INSCRIBED CONVEX SETS AND DISTANCE MAPS
Application to Shape Classification and Spatially Adaptive Image Filtering

Frédérique Robert-Inacio
IM2NP - UMR CNRS 6242, ISEN-Toulon, Place Pompidou, Toulon, France

Keywords: Distance map, Inscribed convex set, Shape classification, Similarity parameter, Spatially adaptive filtering.

Abstract: This paper presents two original applications related to discrete distance maps. Based on the relation linking inscribed convex sets and discrete distance maps, the first application is a spatially adaptive filtering method which is set up for both grey-level and color images. This spatially adaptive filter is really efficient in performances and computation time. Furthermore a new mean of computation for the Asplund distance as well as a method for determining the similarity degree between shapes are also presented. The similarity parameter enables a quantitative shape classification with respect to a set of reference shapes.

1 INTRODUCTION

The discrete distances are widely used in image processing. For example, they can be used to determine Voronoï diagram around objects or the medial axis and skeleton of a shape and so on, as well as they can achieve elementary morphological operations such as dilation or erosion (Soille, 1999)(Borgefors et al., 1999) in 2D or in 3D. But another kind of information in relation with inscribed convex sets can be extracted from distance maps.

This paper deals with two of the applications involving inscribed convex sets: on the one hand, a spatially adaptive filter is set up by determining a customized filtering window size at each point of an image in grey-levels or in color, and on the other hand, a simplified method for the determination of the Asplund distance and of a degree of similarity between shapes is described.

2 DISCRETE DISTANCE MAPS

A discrete distance map (DDM) is computed from a binary image including a background and one or several objects. On the DDM, the distance to the nearest object is estimated at each point of the background and each point of the object set is set to 0. There exist several algorithms evaluating the Euclidean distance in a discrete space (Danielsson, 1980). But several works presented discrete distances on $\mathbb{Z}^2$ as approximations of the Euclidean distance, as described in the following section.

2.1 Approximation of the Euclidean Distance

At the beginning, discrete distances such as Manhattan distance or chessboard distance, were defined in order to coarsely estimate the Euclidean distance between points on a grid (Rosenfeld and Pfaltz, 1968)(Borgefors, 1986). These approximations were refined by the elaboration of the chamfer distances. Let us define the discrete distances.

Let $A(x_A, y_A)$ and $B(x_B, y_B)$ be two points of $\mathbb{Z}^2$.

The Manhattan distance is defined by:

$$d_{Mh}(A,B) = |x_A - x_B| + |y_A - y_B| \qquad (1)$$

The chessboard distance is defined by:

$$d_{Cb}(A,B) = max\left(|x_A - x_B|, |y_A - y_B|\right) \qquad (2)$$

Notice that $d_{Mh}(A,B) \geq d_{Cb}(A,B), \forall (a,B) \in \mathbb{Z}^2$.

A family of discrete distances called chamfer distances also gives an approximation of the Euclidean distance. The principle of computation for such distances is based on the determination of a minimal path leading from A to B. This path is made of several elementary displacements, each of them being weighted. Examples of weights for the main directions on the grid are given in Fig. 1 and the corresponding estimations of Euclidean distance values are given in Table 1.

Robert-Inacio F. (2009).
INSCRIBED CONVEX SETS AND DISTANCE MAPS - Application to Shape Classification and Spatially Adaptive Image Filtering.
In *Proceedings of the Fourth International Conference on Computer Vision Theory and Applications*, pages 60-66
DOI: 10.5220/0001796200600066

Directions	OA	OB	OC	OD	OE
Chamfer 3x3	3				4
Chamfer 5x5	5		11		7
Chamfer 7x7	12	38	27	43	17

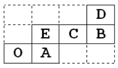

Figure 1: Weights and elementary displacements for the chamfer distances with a neighborhood width of 3, 5 and 7.

Table 1: Distances from O for the chamfer distance with different neighborhood widths.

Point/Width	3	5	7
A	1	1	1
B	10/3	16/5	19/6
C	7/3	11/5	9/4
D	11/3	18/5	43/12
E	4/3	7/5	17/12

The previous examples can be extended to an arbitrary dimension of neighborhood, in order to be more accurate in the estimation of the Euclidean distance, with the main drawback that it makes the computation time increase.

