LINEAR IMAGE REPRESENTATION UNDER CLOSE LIGHTING FOR SHAPE RECONSTRUCTION

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Abstract: In this paper, we propose a method for representing intensity images of objects illuminated by near point light sources. Our image representation model is a linear model, and thus, the 3D shape of objects can be recovered linearly from intensity images taken from near point light sources. Since our method does not require the integration of surface normals to recover 3D shapes, the 3D shapes can be recovered, even if they are not smooth unlike the standard shape from shading methods. The experimental results support the efficinecy of the proposed method.

1 INTRODUCTION

In recent years, the photometric properties of camera image have been studied extensively for reconstructing 3D shape of objects and for generating photorealistic CG images (Shashua, 1997; Hayakawa, 1994; Mukaigawa et al., 2006; Iwahori, 1990; Kim and Burger, 1991; Sato et al., 2006; Okabe and Sato, 2006). It has been shown by Shashua (Shashua, 1997) that if we assume point light source located at infinity and if there is no specular reflection, we can generate arbitrary images from the linear combination of three basis images taken by three different light sources. Mukaigawa et al. (Y.Mukaigawa et al., 2001) proposed a method called image linearization which enables us to generate arbitrary images from three basis images, even if specular reflection and/or shadows exist in images. The photometric properties of each image point, such as specular reflection, diffuse reflection and shadow, can also be classified by using the image linearization (Mukaigawa et al., 2006).

On the other hand, many method have been proposed for reconstructing 3D shape of objects from image intensities. In general, three or more than three images are enough for recovering the surface normal at each image point, and the 3D shape of an object can be recovered by integrating the surface normal, if the 3D shape is differentiable (Hayakawa, 1994).

Unfortunately, these methods assume that the point light sources are located at infinity, and they cannot be applied if the point light sources are close to the object, i.e. near point light sources. This is because the images generated by near light sources include non-linear components, and they cannot be represented linearly. However, images generated by near point light source include much more information on the 3D geometry than those generated by infinite point light sources, and thus their analysis is very important.

Iwahori et al. (Iwahori, 1990) proposed a method for computing surface normal and depth of a Lambertian surface illuminated by a known near light source. This method solves non-linear equations assuming that the point light source exists in the direction of surface normal at a point where the image intensity is maximum. Kim (Kim and Burger, 1991) analyzed the uniqueness of the solution to the non-linear equations. Although these methods enable us to recover less ambiguous shape information, they require large computational cost and may not provide us optimal solutions.

For avoiding these problems, Sato et al. (Sato et al., 2006; Okabe and Sato, 2006) proposed a method for linearizing images with near light sources by dividing the images into small sub-images and assuming parallel light in these sub-images. However, the computational costs of these methods are also large, since they require iterative algorithm. Furthermore, the accuracy of recovered geometry is not so good, since only the local constraints are used in each sub-image.

In this paper, we propose a method for linearly representing images with near light source. We show that linear representation of a near point light source

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image is possible without dividing the image into sub-images. We also show that the 3D shape of objects can be recovered directly by using the proposed linear representation without integrating local surface normals. Thus, the 3D shape can be recovered accurately, even if the shape is not smooth. Also, the computational cost is very small, since the 3D shape can be recovered linearly by using the proposed linear representation.

2 LINEAR REPRESENTATION OF NEAR LIGHT SOURCE IMAGES

2.1 Image Representation under Infinite Light Source

Let us consider a camera, an object and a light source in the 3D space. In this research, we assume that the relative position and orientation between the camera and the object are fixed, and images are taken changing the position of the light source. We also assume Lambertian surface for objects in the scene.

Under the assumption of Lambertian surface, the intensity, I, of the surface can be described by using the surface normal, $\mathbf{n}$, and the direction of light, $\mathbf{s}$, as follows:

$$I = \rho E \max(\mathbf{n}^\top \mathbf{s}, 0) \tag{1}$$

where, ρ denotes the albedo, and E denotes the intensity of light source. If we assume light source at infinity, the direction of light source is constant at any point on the object surface. Thus, if there is no shadow on the object surface, the whole image of the object can be represented as follows:

$$\mathbf{I} = \begin{bmatrix} I_1 \\ \vdots \\ I_i \\ \vdots \\ I_N \end{bmatrix} = E \begin{bmatrix} \rho_1 \mathbf{n}_1^\top \\ \vdots \\ \rho_i \mathbf{n}_i^\top \\ \vdots \\ \rho_N \mathbf{n}_N^\top \end{bmatrix} \mathbf{s} = E\mathbf{B}\mathbf{s} \tag{2}$$

