A NOISE REMOVAL MODEL WITH ANISOTROPIC DIFFUSION BASED ON VISUAL GRADIENT

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Abstract: In recent years considerable amount of researchers have been devoted to anisotropic diffusion method and achieved a series of important development. However, human visual system which perceived and interpreted images has been paid little attention to in all these models. In this paper, we define a visual gradient, which is looked as a generalization of the image gradient. After that we substitute the visual gradient for the image gradient in the anisotropic diffusion model to keep to some extent consistent with human visual system for the first time. Finally numerical results show the proposed method's performance.

1 INTRODUCTION

Since Perona (Perona and Malik, 1990) introduced the anisotropic diffusion to image processing and proposed a multiscale smoothing edge detection model first, a considerable amount of researchers have been devoted to theoretical and practical understanding of this and related methods. The idea of anisotropic diffusion is that if the gradient of a point is large the smoothing process will be low and therefore the exact localization of the edges will be preserved. Accordingly, anisotropic diffusion has a good property of eliminating noise while preserving high frequency components.

However, it has some disadvantages. For example, Perona's model is ill-posed and unstable and it is difficult to confirm the model's parameters. In the recent years there are many modified versions of Perona's model have been presented. Nordström (Nordström, 1990) proposed a biased diffusion to regulate the ill-posed nature of the function. Catté et al. (Catté et al., 1992) have given a thorough derivation of the process and proposed a stable scheme for implementation, eliminating the problem of choosing the number of required iterations. Other researches have been achieved by such as Alvarez et al. (Alvarez et al., 1992), You (Yu-Li, Y and Wenyuan 1996) and Barcelos et al. (Barcelos et al. , 2003).

As we all know, all images are eventually perceived and interpreted by the HVS (Human Visual System). But these modified models consider little the influence of human visual system. In fact, Perona's anisotropic diffusion model bases on the gradient of the image which is not integrated the information of human visual system. So we define the visual gradient by using the properties of HVS and develop an anisotropic diffusion model based on visual gradient to remove image noise without losing the boundaries or edges.

2 ANISOTROPIC DIFFUSION

Let u be the representation of the reconstructed image. This representation can be defined as a function of $\Omega \subset \mathbb{R}^2 \to \mathbb{R}$ that associate the pixel $(x, y) \in \mathbb{R}^2$ to its grey level image $u(x, y)$; Ω is the image support (generally, a rectangle). Perona (1990) substituted the standard heat equation by the following anisotropic diffusion equation (for the sake of briefness, in all models we omit boundary and initial conditions):

$$u_t = div(f(|\nabla u|)\nabla u) \tag{1}$$

where f is a non-increasing smooth function such that $f(s) \geq 0$, $f(0) = 1$ and $\lim_{s\to\infty} f(s) = 0$. The idea is that if the gradient is large, then the diffusion will be low, therefore the exact localization of the edges will be preserved.

However, this model still has several theoretical and practical difficulties. For instance, if the image

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is very noisy, the gradient will be very large; as a result the function will be close to zero at almost every point. Consequently almost all noise will remain when we use this model smooth the gravely noisy image.

Aiming to this operator's theoretical and practical difficulties, considerable amount of researchers proposed many modified models. We list some typical modified models and study their properties.

Catté (1992) proposed and studied the following model:

$$u_t = div(g(|\nabla G_\sigma * u|)\nabla u) \qquad (2)$$

Where $\nabla G_\sigma * u$ denotes a convolution of the image at time t with a Gaussian kernel of scale σ which is to be given a priori. This model alleviates Perona's model's ill-posedness, but it inducts a new parameter σ .Moreover proper selection of this parameter is critical to the success of the proposed anisotropic diffusion.

From a geometric point of view, Alvarez et al. (1992) modify the diffusion operator in a way that the diffusion process becomes more intense along the edges and less intense along the perpendicular direction of the edges:

$$u_t = g(|\nabla G_\sigma * u|)|\nabla u| div(\frac{\nabla u}{|\nabla u|}) \qquad (3)$$

After that ,there are other modified model In (Yu-Li,1996; Barcelos et al.,2003. etc.) Following the idea in (Perona, 1990; Nordström, 1990; Alvarez, et al., 1992), Barcelos et al. (2003) proposed the well-balanced equation:

$$u_t = g|\nabla u| div(\frac{\nabla u}{|\nabla u|}) - \lambda(1-g)(u-I) \qquad (4)$$

All these modified models have the same difficulty of selection of Gauss kernel G_σ .As a matter of the fact, using $\nabla G_\sigma * u$ is a regularization process which complicates the model and increases calculation. Furthermore their studies may be more reasonable if they had paid attention to the properties of human visual system.