2.2 Associated Convex Sets

Fig. 2 shows DDM computed from a binary image containing a unique object: the center point. DDM are given for the Euclidean distance and the discrete distances defined above. Furthermore sets of equidistant points to the object set at a distance of 50 are drawn in white in Fig. 2. If we consider that the circle $C_{\mathcal{E}}(M,R)$ of center M and radius R, is the set of equidistant points from M at a Euclidean distance of R, then, by extension, sets of white points of Fig. 2 are "circles" of radius 50 for the corresponding distances. In this way, we can associate the shape of these "circles" with the corresponding distance. Table 2 summarizes the correspondence. Note that all of these shapes are regular. Fig. 3 presents an example of inscribed "circles" into a particular shape.

3 INSCRIBED CONVEX SETS

By definition a DDM gives at each point x of the background its distance to the nearest object. In the Euclidean case, if $DDM(x)$ is the value reached at point x, the disk $D(x,DDM(x))$ of center x and radius $DDM(x)$ is totally included in the background. Furthermore $DDM(x)$ is the distance from x to the object set. That means that $D(x,DD(x))$ is the greatest disk centered at x and totally included in the background. This remark can be generalized to the convex sets associated with the discrete distances defined above. In this way if X is the shape corresponding to a discrete distance, $X(x,DDM(x))$ is the greatest homothetic set of X centered at x and totally included in the background.

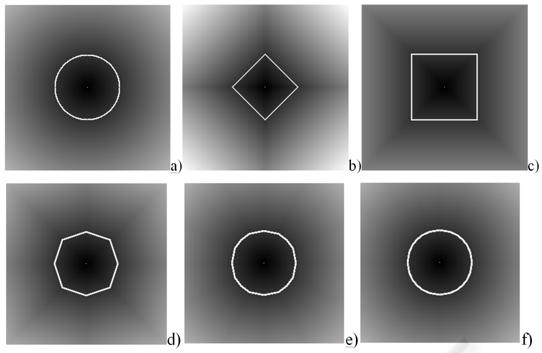

Figure 2: DDM for a) the Euclidean distance, b) the Manhattan distance, c) the chessboard distance, d) the 3x3 chamfer distance, e) the 5x5 chamfer distance and f) the 7x7 chamfer distance.

Let us now consider a set Y and compute a particular DDM by considering the outside of Y (or its complementary set) as the object and Y as the background. The maximum value, $DDM(x_{max})$, reached on the DDM gives the scale ratio corresponding to an inscribed set of shape X into Y. Thus, $DDM(x_{max})$ is the scale ratio applied to the shape X to obtain the inscribed homothetic set of X into Y, if we consider that X is the reference shape at scale 1. Depending on the shape Y, x_{max} is unique or not, in other words, there can exist several positions to center the inscribed homothetic set of X into Y.

4 SPATIALLY ADAPTIVE FILTERING

4.1 Adaptive Sliding Window Size

Sliding windows used for filtering are generally square-shaped and these squares are oriented at 0^o. But such squares are "circles" for the chessboard distance. That is why we are going to use a DDM based on the chessboard distance to design the sliding window associated with a given point. As the chessboard DDM allows to determine the greatest homothetic square totally included in the background, it is sufficient to compute an appropriate binary image describing objects and background, in order to obtain window sizes depending on the location and stored in the DDM.

For grey-level images, the binary image can be chosen as a thresholded gradient image (TG). In this

Table 2: Correspondance between discrete distances and associated convex shapes.

Distance	Euclidean	Manhattan	Chessboard	Chamfer 3x3	Chamfer 5x5	Chamfer 7x7
Shape	Circle	Square (45^0)	Square (0^0)	Octagon	Dodecagon	24-edge Polygon

