CLASSIFYING AND COMPARING REGULAR TEXTURES FOR RETRIEVAL USING TEXEL GEOMETRY

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Abstract: Regular textures can be modelled as consisting of periodic patterns where a fundamental unit, or texel, occurs repeatedly. This paper explores the use of a representation of texel geometry for classification and comparison of regular texture images. Texels are automatically extracted from images and the distribution of texel shape and orientation is modelled. The application of this model to image retrieval and browsing is discussed using examples from a database of art and textile images.

1 INTRODUCTION

Regular textures can be modelled as consisting of periodic patterns where a fundamental unit (texel) occurs repeatedly. Texture periodicity analysis has attracted much attention recently and has been used for texture tracking (Lin et al., 2007), synthesis (Charalampidis, 2006), and retrieval (Liu et al., 1996; Lin et al., 1999; Lee et al., 2005).

In common with much of the previous work, this paper focuses on the study of so-called *wallpaper* patterns. There exist 17 wallpaper groups which together account for all patterns generated by two linearly independent vectors (Liu et al., 2004). Here, regular textures generated by translation only are considered, as shown in Figure 1. A pair of vectors with shortest length (two linearly independent directions), $(\mathbf{t}_1, \mathbf{t}_2)$ define a parallelogram which is called the texel. The texel repeatedly tiles the image to form a lattice structure. $\mathbf{t}_1$ and $\mathbf{t}_2$ define the size, shape, and orientation of the texel.

Texel extraction is key to understanding regular texture. Starovoitov et al. (1998) used features derived from co-occurrence matrices to extract the texel. Charalampidis (2006) achieved this in the frequency domain based on the assumption that fundamental frequencies hold the basic structure information of regular texture. Lin et al. (1997) obtained texels by detecting salient peaks in the autocorrelation (AC) function of a texture image. Liu et al. (2004) extended the work of Lin et al. (1997) by adopting more dominant peaks of the AC function.

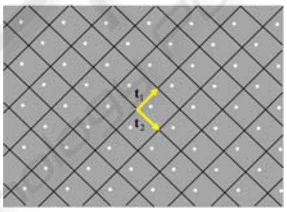

Figure 1: A wallpaper pattern example with its two placement vectors and lattice structure.

Several applications are based on the results of texel extraction from regular texture. Chetverikov (2000) and Leu (2001) measured the regularity degree of images using features derived from the AC function and similarity among texels, respectively. The regularity measurement can be applied to classify regular and irregular texture images. Texture image retrieval and browsing systems have been proposed in which the features used are related to texture periodicity (Liu et al., 1996; Lin et al., 1999; Lee et al., 2005). Charalampidis (2006) implemented texture synthesis using extracted texels. Lin et al. (2006) designed a geometric regularity score that depended on both the magnitudes and directions of $\mathbf{t}_1$ and $\mathbf{t}_2$ to evaluate various texture synthesis algorithms. Recently, Hays et al. (2006) and Lin et al. (2007) extended regular texture models to extract and track texels of *near*-regular texture, respectively.

As can be seen from Figure 1, texel geometry indicates the spatial arrangement of a regular texture. Lin et al. (2006) adopted texel geometric information for the purpose of comparing synthesized texels and original texels. The

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comparison was based on a Euclidean distance without taking the intrinsic distribution of texels into consideration.

This paper presents a method that takes advantage of the geometric information from texels to retrieve and browse images. Firstly, texels are automatically extracted from regular texture images. Then, aspects of the texel geometry are represented as feature vector. Based on the distribution of a collection of images in the resulting feature space, clusters are defined such that each cluster corresponds to a type of texel. Each cluster is modelled as Gaussian and Bayes' rule is used to estimate the probability that a regular texture has a certain texel type. The estimated distributions are also used to measure similarity between texels. Finally, the proposed techniques are applied to perform image retrieval and browsing.

The main contributions of this paper are: 1) A 3D representation is proposed to characterize texel geometry and embody the spatial arrangement information of regular texture in a manner that is invariant to translation and scaling in the image plane. 2) Texel clusters are defined and modelled based on the distribution of a collection of data. 3) Instead of using Euclidean distance, texel comparisons are made based on the probabilities that the image belongs to each cluster and the intrinsic cluster distributions. Finally, we show how these methods can be applied to image retrieval and browsing.

