compute a qualitative pessimistic decision problem. This problem is viewed as the one of computing inconsistency degrees of particular bases in the framework of possibilistic logic. The application exploits the syntactic counterparts of data fusion techniques. Our approach avoids the use of the ATMS to compute the pessimistic optimal qualitative decision developed in (Dubois et al., 1999) which is known to be a hard problem.

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