3 ANISOTROPIC DIFFUSION BASED ON HVS

3.1 Properties of the HVS

As information carriers, all images are eventually perceived and interpreted by human visual system. As a result, human vision psychology and psychophysics play an important role in the successful communication of image information. This is an important new area that needs to be further explored, this paper try to have a first attempt of integrating human visual information into anisotropic diffusion model.

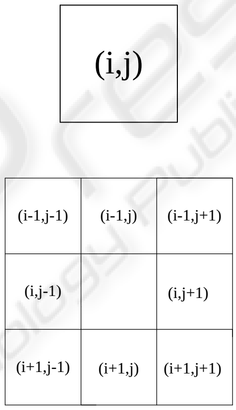

Figure 1: point (i, j) and its eight neighbors.

Weber's law (Pratt,1991) is the famous portrait of the function of human visual system in spatial domain. This law reveals the universal influence of the background stimulus u on human's sensitivity to the intensity increment δg , or the so-called WPD (Weber Perceptive Different) which is denoted as w . It claims that the fraction $\delta g / \bar{g}$ is a constant at a great range of luminance:

$$\delta g / \bar{g} = const \qquad (5)$$

But when the background luminance is very strong or week the fraction is not a constant. In other words, the WPD w is a function of the background luminance $\bar{g}$ .The expression is as following (Lihua, 2005):

$$w(\bar{g}) = \begin{cases} \frac{20-12\bar{g}}{88} & 0 \le \bar{g} < 88 \\ 0.002(\bar{g}-88)^2 + 8 & 88 \le \bar{g} \le 138 \\ \frac{7(\bar{g}-138)}{255-138} + 8 & 138 < \bar{g} \le 255 \end{cases} \qquad (6)$$

In(6) a small neighborhood around each pixel, i.e. a pixel 3×3 (or 5×5 et al.) region is considered (see Figure 1). The region consists of the central pixel (i, j) which gray level is denoted using $g(i, j)$ and a surround consisting of the eight neighboring with a mean gray level, we use the mean gray value, which is denoted using $\bar{g}(i, j)$, of the surrounding eight neighboring points standing for the background luminance:

$$\bar{g}(i,j)=\frac{1}{8}\sum_{l=-1,l\neq0}\sum_{k=-1,k\neq0} g(i+l,j+k) \tag{7}$$

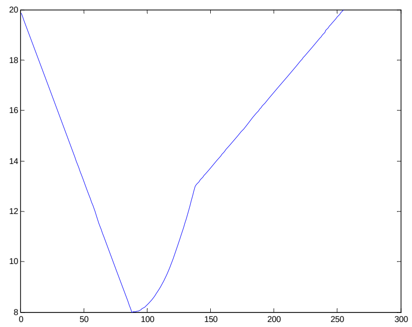

Figure 2: the Weber perceptive different function.

3.2 Visual Gradient

As we all know, anisotropic diffusion model bases on the Gradient of the image. In order to using the information of human visual system, we integrate expression (13) into the gradient and define visual gradient:

$$G_v(g)=\alpha\cdot\nabla g / w(\bar{g}) \tag{8}$$

where ∇g denotes the gradient of the image and $w(\bar{g})$ is the WPD in the background $\bar{g}$, α is an adjustable parameters.

From Figure 2 we know that at the background luminance 88, the WPD is the minimum, that is, human visual system is most sensitive to the gradient, so the corresponding visual gradient must be greater than the gradient. However, at the background luminance 255, human visual system's sensitivity is weakest, so the corresponding visual gradient must be greater than the gradient.

In fact, in the definition of visual gradient if $\alpha\cdot w(\bar{g})\equiv1$, the visual gradient degenerates to general gradient. Therefore the visual gradient is essentially a generalization of image gradient.

3.3 New Model and its Properties

Base on the definition of the visual gradient which is closer to human visual system than gradient, we substitute the visual gradient for gradient in anisotropic diffusion model's diffusion function and get the new model based on the visual gradient:

$$\begin{cases} u_t = div(f(G_v(g))\nabla g) \\ G_v(g)=\alpha\cdot w(\bar{g})\cdot\nabla g \end{cases} \tag{9}$$

where $G_v(g)$, g denote correspondingly the visual gradient, gray level of an arbitrary point and $\bar{g}$ denotes its eight neighbors' mean gray level. In the discrete numerical implementation, they are substituted by $G_v(g(i,j))$, $g(i,j)$ and $\bar{g}(i,j)$. Diffusion function is a non-increasing smooth function such that $f(s)\geq0$, $f(0)=1$ and $\lim_{s\to\infty} f(s)=0$ .Two choices suggested by Perona are

$$f(s)=\exp\left[-(s/k)^2\right] \tag{10}$$

and

$$f(s)=1/(1+(s/k)^2) \tag{11}$$

where k is a constant to be tuned for a particular application.

In the proposed method, we use visual gradient rather than image gradient controlling the anisotropic diffusion model's diffusion coefficient.