Thus, an image under arbitrary light source can be described as follows:

$$\mathbf{I} = E[\mathbf{I}_1 \;\; \mathbf{I}_2 \;\; \mathbf{I}_3]\mathbf{s}' = E\mathbf{B}\mathbf{A}\mathbf{A}^{-1}\mathbf{s} \tag{3}$$

where, $\mathbf{A}$ denotes a 3×3 matrix which consists of three vectors of light direction of $\mathbf{I}_1$, $\mathbf{I}_2$ and $\mathbf{I}_3$, and $\mathbf{s}' = \mathbf{A}^{-1}\mathbf{s}$.

As shown in (3), arbitrary images under infinite light sources can be described linearly by using three basis images. However, if the light source is close to the 3D object, the illumination model described in (3) is no longer valid. In the next section, we consider the illumination model under near light sources.

2.2 Image Representaion under Near Light Sources

If the light source is close to the 3D object, the illumination model described in (3) is no longer valid. If we consider, the position of the light source, $\mathbf{S}$, a point on the surface, $\mathbf{X}$, and the surface normal, $\mathbf{n}$, at the point $\mathbf{X}$, then the image intensity I at the surface point $\mathbf{X}$ illuminated by a near light source $\mathbf{S}$ can be described as follows:

$$I = \frac{E}{||\mathbf{S}-\mathbf{X}||^2} \frac{\rho \mathbf{n}^\top (\mathbf{S}-\mathbf{X})}{||\mathbf{S}-\mathbf{X}||} \tag{4}$$

In (4), $(\mathbf{S}-\mathbf{X})/||\mathbf{S}-\mathbf{X}||$ describes the direction of light source, and $E/||\mathbf{S}-\mathbf{X}||^2$ corresponds to the attenuation of image intensity according to the distance between the surface point and the light source.

As shown in (4), we can no longer describe the illumination model linearly, since the direction of light source at each surface point is not constant, and the image intensity at a surface point depends on the distance between the light source and the surface point as well as the relative orientation between the light and the surface normal. Thus, the existing methods require non-linear optimization for recovering the 3D shape from images taken under near light sources.

2.3 Linear Image Representation under Near Light Sources

In this section we show a method for linearly representing images observed under near light sources. We assume that the attenuation of image intensity caused by changes in distance between the surface point and the light source is negligible. This assumption is valid if the depth of the object is relatively small comparing with the distance to the object. In this case, the image intensity I at the surface point $\mathbf{X}$ illuminated by a near light source $\mathbf{S}$ can be described as follows:

$$I = E \frac{\rho \mathbf{n}^\top (\mathbf{S}-\mathbf{X})}{||\mathbf{S}-\mathbf{X}||} \tag{5}$$

Although the direction of light source $(\mathbf{S}-\mathbf{X})/||\mathbf{S}-\mathbf{X}||$ is different at each surface point, the position of the light source $\mathbf{S}$ is identical for any surface point. If we take the square of the image intensity I, we have I^2 from (5) as follows:

$$I^2 \;\; = \;\; \rho^2 E^2 \frac{\mathbf{n}^\top (\mathbf{S}-\mathbf{X}) \mathbf{n}^\top (\mathbf{S}-\mathbf{X})}{(\mathbf{S}-\mathbf{X})^\top (\mathbf{S}-\mathbf{X})} \tag{6}$$

Now, let us consider a vector $\mathbf{S}^2$ which consists of the quadratic terms of $\mathbf{S}$ as follows:

$$\mathbf{S}^2 = \begin{bmatrix} S_x^2 \\ S_y^2 \\ S_z^2 \\ S_xS_y \\ S_xS_z \\ S_yS_z \\ S_x \\ S_y \\ S_z \end{bmatrix} \quad (7)$$

where, S_x, S_y and S_z denote x, y and z coordinates of the light source position $\mathbf{S}$. Then, (6) can be rewritten as follows:

$$\lambda \widetilde{I^2} = \mathbf{P}\widetilde{\mathbf{S}^2} \quad (8)$$

where, $\widetilde{I^2}$ and $\widetilde{\mathbf{S}^2}$ represent the homogeneous coordinates of I^2 and $\mathbf{S}^2$ as follows:

$$\widetilde{I^2} = \begin{bmatrix} I^2 \\ 1 \end{bmatrix} \qquad \widetilde{\mathbf{S}^2} = \begin{bmatrix} \mathbf{S}^2 \\ 1 \end{bmatrix} \quad (9)$$

$\mathbf{P}$ is a 2×10 matrix, which includes the surface normal and the coordinates of the surface point.

