

NOVEL DIGITAL DIFFERENTIATOR AND CORRESPONDING FRACTIONAL ORDER DIFFERENTIATOR MODELS

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Abstract: This paper proposes a novel first order digital differentiator. The differentiator is obtained by linear mixing of Al-Alaoui operator (Al-Alaoui, 1993) and wide band differentiator (Hsue, 2006). MATLAB simulation results of the proposed differentiator for various sampling frequencies have been presented. The magnitude results are in close conformity to the theoretical results for approximately 78% of the full range. The phase of the new differentiator is almost linear, with a maximum phase error of 8.24°. We have also proposed new operator based fractional order differentiator models. These models are obtained by performing the Taylor series expansion and continued fraction expansion of the proposed operator. Comparisons of the suggested models with the existing models of half differentiators show perceptible improvement in performance of the fractional order circuit. MATLAB simulation results show that the magnitude response of the proposed half differentiator matches with the theoretical results of continuous-time domain half differentiator for almost the whole frequency range and the phase approximates a constant group delay which is desirable for many applications. The major purpose of this paper is to emphasize that fractional order control systems are better than the conventional order systems as the system control performance is enhanced.

1 INTRODUCTION

There are many design approaches for obtaining digital differentiators. Al-Alaoui (Al-Alaoui, 1995) used Simpson's rule to develop stable differentiators. In another paper (Al-Alaoui, 1993), Al-Alaoui has used a linear combination of Simpson's rule and trapezoidal rule to develop differentiator models. Tseng (Tseng, 2001) has proposed the design of fractional order digital FIR differentiator by solving linear equations of Vandermonde form and in (Tseng, 2007), he has proposed the design of FIR and IIR fractional order Simpson digital integrators using binomial series expansion. Zhao *et al.* (Zhao et

al., 2005) have proposed a method for design of fractional order FIR differentiators in frequency domain and have presented simulation results to validate their technique. In (Bhattacharya and Antoniou, 1995), Bhattacharya *et al.* have designed digital differentiators using neural networks. B. Kumar *et al.* (Kumar and Roy, 1988), (Kumar and Roy, 1989), (Reddy et al., 1991) have designed digital differentiators for low, high and midband frequencies respectively. Khan *et al.* (Khan et al., 2004) have proposed higher degree FIR differentiators based on Taylor series expansion. In (Hsue et al., 2004), the bilinear rule is modified to develop 1st and 2nd order having operating frequencies larger than 10 GHz. Schneider *et al.*

(Schneider et al., 1991) have proposed new 2nd and higher order stable s-to-z mapping functions and explored the sources of error in the higher order mapping functions. Work on fractional order systems has been done in (Chen and Moore, 2002),(Chen and Vinagre, 2003),(Xue and Chen, 2002),(Varshney et al., 2002).

In this paper, a new first order s-to-z transformation is proposed which is obtained by using the Al-Alaoui operator and the Hsue *et al.* operator. The idea was to linearly mix two well known approaches to obtain a differentiator which would also follow the ideal differentiator for a large range of frequencies. Both differentiators being of first order and approximating the ideal differentiator for a large range of frequencies, the proposed differentiator results are found to be in close conformity with those of the ideal differentiator. The differentiator models are developed for different values of sampling frequency and their performance compared. The half differentiator models obtained by discretization of the proposed operator are developed and their performance compared with existing half differentiator models (Chen and Moore, 2002) as well as the theoretical result of continuous-time domain half differentiator. MATLAB simulation results are presented to validate the effectiveness of the proposed operator and its differentiator models.

The paper is organized as follows: the new operator is proposed in Section 2. In Section 3, we have developed the fractional order differentiator models for $s = 1/2$. In Section 4, the MATLAB simulation results of the proposed operator and the half differentiators are presented and compared with their ideal counter parts. Section 5 concludes the paper.