The rest of the paper is organized as follows. Section 2 summarises the texel extraction algorithm. Section 3 proposes a model of texel types based on the distribution of a collection of images. Section 4 applies the model to image browsing and retrieval. Experiments are presented in Section 5. Finally, conclusions are drawn in Section 6.

2 TEXEL EXTRACTION

A previously published method (Han et al., 2008) was used to extract texels, i.e. to estimate ($\mathbf{t}_1, \mathbf{t}_2$). The algorithm is described here briefly for completeness.

The texel extraction algorithm contains two steps: texel hypotheses generation and hypothesis comparison. The first step begins by computing the AC function. Peaks in AC functions are always associated with texture periodicity. Following the ideas of Lin et al. (1997) and Liu et al. (2004), salient AC peaks are selected and used to obtain texels. Changing the number of peaks considered can result in different texel candidates.

The second step compares all of the texel candidates obtained from the first step using a Bayesian model comparison framework. Let I be an image and $H \equiv (\mathbf{t}_1, \mathbf{t}_2)$ denote a texel hypothesis for I, H_k the k^{th} in a set of hypotheses, and M_k a statistical model defined based on H_k with parameters θ_k. Texel extraction can be formulated as choosing the most probable texel hypothesis given the image. By Bayes' theorem, the posterior probability is proportional to the likelihood of the hypothesis times a prior:

$$p(H_k \mid I) = \frac{p(I \mid H_k)\,p(H_k)}{p(I)} \propto p(I \mid H_k)\,p(H_k) \qquad (1)$$

In the absence of prior knowledge favouring any particular hypothesis, the prior is taken to be uniform. For each H_k, we define a unique M_k deterministically so $p(M_k \mid H_k)$ is a delta function. Hence,

$$p(H_k \mid I) \propto p(I \mid M_k) = \int p(I \mid \theta_k, M_k)\,p(\theta_k \mid M_k)\,d\theta_k \qquad (2)$$

The integral in Eq. (2) can be approximated using Bayes Information Criterion (BIC). The details of BIC approximation can be found in Raftery (1995). The BIC for the model is:

$$BIC(M) = -\log p(I \mid \hat{\theta}, M) + (d/2)\log N \approx -\log p(I \mid M) \qquad (3)$$

where d is the number of parameters and $\hat{\theta}$ is a maximum likelihood parameter estimate.

The hypothesis with the model that has the largest marginal likelihood is selected. Using the BIC approximation, hypothesis H_k is selected by

$$\hat{k} = \underset{k}{\operatorname{argmax}}\, p(H_k \mid I) = \underset{k}{\operatorname{argmin}}\{BIC(M_k)\} \qquad (4)$$

The texel model M_k should be able to account for both regularity from periodic arrangement and statistical photometric and geometric variability. Here a Gaussian with covariance matrix of the form $\sigma^2\mathbf{I}$ was used to model a texel's appearance. The reader is referred to Han et al. (2008) for further details.

3 TEXEL GEOMETRY

This paper focuses on modelling the geometry of a texel, ($\mathbf{t}_1, \mathbf{t}_2$), and not its pixel values. We opt for a representation that is scale invariant since the physical scale of the imaged objects is unknown.

The following three features are used to describe the spatial arrangement:

- α : the angle between $\mathbf{t}_1$ and the image x-axis;
- ϕ : the angle between $\mathbf{t}_1$ and $\mathbf{t}_2$;
- r : the ratio of lengths, i.e. $r = \frac{|\mathbf{t}_1|}{|\mathbf{t}_2|}$.

Note that the angle between $\mathbf{t}_1$ and the x-axis is not larger than the angle between $\mathbf{t}_2$ and the x-axis, by construction. Figure 2 shows an example. The value of α ranges from 0 to 90 degrees. $\mathbf{t}_2$ is the texel vector that subtends the smallest angle with $\mathbf{t}_1$, and that angle is ϕ . The value of ϕ for a wallpaper pattern always lies between 60 and 90 degrees.

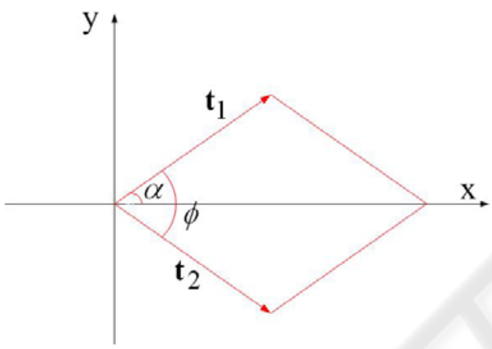


Figure 2: An example of the geometry of a texel.