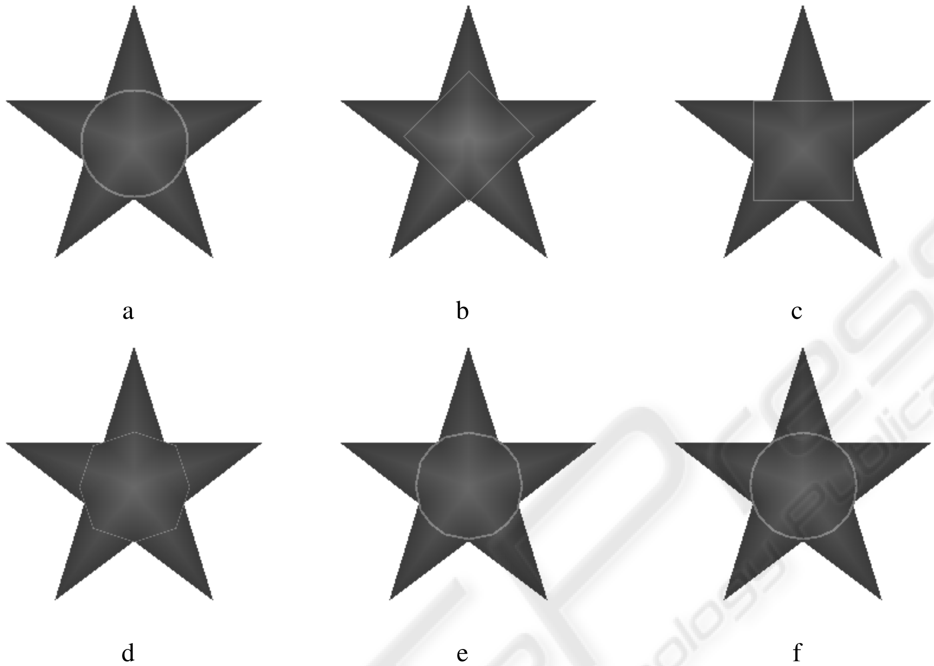

Figure 3: Inscribed ”circles” for a) Euclidean, b) Manhattan, c) Chessboard, d) Chamfer 3x3, e) Chamfer 5x5 and f) Chamfer 7x7 distances.

way high frequency areas are preserved while homogeneous regions are smoothed. The threshold value is chosen by taking into account the maximal amplitude of noise that must be filtered.

For color images, we compute a color distance map (CDM) that will be thresholded. This CDM is derived from two color difference images. In other words, the Euclidean distance between the color vector of point $P(x,y)$ and those of its right neighbor $P_r(x+1,y)$ is computed on the original color image to obtain the X-CDM. As well, the Y-CDM is obtained by computing the Euclidean distance between the color vector of $P(x,y)$ and those of its top neighbor $P_t(x,y+1)$. A threshold value v is then chosen according to the amplitude of noise to be filtered. Actually noise of amplitude lower or equal to th will be removed whereas noise of greater amplitude will be consider as relevant information. Thus the X-CDM and Y-CDM are thresholded at v and the thresholded CDM is obtained by mixing these two binary images in order to keep all data representing high color distances.

Then the chessboard DDM is computed from one of those binary images by considering points of high frequency or of high color distance as the object set. In this way, at each point of the background, the DDM gives the half width of the maximal square totally included in the background. If $ww(P)$ is the maximal window width at point $P(x,y)$, $ww(P)$ depends on the location of P and it is the adaptive size of the sliding window and:

$$\forall P(x,y) \in \mathbb{Z}^2, ww(P) = 2 \times DDM(P) + 1 \quad (3)$$

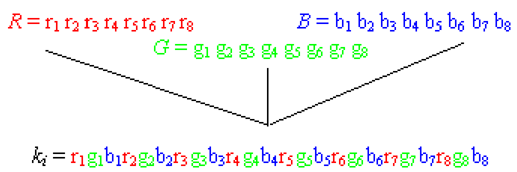

Figure 4: Bit-mixing paradigm.

4.2 Median Filter

For grey-level images the median filter is achieved by determining the median value for each sliding window. Let us note that a sliding window contains $ww(P)^2$ points and that this number is an odd value. Grey-level values belonging to a sliding window are

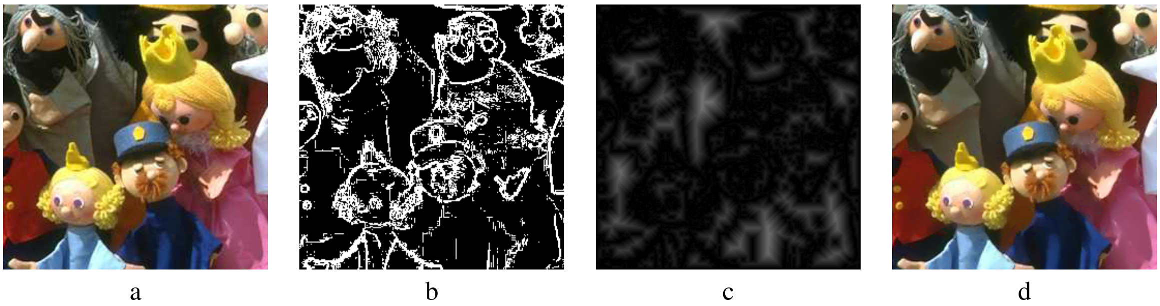

Figure 5: Adaptive median filtering process: a) original image, b) thresholded CDM, c) chessboard DDM, d) filtered image.

Figure 6: Adaptive median filtering results for different values of th: a) $th = 5$, b) $th = 10$, c) $th = 20$, d) $th = 30$.

sorted and the median value is determined as the $\left(\frac{ww(P)^2+1}{2}\right)^{th}$ one.