2 PROPOSED NEW OPERATOR

The Al-Alaoui operator based integrator in z-domain is

$$H_{al}(z) = \left[\left(\frac{7T}{8} \right) \left(\frac{1 - z^{-1/7}}{1 - z^{-1}} \right) \right] \quad (1)$$

and the integrator obtained by inverting the transformation of a wide-band differentiator in (Hsue, 2006) is

$$H_{Hsue}(z) = \left[\left(\frac{T}{2} \right) \left(\frac{1 + 0.1658z^{-1}}{1 - z^{-1}} \right) \right] \quad (2)$$

where T is the sampling period.

To obtain a differentiator that fits better the ideal differentiator over the entire normalized frequency band, linear mixing of Al-Alaoui differentiator and the wide-band differentiator is performed. The procedure is as follows: first, the transfer functions of the two integrators of eqns. (1, 2) are linearly mixed as in eqn. (3).

$$\begin{aligned} H_{new}(z) &= \alpha H_{Hsue}(z) + (1 - \alpha) H_{al}(z) \\ &= \alpha \left[0.28 \frac{(1 + 0.1658z^{-1})}{(1 - z^{-1})} \right] + (1 - \alpha) \left[\frac{7T}{8} \frac{(1 - z^{-1/7})}{(1 - z^{-1})} \right] \end{aligned} \quad (3)$$

where α , ($0 < \alpha < 1$) determines the contribution of each operator in the new operator.

Second, the transfer function of eqn. (3) is inverted and the resulting transfer function of the new digital differentiator is

$$G_{new}(z) = \frac{(z - 1)}{z(0.875 - 0.375\alpha T) + (0.12495 - 0.04205\alpha T)} \quad (4)$$

Using Jury's stability criterion, the differentiator $G_{new}(z)$ was found to be stable for the condition $\alpha T < 2.25$; $\forall (0 < \alpha < 1)$. Choosing $T = 0.05s$, (sampling frequency = $2\pi * 1/T = 125.7 \text{ rad/sec}$), the transfer function of the new differentiator is:

$$G_{new}(z) = \frac{(z - 1)}{z(0.875 - 0.01875\alpha) + (0.12495 - 0.21025\alpha)} \quad (5)$$

Now, α is varied from 0 to 1 in increments of 0.1. The magnitude response of the proposed differentiator is plotted for different values of α as shown in Fig 1.

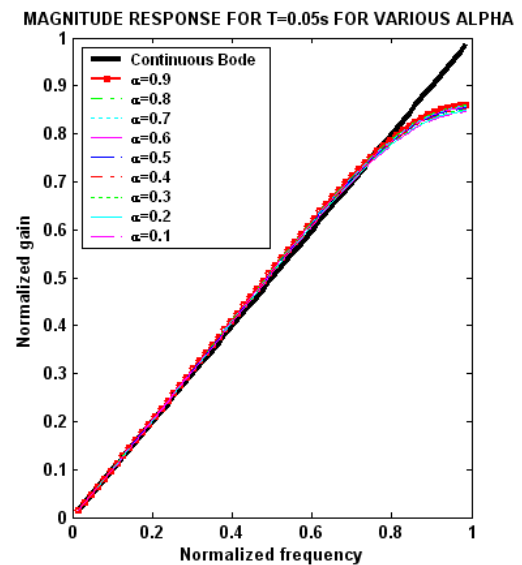


Figure 1: Magnitude response of new operator for various α with $T=0.05s$.

The percentage relative magnitude error of the new differentiator is compared with the magnitude response of the ideal differentiator and plotted in Fig 2.

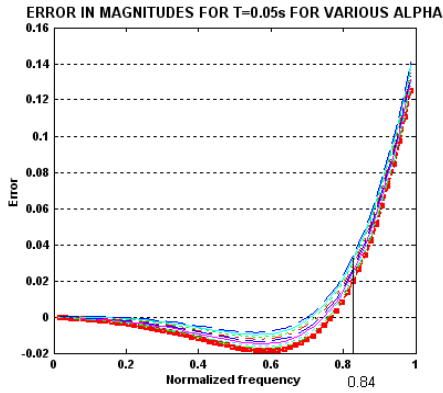


Figure 2: Relative magnitude error as compared to the continuous-time differentiator for various α .