The three-dimensional feature vector (α, ϕ, r) is automatically extracted from each regular texture image using the method described in Section 2. A distribution of 200 images in this 3D feature space is shown in Figure 3. See Section 5 for details of the dataset. Inspection of this distribution suggests clusters. Each cluster can be considered to correspond to a type of texel. Specifically, five clusters of texels might be defined according to the following five rules:

- Cluster 1: texels are rectangles with $\alpha \approx 0^\circ$, $\phi \approx 90^\circ$;
- Cluster 2: texels are parallelograms with $\alpha \leq 45^\circ$, $\phi < 90^\circ$;
- Cluster 3: texels are parallelograms with $\alpha > 0^\circ$, $\phi \approx 90^\circ$;
- Cluster 4: texels are parallelograms with $\alpha > 45^\circ$, $\phi < 90^\circ$;
- Cluster 5: texels are parallelograms with $\alpha \approx 0^\circ$, $\phi < 90^\circ$.

Any image in the dataset can be classified into a cluster based on the defined rules. However, a model of the cluster *distributions* is more useful, enabling the clusters to be parameterised and meaningful texel similarity measures to be defined. Each cluster can be modelled as a three-dimensional Gaussian distribution with a probability density function

$$p(\mathbf{x} | C_i) = (2\pi)^{-3/2} |\mathbf{\Sigma}|^{-1/2} \cdot \exp\left\{-\frac{1}{2}(\mathbf{x}-\boldsymbol{\mu}_i)^T \mathbf{\Sigma}_i^{-1} (\mathbf{x}-\boldsymbol{\mu}_i)\right\} \tag{5}$$

where $\mathbf{x} = (\alpha, \phi, r)$ denotes the feature vector of an image, $C_i, i \in \{1,2,3,4,5\}$, denotes the cluster index or class, $\boldsymbol{\mu}_i$ denotes the mean for class *i*, and $\mathbf{\Sigma}_i$ denotes the covariance matrix for class *i*. The parameters $\boldsymbol{\mu}_i$ and $\mathbf{\Sigma}_i$ can be estimated using maximum likelihood. The class posterior probability can then be estimated via Bayes' theorem,

$$P(C_i | \mathbf{x}) = \frac{P(\mathbf{x} | C_i) P(C_i)}{P(\mathbf{x})} \tag{6}$$

where the prior $P(C_i)$ can be estimated from the frequencies of the classes in the data.

4 IMAGE BROWSING AND RETRIEVAL

Due to the rapidly growing number of digital images in our lives, there is a great need for effective image retrieval techniques. Content-based image retrieval using image features such as color, shape, and texture can be effective when the user has a query image to hand. However, when the user's intention is ambiguous, image browsing can be more useful. Browsing supposes that the images can be categorized and ordered in meaningful ways. In the case of retrieval and browsing of images exhibiting regular texture, the spatial arrangement is obviously quite an important feature. In this section, we illustrate how the technique for describing and modelling texel geometry (the spatial arrangement of regular texture) can be applied to content-based retrieval and browsing.

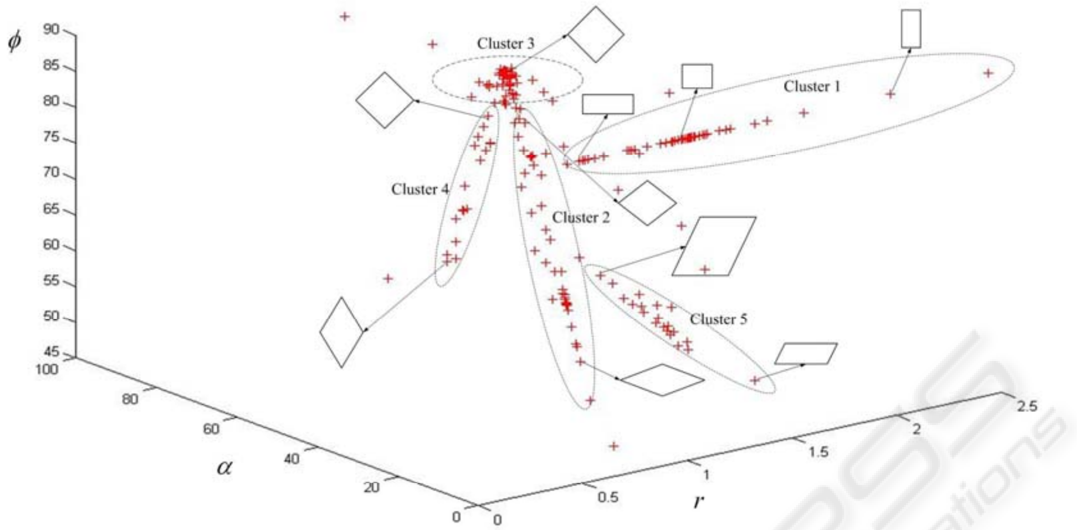


Figure 3: A distribution of 200 regular texture images in the 3D texel feature space.