As shown in (8), images taken under near light sources can be represented linearly by using the quadratic term of light source position. Since the matrix $\mathbf{P}$ includes the shape information of the object, the 3D shape of the object can be recovered linearly by using (8). We call $\mathbf{P}$ an *intensity projection matrix* in the following part of this paper.

2.4 Computing Intensity Projection Matrix from Images

We next consider a method for computing the intensity projection matrix $\mathbf{P}$ from camera images. If we know the light source position $\mathbf{S}$, the following equation can be derived by eliminating λ in (8):

$$\begin{bmatrix} \widetilde{\mathbf{S}^2}^\top & -I^2\widetilde{\mathbf{S}^2}^\top \end{bmatrix} \begin{bmatrix} \mathbf{p}_1 \\ \mathbf{p}_2 \end{bmatrix} = 0 \quad (10)$$

where, $\mathbf{p}_1$ and $\mathbf{p}_2$ are ten vector, and $\mathbf{P} = [\mathbf{p}_1, \mathbf{p}_2]^\top$.

Thus, if we have M images taken under M different light sources, we have the following equation:

$$\mathbf{M} \begin{bmatrix} \mathbf{p}_1 \\ \mathbf{p}_2 \end{bmatrix} = \mathbf{0} \quad (11)$$

where, $\mathbf{M}$ is a $M \times 20$ matrix as follows:

$$\mathbf{M} = \begin{bmatrix} \widetilde{\mathbf{S}_1^2} & -I_1^2\widetilde{\mathbf{S}_1^2}^\top \\ & \vdots \\ \widetilde{\mathbf{S}_M^2} & -I_M^2\widetilde{\mathbf{S}_M^2}^\top \end{bmatrix} \quad (12)$$

Thus, $\mathbf{p}_1$ and $\mathbf{p}_2$ can be obtained by simply solving the linear equation (11). Therefore, if we have 19 or more than 19 images, we can compute the intensity projection matrix $\mathbf{P}$. Note, the light sources must be linearly independent in the 9 vector space $\mathbf{S}^2$.

2.5 Recovering Light Source Information

Up to now, we derived a method for computing the intensity projection matrix $\mathbf{P}$ in the case where the light source positions $\mathbf{S}$ are available. In this section, we consider a method for recovering the light source position $\mathbf{S}$ in the case where the intensity projection matrix $\mathbf{P}$ is given.

If we have $\mathbf{P}$, then the following equation on $\mathbf{S}$ can be obtained from (8):

$$\begin{bmatrix} \mathbf{p}_1^\top - I^2\mathbf{p}_2^\top \end{bmatrix} \tilde{\mathbf{S}}^2 = 0 \quad (13)$$

Since $\mathbf{S}$ is constant for all the points in an image, the 9 vector $\mathbf{S}^2$ can be computed from minimum of 9 points in the image, and the light source position $\mathbf{S}$ can be recovered.

3 RECOVERING 3D SHAPE FROM INTENSITY PROJECTION MATRIX

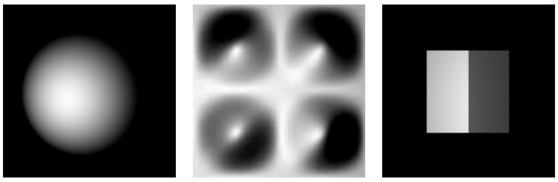
We next consider a method for recovering the 3D shape of objects by using the linear representation of intensity images.

In section 2.4, we showed a method for computing the intensity projection matrix $\mathbf{P}$. Since the intensity projection matrix $\mathbf{P}$ includes the 3D coordinates $\mathbf{X}$ and the surface normal $\mathbf{n}$ at each surface point of objects, we can recover the 3D shape of objects from $\mathbf{P}$.