Observations show that best matching with ideal differentiator were for $\alpha = 0.9$. The error is within 2% upto 0.84 of the Nyquist frequency. Fig 3 shows the phase of the new differentiator for different α . The response is almost linear with a maximum phase of 8.24° at 0.55 of the Nyquist frequency. The ideal linear phase corresponds to an ideal differentiator with half a sample of delay. These results are comparable with those of Al-Alaoui operator based differentiator as suggested in (Al-Alaoui, 1993).

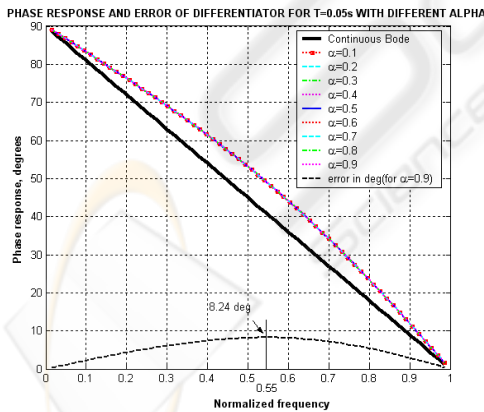


Figure 3: Phase of new operator for various α and corresponding linear phase differentiator and phase error for $\alpha=0.9$.

Using four values of T viz. $0.05s$, $0.00625s$, $0.001s$ and $0.000625s$, and with $\alpha = 0.9$, the transfer functions of the new differentiator are:

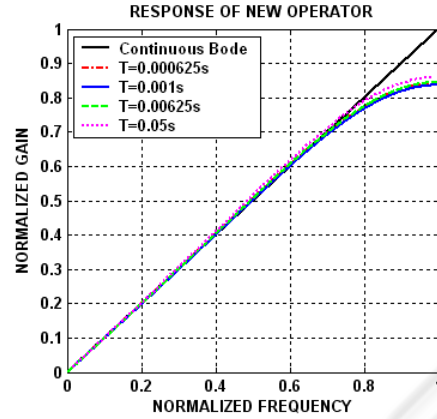


Figure 4: Magnitude response of new operator for different T .

$$G_1(z)|_{T=0.05s} = \frac{23.1901(z-1)}{(z+0.1434)} \quad (6)$$

$$G_2(z)|_{T=0.001s} = \frac{1126.263(z-1)}{(z+0.1428)} \quad (7)$$

$$G_3(z)|_{T=0.00625s} = \frac{182.0131(z-1)}{(z+0.1428)} \quad (8)$$

$$G_3(z)|_{T=0.000625s} = \frac{1814.4141(z-1)}{(z+0.1428)} \quad (9)$$

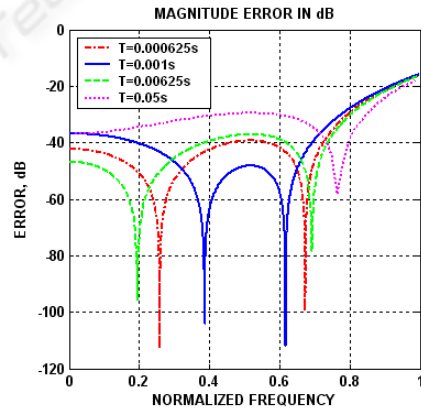


Figure 5: Magnitude error (in dB) as compared to the continuous-time differentiator for different T .

The magnitude response of eqns. (6-9) are plotted and compared with the magnitude of ideal differentiator (Fig 4). The relative magnitude errors are plotted in dB in Fig 5 and in percentage in Fig 6. The phase response of the new differentiator and the relative phase error is plotted in Fig 7.