4.1 Browsing a Texel Class

One approach to organising an image database for browsing is to categorise the images and to then display images within a category in a meaningful way.There are then two problems: (i) how to categorise images, and (ii) how to lay out images within a category meaningfully for display. It is proposed that regular texture images can be categorised according to texel geometry. As shown in Figure 3, data points within a cluster tend to be scattered along a one-dimensional trend. This corresponds to the direction of maximal intra-class variance which is given by the principal component of the class distribution. This direction gives a good feature for intra-class discrimination and motivates projecting data onto these principal components. Ranking images according to the projected values will reflect the intra-class variation of texel geometry. More formally,

1. Given a set of training images from a texel class, estimate the class mean $\boldsymbol{\mu}$, the covariance matrix $\boldsymbol{\Sigma}$, and the eigenvector $\mathbf{v}$ of this matrix that corresponds to the largest eigenvalue λ.
2. For each test image from the same texel class, project the texel $\mathbf{x} = (\alpha, \phi, r)$ onto the first eigenvector: $y = \mathbf{v}(\mathbf{x} - \boldsymbol{\mu})^{\mathbf{T}}$.
3. List the images in ascending order of their y value.

Figure 4 shows an example of browsing an image class. The original image is shown first, followed by its lattice structure. In this category, there were a total of 60 images. Due to limited space, only a few images are shown. The first row shows the three images placed at the front of the ranking list, the second row shows the middle three, and the third row shows the last three images. Texels of this image class differ mainly in the value of ϕ which varies from approximately 60 degrees to approach 90 degrees. Projecting the image data to the trend of the first principal component retains the 'most important' variation. Images in this category are thus sorted in order of increasing ϕ.

4.2 Image Retrieval

Key to image retrieval and browsing is to measure similarity between images, whether between a query and the database for retrieval, or between images in the database for structuring the data for indexing and visualisation. Consider the case of query-by-example in which a query image Q is to be compared to an image A from the database. Assume that A has been classified as belonging to the class C_j, $j \in \{1,2,3,4,5\}$. The similarity between

Q and A is estimated as follows.

1. Calculate the probability $P(C_j \mid Q)$ that Q belongs to class C_j. (See Eqs. (5) and (6)).
2. Project Q and A onto the principal component of class C_j to obtain $y_Q = \mathbf{v}_j (Q - \boldsymbol{\mu}_j)^{\mathbf{T}}$ and $y_A = \mathbf{v}_j (A - \boldsymbol{\mu}_j)^{\mathbf{T}}$.
3. The similarity of Q to A is computed as:

$$S_{QA} = \frac{1}{Z} \exp\left(-\frac{(y_Q - y_A)^2}{2\lambda_j} \right) P(C_j \mid Q) \tag{7}$$

where $Z = (2\pi)^{1/2} \lambda_j^{1/2}$

The above processing is repeatedly performed to every image in the database to yield a similarity to the query. Then, the images are ranked in decreasing order of similarity to the query. The similarity measure in Eq. (7) takes into account the probability that the query belongs to the same class and the distance between the images in that class (appropriately scaled).

Figures 5 and 6 show examples of the proposed image retrieval algorithm. In each of these Figures, the query image is shown at the top-left, and the top eight returned images are shown. The images are ordered from left to right and from top to bottom. Recall that the texel geometry is represented in a way that is scale invariant. Therefore, the similarity measure is in terms of shape and orientation. As can be seen, the returned images have their basic texture units repeated in similar ways to the query images.

5 EXPERIMENTS

Three experiments were performed to evaluate the proposed methods. The first experiment tested the performance of the texel extraction algorithm. The second experiment tested the ability of the Gaussian cluster models to yield correct classification of texels. The final experiment explored the ability of the principal components to represent the clusters.