Ours idea is that in the different background the gradient is also different and the WPD must obey some rule. Triggered by this idea, we use the expression (6) generalizing the gradient to keep to some extent consistent with human visual system.

4 EXPERIMENTAL RESULTS

We use Perona's discrete scheme(Perona, 1990) and obtain the formulation:

$$u_{i,j}^{n+1}=u_{i,j}^{n}+\lambda f(G_v{}^n(i,j))\left[\nabla_N u+\nabla_S u +\nabla_E u+\nabla_W u\right]_{i,j}^{n} \tag{12}$$

where $0<\lambda<1/4$ for the numerical scheme to be stable and

$$\nabla_N u_{i,j}=u_{i-1,j}-u_{i,j}, \nabla_S u_{i,j}=u_{i+1,j}-u_{i,j}$$
$$\nabla_E u_{i,j}=u_{i,j+1}-u_{i,j}, \nabla_W u_{i,j}=u_{i,j-1}-u_{i,j}$$

The conduction function is updated at every iteration as the function of the visual gradient(10) (11):

$$f\left(G_v{}^n(i,j)\right)=\exp\left[-\left(G_v{}^n(i,j)/k\right)^2\right]$$

or

$$f(G_v^n(i,j)) = 1/(1+(G_v^n(i,j)/k)^2)$$

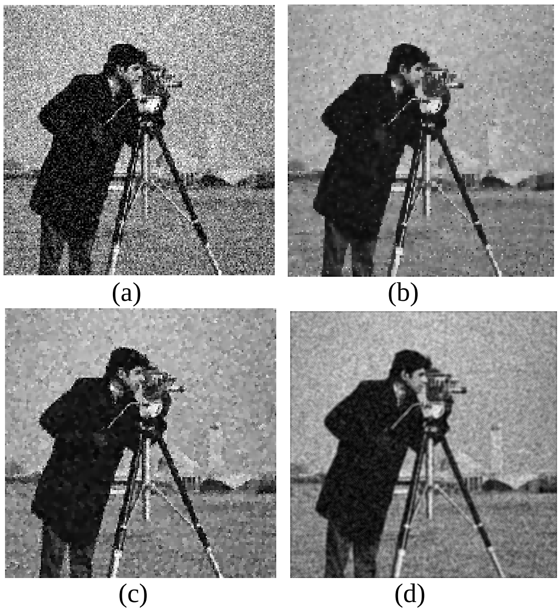

Figure 3: different noise removal models' experimental results. (a) blurred image by Gauss noised with zero mean and 0.01 variance, (b) filtered by Perona's model with iteration 14,(c) smoothed by ALM model with iteration 25,(d)smoothed by the proposed model with iteration 8.

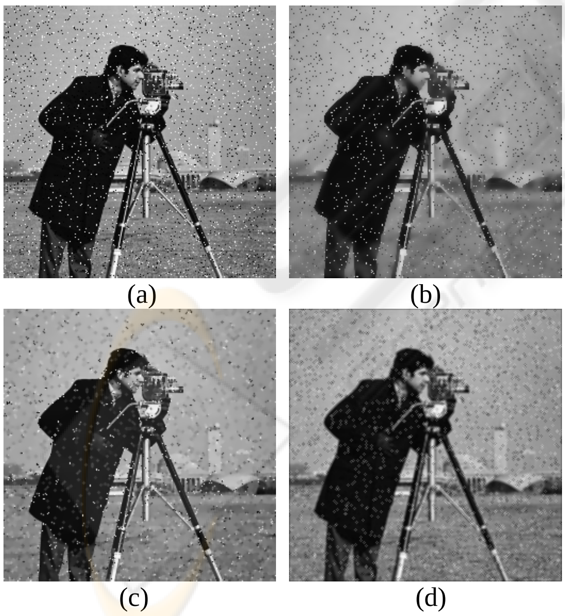

Figure 4: different noise removal models' experimental results.(a) noised image by salt & pepper noise which density is 0.05, (b) smoothed by Perona's model with iteration 17, (c) filtered by ALM model with iteration 45, (d) smoothed by the proposed model with iteration 10.

In the experiments, we applied Perona's model, ALM model (Alvarez, et al., 1992) and the proposed model to smooth the cameraman image with zero mean and 0.02 variance noise (Figure 3). In Figure 4 we use salt & pepper noise which density is 0.05 blurred the origin image. Among the different filtered images the reconstructed image using the proposed model keep to more extent consistent with human visual system.

5 CONCLUSIONS

In this work, an anisotropic diffusion model for image smoothing based on the visual gradient is presented. Our model uses a visual gradient which is a generalization of the image gradient. Numerical results show the proposed method's performance.

As a tentative study of integrating HVS information into anisotropic diffusion model for the first time, the proposed model's performance is expected to be improved in the further researches.

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