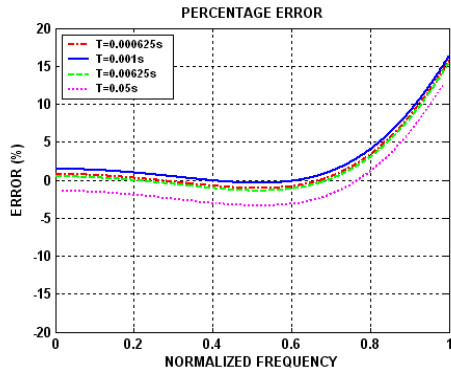


Figure 6: Relative magnitude error (in percentage) for different T.

3 FRACTIONAL ORDER DIFFERENTIATOR MODELS

Next the fractional order differentiator models based on the proposed operator are suggested. Discretization is the key step in the digital implementation of the fractional order controller containing s^r where $(r \in R)$; $0 < r < 1$. The discretization of fractional order differentiator can be expressed by a generating function $s = \omega(z^{-1})$. The generating function is used for obtaining the coefficients and the form of the approximation (Chen and Moore, 2002).

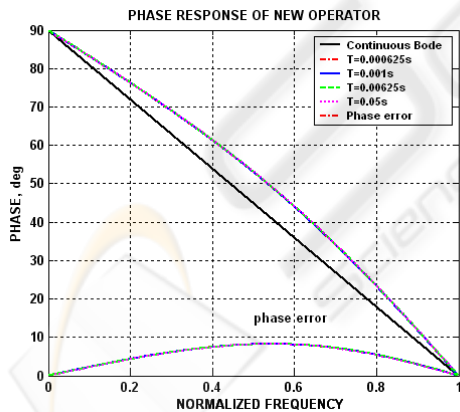


Figure 7: Phase response for different T, corresponding linear phase differentiator and phase error.

In this paper, we have developed the models of half differentiator for various sampling periods using direct discretization method. We have discretized the fractional order derivative using Taylor series expansion (TSE) and continued fraction expansion (CFE).

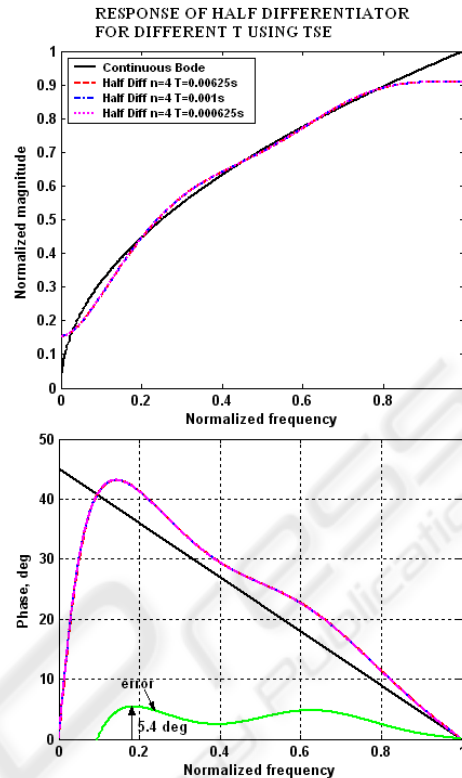


Figure 8: Response of half differentiator obtained by Taylor series expansion of the new operator for different values of T.

In the first method, the TSE of the numerator and denominator polynomials of the transfer function of eqns. (6-9) are performed. Truncating the length of the numerator and denominator expansions, the approximate models of half differentiator for $n = 3 \text{ to } 5$ are obtained. In the second method, continued fraction expansion technique is used to expand the new operator. The continued fraction expansion uses the MATLAB command 'cfrac' (Chen and Moore, 2002), to obtain the models of half differentiator for $n = 3 \text{ to } 5$ by collecting the coefficients of the numerator and denominator polynomials.