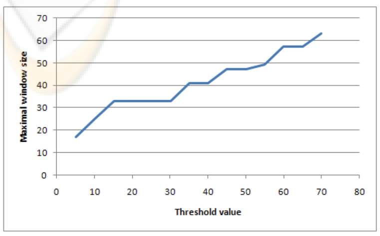

Figure 7: Maximal window size according to th.

For color images (Robert-Inacio and Dinet, 2006), it is not possible to sort color vectors as they belong to $\mathbb{Z}^3$ which does not have total order. So a bit-mixing paradigm (Lambert and Macaire, 2000)(Fig. 4) is used in order to associate a 24-bit integer value $v_{24}(r,g,b)$ to each RGB color vector (r,g,b). In this way it is possible to arrange in order color vectors of the sliding window according to their $v_{24}(r,g,b)$, and then, to determine the median value. This way to sort colors can be unsatisfactory from a theoretical point of view, but it gives efficient results in practice. Fig. 5 shows the whole process on a color image.

Figure 8: Median filtering results for different window widths: a) 3x3, b) 5x5, c) 7x7, d) 9x9.

4.3 Experimental Results

Fig. 6 shows filtered images obtained for 4 different threshold values of the CDM. We can note that some degradation in hair areas. The maximal size for sliding windows is respectively of 17, 25, 33 and 33 for th = 5, 10, 20 or 30. Fig. 7 shows the evolution of the maximal window size according to the threshold value of the CDM, for this particular image. The higher th, the smoother the result, with the drawback of loosing relevant information. Fig. 8 shows the rapid degradation of the filtered image in case of fixed window size on the whole image.

5 SHAPE CLASSIFICATION

Shape classification can be achieved by using qualitative or quantitative tools. In the second case, the classification is made with respect to one or several criteria. In order to do that, shape parameters or distance between shapes can be used. In this section, we recall the definition of the Asplund distance between shapes and of a similarity parameter estimating the degree of likeliness of the shape under study to a reference shape.

5.1 Asplund Distance

The Asplund distance (Serra, 1988)(Robert-Inacio, 2007) between two compact sets X and Y of $\mathcal{R}^2$, $d_A(X,Y)$, is defined by:

$$d_A(X,Y) = ln\frac{k(X,Y)}{K(X,Y)} \quad (4)$$

where:

$$k(X,Y) = inf\{k > 0, Y \subset_t k.X\} \quad (5)$$

$$K(X,Y) = sup\{k > 0, k.X \subset_t Y\} \quad (6)$$

$\subset_t$ means that it exists $x \in \mathcal{R}^2$ such that $x + X \subset Y$. In other words, $k(X,Y)$ is the scale ratio so that $k(X,Y).X$ is circumscribed to Y and $K(X,Y)$ is the scale ratio so that $K(X,Y).X$ is inscribed into Y. We can also remark that:

$$K(X,Y) = k(Y,X) \quad (7)$$

5.2 Similarity Parameter

The similarity parameter *SP* is defined for any pair of convex sets X and Y of $\mathcal{R}^2$ by the following formula (Robert, 1998)(Robert-Inacio, 2007):

$$SP(X,Y) = \frac{k(X,Y)}{k(Y,X)}.\frac{\mu(X)}{\mu(Y)} \qquad (8)$$

where μ is the surface area measure.

5.3 Implementation

In this section, let us consider the case where X is a "circle" for a discrete distance. It is then very easy to determine the ratio to inscribe X into Y, by computing the corresponding DDM inside Y. In this case, the maximal value is $k(X,Y)$.

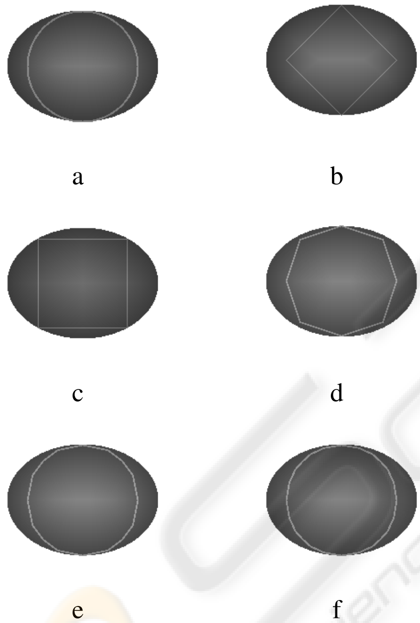

Figure 9: Inscribed "circles" for a) Euclidean, b) Manhattan, c) Chessboard, d) Chamfer 3x3, e) Chamfer 5x5 and f) Chamfer 7x7 distances.