A dataset of 200 regular texture images was used for evaluation, comprising 147 images of textiles from a commercial archive and 53 images taken from three public domain databases (the Wikipedia Wallpaper Groups page, a Corel database, and the CMU near-regular texture database). The images ranged in size from 300×225 pixels to 2648×1372 pixels. The number of texel repeats per image ranged from five to a few hundreds. This data set includes images that are challenging because of (i) appearance variations among texels, (ii) small geometric deformations, (iii) texels that are not distinctive from the background and are large non-homogeneous regions, (iv) occluding labels, and (v) stains, wear and tear in some of the textile images.

5.1 Evaluation of Texel Extraction

Two volunteers (one male and one female) qualitatively scored and rank ordered the algorithms. In cases of disagreement, they were forced to reach agreement through discussion. (Disagreement happened in very few cases). The observers were shown extracted texels overlaid on images and were asked to label each texel as obviously correct (OC), obviously incorrect (OI), or neutral. They were to assign OC if the texel was exactly the same or very close to what they expected, OI if the result was far from their expectations, and neutral otherwise. In our texel extraction algorithm, variance of the Gaussian model was the only free parameter and it was set as $\sigma^2 = 100$. The numbers of OC, OI, and neutral results were *164*, *17* and *19*, respectively. Thus, the accuracy of texel extraction was *164/200=82%*. Figure 7 shows some example results.

5.2 Evaluation of Gaussian Model

A classification experiment was performed to assess the suitability of the assumption of Gaussian clusters. Images were classified as belonging to the cluster with the largest posterior probability as computed using Equations (5) and (6).

The data set of *200* images was divided into two disjoint sets of *100* images each. One was used as a training set and the other as a test set. The experiment was then repeated after switching the training and test sets. Training data and ground truth were labelled using the rules in Section 3.2. The classification rates for the two test sets were *91%* and *96%* giving an average rate of *93.5%*. The confusion matrix averaged over the two test sets is shown in Table 1. Regular textures from classes 2, 3, and 4 were more likely to be misclassified, as would be expected from inspection of Figure 3.

5.3 Evaluation of Texel Comparison

It was proposed that texels be represented by projection onto their class-specific principal component. The intra-class distribution is thus modelled as a 1D Gaussian. An experiment was

Figure 4: Image browsing example for class 4.

Figure 5: Query-by-example based on texel geometry. The query is top-left followed by the seven best matches.

Figure 6: Query-by-example based on texel geometry. The query is top-left followed by the seven best matches.

performed to explore the effect of this projection. Texels from class *i* can be generated from this model by:

$$\mathbf{x} = \boldsymbol{\mu}_i + a\mathbf{v}_i \tag{8}$$

where a is an appropriately set weight. Weights with large magnitudes result in texels far from the mean. In practice, data will fall in a range such as

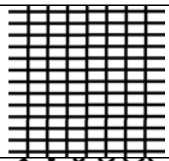
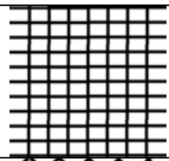
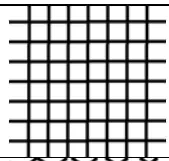
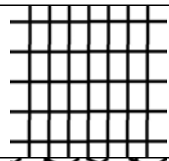
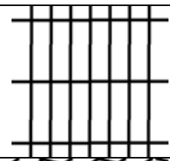
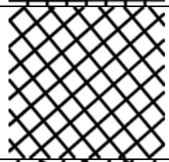
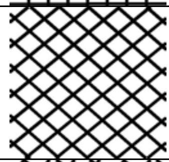
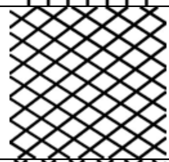
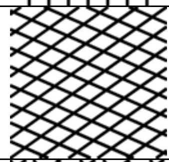
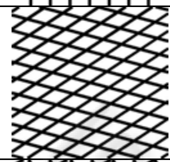
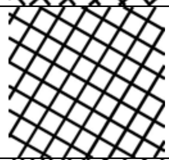
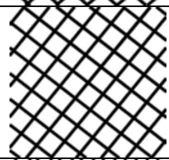
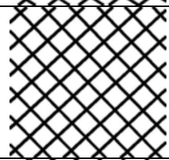
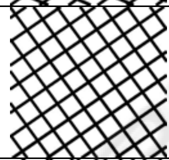
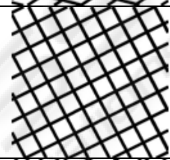
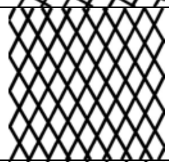
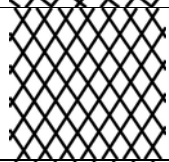
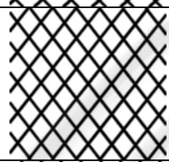
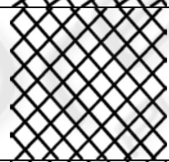
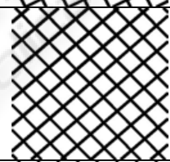
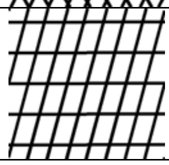
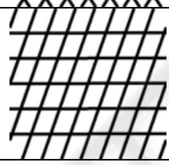
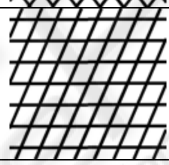
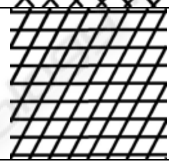
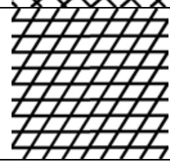
$$-3\sqrt{\lambda} \le a \le 3\sqrt{\lambda} \tag{9}$$