3.1 Discretization of New Operator using Taylor Series Expansion

The proposed new operator for $T = 0.001s$ is

$$s = \frac{1126.263(z-1)}{(z+0.1428)} \quad (10)$$

For half differentiator

$$s^{1/2} = \left(1126.263 \left(\frac{z-1}{z+0.1428} \right) \right)^{1/2} \quad (11)$$

Expanding the above eqn. (11) using Taylor series expansion the first 11 terms of the expansion are

$$G_{11}(z) = \frac{\begin{pmatrix} z^{11} - 0.5z^{10} - 0.125z^9 - 0.0625z^8 - 0.0391z^7 \\ -0.0273z^6 - 0.0205z^5 - 0.0161z^4 - 0.0131z^3 \\ -0.0109z^2 - 0.0093z - 0.008 \end{pmatrix}}{\begin{pmatrix} 4.0301e-12z^{11} - 3.268e-11z^{10} + 2.6925e-10z^9 \\ -2.627e-9z^8 + 1.9503e-8z^7 - 1.7384e-7z^6 \\ +1.6232e-6z^5 - 1.624e-5z^4 + 1.8197e-4z^3 \\ -0.0025z^2 + 0.0714z + 1 \end{pmatrix}} \quad (12)$$

Truncating the length of the expansion, the third order half differentiator model is

$$G_3 = \frac{\begin{pmatrix} 33.51z^3 - 16.75z^2 - 4.189z - 2.094 \end{pmatrix}}{\begin{pmatrix} z^3 + 0.07136z^2 - 0.002499z + 0.0001819 \end{pmatrix}} \quad (13)$$

PERCENTAGE ERROR IN MAGNITUDE FOR HALF DIFFERENTIATOR FOR DIFFERENT T (USING TSE)

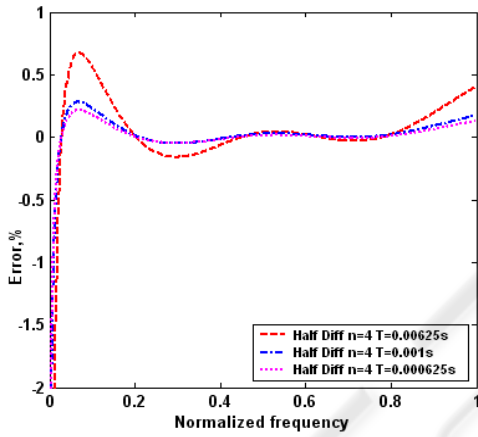


Figure 9: Percentage magnitude error for half differentiator obtained by Taylor series expansion of the new operator for different values of T.

Similar method is used to obtain the half differentiator models for $T = 0.00625s$, $0.000625s$ (Table I). Fig 8 shows the magnitude response and the group delay for the models of half differentiators obtained using TSE for various sampling frequencies. The relative error in phase is also shown in the same figure. The relative magnitude error in percentage is given in Fig 9.

3.2 Discretization of New Operator using Continued Fraction Expansion

The transfer functions of eqns. (6-9) are expanded with $r = 0.5$ using continued fraction expansion to obtain the half differentiator models. The half differentiator models for $T = 0.001s, 0.00625s$ and

$0.000625s$ are listed in Table 1. The magnitude response and group delay for the models of half differentiators obtained using CFE are plotted for various sampling frequencies in Fig 10. The relative error in phase is also plotted in Fig 10. The relative magnitude error (in percentage) is given in Fig 11.