DDM can also be useful in the particular case where Y is symmetrical with respect to a point c. In this case the center point c is the center of the homothetic set of X, circumscribed to Y. It is then sufficient to compute another DDM by considering a set object reduced to c, $DDM(c)$. And the maximal value reached on the boundary of Y by $DDM(c)$ is the scale ratio to apply to X (shape at scale 1) so that it is circumscribed to Y. In other cases algorithms such that the circumscribed disk algorithm and its extension to convex sets must be used. Fig. 9 and 10 illustrate the computation of inscribed and circumscribed convex sets X in the case where Y is convex and symmetrical.

We can note that the circumscribed disk algorithm and its extension to convex sets and the DDM can be extended to the third dimension (Jones et al., 2006). Furthermore other distance maps can generate other "circles" and then comparison to these new shapes can be achieved (Strand et al., 2006).

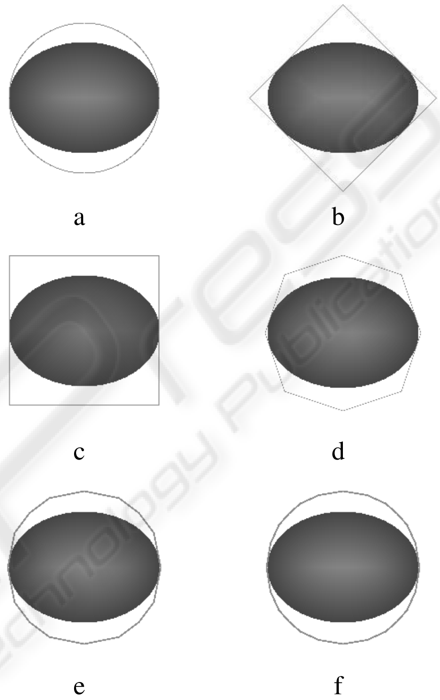

Figure 10: Circumscribed "circles" for a) Euclidean, b) Manhattan, c) Chessboard, d) Chamfer 3x3, e) Chamfer 5x5 and f) Chamfer 7x7 distances.

6 CONCLUSIONS

Applications involving discrete distance maps are numerous and most of them are related to mathematical morphology. In this paper we have presented a spatially adaptive median filter using discrete distance maps in order to determine the greatest size of sliding window at each point of the image. This filter enables a better smoothing of quite homogeneous areas while preserving regions where gradient or color distance is relevant. This method gives satisfactory results.

In a second time, we have described an application related to shape classification. Discrete distance maps can be used in this case, if one of the shapes to compare is a "circle" for a given discrete distance. In this way, the computation of the inscribed shape into the other one is simplified as it is achieved by determining the maximal value on the corresponding discrete distance maps. The circumscribed shape can also be determined from a particular discrete distance map if the other shape is symmetrical. Furthermore results concerning shape classification can be easily extended to the 3D.

REFERENCES

Borgefors, G. (1986). Distance transformations in digital images. *CVGIP*, 34:344–371.

Borgefors, G., Nystrom, I., and SannitiDiBaja, G. (1999). Computing skeletons in three dimensions. *Pattern Recognition*, 32:1225–1236.

Danielsson, P. (1980). Euclidean distance mapping. *CVGIP*, 14:227–248.

Jones, M., Baerentzen, J., and Sramek, M. (2006). 3d distance fields: a survey of techniques and applications. *IEEE trans. on visualization and computer graphics*, 12/4:581–599.

Lambert, P. and Macaire, L. (2000). Filtering and segmentation : the specificity of color images. In *Conference on Color in Graphics and Image Processing*, pages 57–64, Saint-Etienne, France.

Robert, F. (1998). Shape studies based on the circumscribed disk algorithm. In *IEEE-IMACS, CESA 98*, pages 821–826, Hammamet, Tunisia.

Robert-Inacio, F. (2007). A relation between a similarity parameter, the asplund distance and discrete distance maps. In *IVCNZ 2007*, pages 254–259, Hamilton, New Zealand.

Robert-Inacio, F. and Dinet, E. (2006). An adaptive median filter for colour image processing. In *3rd CGIV*, pages 205–210, Leeds, UK.

Rosenfeld, A. and Pfaltz, J. (1968). Distance functions in digital pictures. *Pattern Recognition*, 1:33–61.

Serra, J. (1988). *Image analysis and mathematical morphology: theoretical advances*. Academic Press.

Soille, P. (1999). *Morphological image analysis*. Springer Verlag, Berlin.

Strand, R., Nagy, B., Fouard, C., and Borgefors, G. (2006). Generating distance maps with neighbourhood sequences. In *DGCI 2006*, volume 4245, pages 295–307, Szeged, Hungary.