where λ is the eigenvalue for eigenvector $\mathbf{v}_i$.
Table 2 shows texels synthesised from each of the five classes by setting $a = 0, \pm\sqrt{\lambda}, \pm 2\sqrt{\lambda}$.

Table 1: Confusion matrix for texel classification.

True \ Predicted	1	2	3	4	5
1	**29.5**	**0.5**	**0.0**	**0.0**	**0.0**
2	**0.0**	**22.0**	**2.0**	**0.0**	**0.0**
3	**0.0**	**1.5**	**21.5**	**0.0**	**0.0**
4	**0.0**	**1.5**	**1.0**	**10.0**	**0.0**
5	**0.0**	**0.0**	**0.0**	**0.0**	**10.5**

Table 2: Synthetic texels generated by the model.

a	$-2\sqrt{\lambda}$	$-\sqrt{\lambda}$	0	$\sqrt{\lambda}$	$2\sqrt{\lambda}$
Class 1					
Class 2					
Class 3					
Class 4					
Class 5					

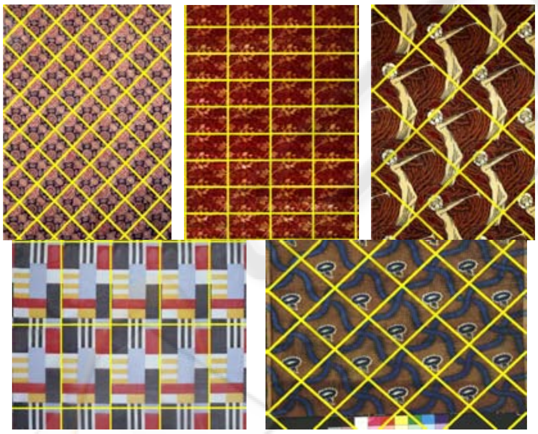


Figure 7: Some results from the texel extraction algorithm.

It can be seen that the major mode of variation for class 1 was the ratio of the lengths of $\mathbf{t}_1$ and $\mathbf{t}_2$. The major mode of variation for class 3 combined the ratio of the lengths of $\mathbf{t}_1$ and $\mathbf{t}_2$, and the direction of $\mathbf{t}_1$. The major mode of variation for classes 2 and 4 involved all three features. These synthetic data suggest that the proposed model is able to capture the variability of each class effectively.

6 CONCLUSIONS

In this paper, a systematic study of the texel geometry of regular textures has been presented. A fully automatic algorithm using Bayesian model comparison was used to extract texels. A feature vector defined on the obtained texel was proposed to characterize the geometry of a texel. The distribution of a set of regular texture images in the feature space was modelled. The proposed model is easy to implement and was applied to guide image browsing and retrieval effectively. Experiments on a collection of regular texture images have demonstrated the promise of the approach.

Various extensions to this work would be interesting to investigate in future work. 1) It would be useful to analyse other regular texture data sets to investigate the breadth of applicability of the proposed clustering model. 2) Evaluations of image retrieval and browsing should be conducted on a large-scale database combining the proposed technique with other features that model the appearance of the texels. 3) The proposed work has been applied to image retrieval and browsing in this paper. We believe it can also be extended to help fabric designers to categorize and manage their digital archives, and provide them with interesting sources to spark and fuel design inspiration.

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