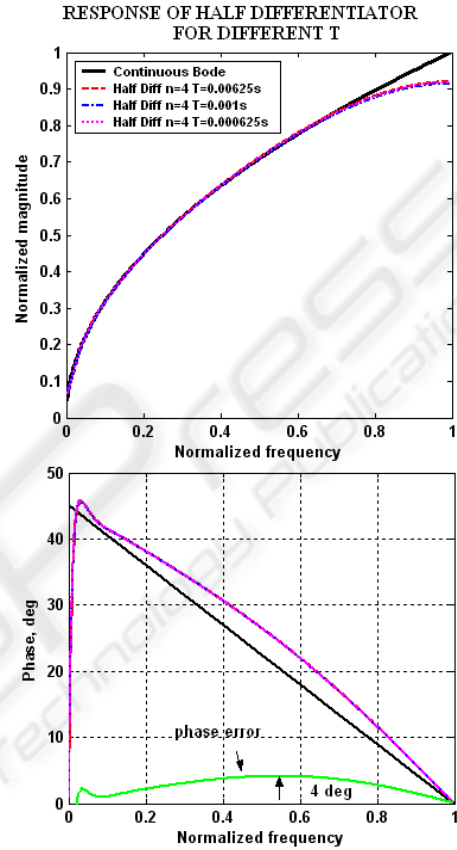


Figure 10: Response of half differentiator obtained by continued fraction expansion of the new operator for different values of T.

4 SIMULATION RESULTS

In this paper, a new operator is proposed by linear mixing of Al-Alaoui operator and the Hsue *et al.* operator. The half differentiator models obtained by discretization of the new operator using Taylor series expansion and continued fraction expansion are also suggested.

The magnitude and phase response of the proposed differentiator are compared with the responses of the ideal differentiator and MATLAB simulation results have been presented to validate the effectiveness of the proposed approach.

Fig 4 shows the magnitude response of proposed differentiator for $T = 0.001s, 0.00625s$ and

0.000625s. The results are compared with the response of ideal differentiator and it matches with the theoretical results for approximately 78% of the frequency range for different sampling frequencies. In Fig 5, the magnitude error is plotted in dB. From the plot, it is observed that the best performance is obtained for $T = 0.000625s$ as error is less than 40dB upto 0.73 of the Nyquist frequency. The results for $T = 0.00625s$ are good for the range from 0 to 0.74, excepting 0.36 to 0.62 of the Nyquist frequency.

PERCENTAGE ERROR IN MAGNITUDE FOR HALF DIFFERENTIATOR FOR DIFFERENT T (USING CFE)

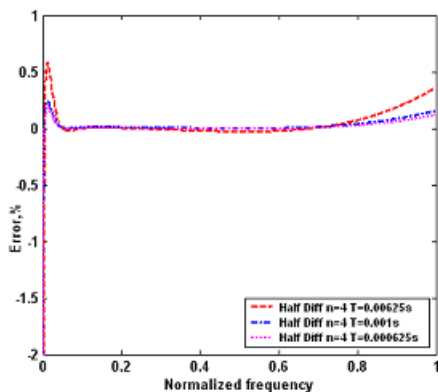


Figure 11: Percentage magnitude error for half differentiator obtained by CFE of the new operator for different values of T.

For $T = 0.001s$, the operational range is limited to the middle frequency range from 0.2 to 0.69 of the Nyquist frequency range. Fig 6 shows the magnitude error in percentage for different sampling frequencies. Fig 7 shows the phase response of the proposed differentiator for different sampling frequencies. It can be seen that the proposed operator has linear phase response with a maximum error of 8.2 deg at 0.55 of the Nyquist frequency.

From the MATLAB simulation results of the half differentiator (Fig 8, 10), it is observed that the magnitude of the models obtained using continued fraction expansion are in close conformity to the theoretical results of half differentiator in continuous-time domain for the full range of frequencies and the phase approximates a constant group delay which is desirable for many applications. The percentage error in magnitude of half differentiator (Fig 9, 11) obtained by continued fraction expansion of the proposed operator is less than 0.5% for the entire range of frequency. Fig 8, 10 reveal that the CFE based models of half differentiator give constant group delay for wider

range of frequency (0.03 to 1 of the Nyquist frequency) as compared to the TSE based half differentiator models (0.15 to 1 of the Nyquist frequency). Moreover the error in phase is less in the CFE based half differentiator models.

In Figs. 12, 13 we present the comparison of the response of the new operator based fifth order half differentiator models with the existing model of fifth order half differentiator based on Al-Alaoui operator for $T = 0.001s$. It is observed that the performance of the new operator based half differentiators is better than that of the Al-Alaoui operator based half differentiator.

COMPARISON OF MAGNITUDE RESPONSES OF FIFTH ORDER HALF DIFFERENTIATORS FOR T=0.001s

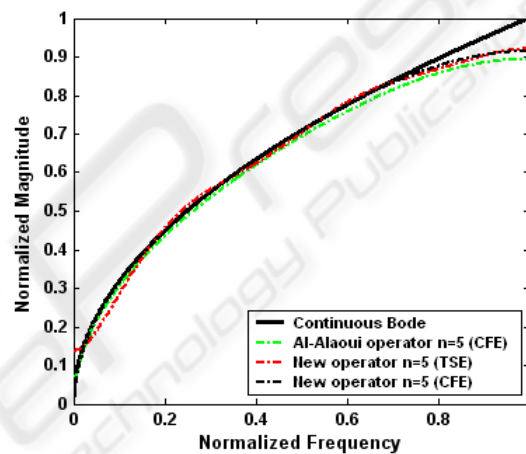


Figure 12: Comparison of magnitude responses (for n=5) of existing half differentiator based on Al-Alaoui operator, the proposed operator and the continuous-time domain half differentiator for $T=0.001s$.

GROUP DELAY OF HALF DIFFERENTIATORS (N=5) FOR T=0.001s

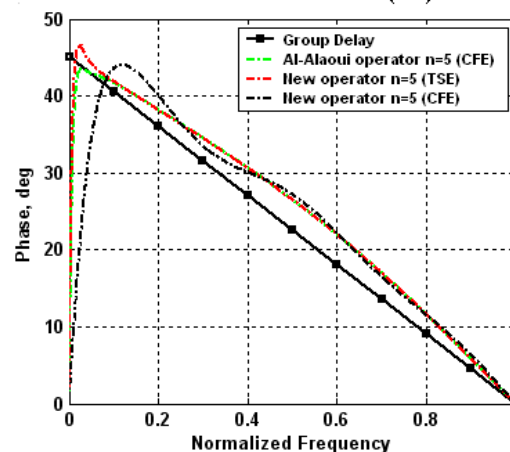


Figure 13: Group delays of the existing half differentiator based on Al-Alaoui operator, and the proposed operator.

5 CONCLUSIONS

In this paper, two well known approaches have been used to develop a new first order s-to-z mapping function. The proposed operator was found to be stable for various sampling frequencies and the magnitude results matched with the ideal differentiator upto 78% of the Nyquist frequency. The phase of the proposed operator also approximates a linear phase of half a sample of delay with a maximum error of 8.24° at 0.55 of the Nyquist frequency.

The half differentiator models obtained by discretization of the proposed operator using continued fraction expansion exhibit better performance in terms of magnitude and phase as compared to those obtained by Taylor series expansion. The above mentioned results of half differentiator validate the effectiveness of the proposed operator. Such modeling finds application in discrete realization of fractional order circuits.

In this paper, z-domain stable models of fractional order differentiators (s^r) have been presented for $r = 0.5$. This method can be further extended to obtain z-domain stable models based on the proposed operator for different r .

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Table 1: Half differentiator models obtained by using CFE and TSE on the proposed operator.

T	HALF DIFFERENTIATOR MODELS USING TAYLOR SERIES EXPANSION	HALF DIFFERENTIATOR MODELS USING CONTINUED FRACTION EXPANSION
0.001s	$G_3 = \left(\frac{33.51z^3 - 16.75z^2 - 4.189z - 2.094}{z^3 + 0.07136z^2 - 0.002499z + 0.0001819} \right)$	$G_3 = \left(\frac{33.56z^3 - 52.74z^2 + 21.23z - 1.273}{z^3 - z^2 + 0.143z + 0.0204} \right)$
	$G_4 = \left(\frac{33.51z^4 - 16.75z^3 - 4.19z^2 - 2.094z - 1.31}{z^4 + 0.07138z^3 - 0.002499z^2 + 0.0001819z - 1.623e - 5} \right)$	$G_4 = \left(\frac{33.56z^4 - 67.12z^3 - 4.11z^2 - 7.244z - 0.06951}{z^4 - 1.429z^3 + 0.49z^2 + 0.005755z - 0.007076} \right)$
	$G_5 = \left(\frac{33.51z^5 - 16.75z^4 - 4.193z^3 - 2.092z^2 - 1.311z - 0.9143}{z^5 + 0.07138z^4 - 0.002499z^3 + 0.0001818z^2 - 1.623e - 5z + 1.623e - 6} \right)$	$G_5 = \left(\frac{33.56z^5 - 81.51z^4 + 67.13z^3 - 20.55z^2 + 1.302z + 0.1337}{z^5 - 1.857z^4 + 1.201z^3 - 0.1226z^2 - 0.02122z + 0.001369} \right)$
0.00625s	$G_3 = \left(\frac{13.49z^3 - 6.745z^2 - 1.687z - 0.8432}{z^3 + 0.07136z^2 - 0.002499z + 0.0001819} \right)$	$G_3 = \left(\frac{13.49z^3 - 21.2z^2 + 8.536z - 0.5116}{z^3 - z^2 + 0.143z + 0.0204} \right)$
	$G_4 = \left(\frac{13.49z^4 - 6.745z^3 - 1.687z^2 - 0.8431z - 0.5276}{z^4 + 0.07138z^3 - 0.002499z^2 + 0.0001819z - 1.623e - 5} \right)$	$G_4 = \left(\frac{13.49z^4 - 26.98z^3 + 16.52z^2 - 2.912z - 0.02795}{z^4 - 1.429z^3 + 0.49z^2 + 0.005755z - 0.007076} \right)$
	$G_5 = \left(\frac{13.49z^5 - 6.744z^4 - 1.688z^3 - 0.842z^2 - 0.5276z - 0.3681}{z^5 + 0.07138z^4 - 0.002499z^3 + 0.0001818z^2 - 1.623e - 5z + 1.623e - 6} \right)$	$G_5 = \left(\frac{13.49z^5 - 32.77z^4 + 26.99z^3 - 8.263z^2 + 0.5233z + 0.05373}{z^5 - 1.857z^4 + 1.201z^3 - 0.1226z^2 - 0.02122z + 0.001369} \right)$
0.000625s	$G_3 = \left(\frac{42.6z^3 - 21.3z^2 - 5.324z - 2.662}{z^3 + 0.0714z^2 - 0.0025z + 0.000182} \right)$	$G_3 = \left(\frac{42.6z^3 - 66.94z^2 + 26.95z - 1.616}{z^3 - z^2 + 0.1429z + 0.0204} \right)$
	$G_4 = \left(\frac{42.6z^4 - 21.3z^3 - 5.326z^2 - 2.662z - 1.666}{z^4 + 0.07138z^3 - 0.002499z^2 + 0.0001819z - 1.624e - 5} \right)$	$G_4 = \left(\frac{42.6z^4 - 85.2z^3 + 52.17z^2 - 9.193z - 0.08823}{z^4 - 1.429z^3 + 0.49z^2 + 0.005755z - 0.007076} \right)$
	$G_5 = \left(\frac{42.6z^5 - 21.29z^4 - 5.33z^3 - 2.659z^2 - 1.666z - 1.162}{z^5 + 0.07137z^4 - 0.002499z^3 + 0.0001819z^2 - 1.624e - 5z + 1.624e - 6} \right)$	$G_5 = \left(\frac{42.6z^5 - 103.5z^4 + 85.2z^3 - 26.09z^2 + 1.652z + 0.1697}{z^5 - 1.857z^4 + 1.201z^3 - 0.1226z^2 - 0.02122z + 0.001369} \right)$