From (6) we find that the 10 components of $\mathbf{p}_2$ can be described as follows:

$$\lambda \mathbf{p}_2 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \\ 0 \\ 0 \\ -2X \\ -2Y \\ -2Z \\ X^2+Y^2+Z^2 \end{bmatrix} \quad (14)$$

where, $\mathbf{X} = [X, Y, Z]^\top$. Thus, the 3D shape $\mathbf{X}$ can be



(a) (b) (c)

Figure 1: Example images used in our experiments. (a) shows the image of a sphere, (b) shows the image of a sinusoidal surface, and (c) shows the image of a triangular prism.

recovered from $\mathbf{p}_2$ as follows:

$$\mathbf{X} = \begin{bmatrix} -p_{27}/2p_{21} \\ -p_{28}/2p_{21} \\ -p_{29}/2p_{21} \end{bmatrix} \quad (15)$$

where, p_{2i} denotes the i-th component of $\mathbf{p}_2$.

It is known that the standard shape from shading under infinite light sources can recover the 3D shape only if the 3D shape is differentiable, since it recovers the 3D shape by integrating surface normals. However, the proposed method recovers the 3D shape directly without using surface normals, and thus it can recover the 3D shape even if the shape is not smooth. This is a big advantage of the proposed method together with the linearity.

4 EXPERIMENTS

We next show the results of some experiments to show the efficiency of the proposed method. In these experiments we used synthetic images and evaluated the proposed method. Fig. 1 shows three example images used in our experiments, which are images of a sphere, a sinusoidal surface, and a triangular prism. The image size is (128×128).

4.1 Recovery of 3D Shape

We first show the results of recovering 3D shape from the proposed method. The light source positions are given in this experiment. The 3D shapes are recovered from 19 images in which the light sources are close to the objects and are different each other. We recovered surface points where the image intensities in 19 images are not equal to 0.

Fig. 2, Fig. 3 and Fig. 4 show the 3D shapes recovered by using the proposed method. In these figures, the blue points show the recovered shapes and the green points show the ground truth. The RMS errors of the estimated shapes were equal to 0 in all the shapes. As shown in Fig. 4, the triangular prism can

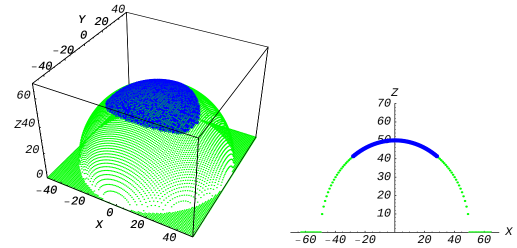


Figure 2: The 3D shape (sphere) recovered by using the proposed method. The blue points show the recovered shape and the green points show the ground truth.

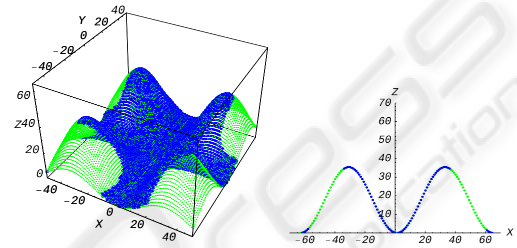


Figure 3: The 3D shape (sinusoidal surface) recovered by using the proposed method. The blue points show the recovered shape and the green points show the ground truth.

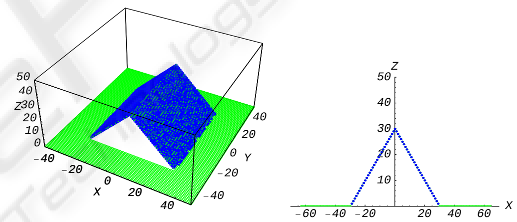


Figure 4: The 3D shape (triangular prism) recovered by using the proposed method. The blue points show the recovered shape and the green points show the ground truth. We find that the proposed method can be applied, even if the 3D shape is not smooth.

also be recovered accurately, and thus we find that the proposed method can be applied, even if the 3D shape is not smooth.

4.2 Generation of Arbitrary Light Source Images

We next generate arbitrary light source images by using the proposed linear representation of image intensity. In this experiment, we compute intensity projection matrix of arbitrary light source positions and generate synthetic images projecting the recovered 3D shapes by the intensity projection matrix. The image generation can be achieved linearly by using the proposed linear representation.

Fig. 5 shows some example images generated by the proposed method. (a1) and (b1) show synthetic

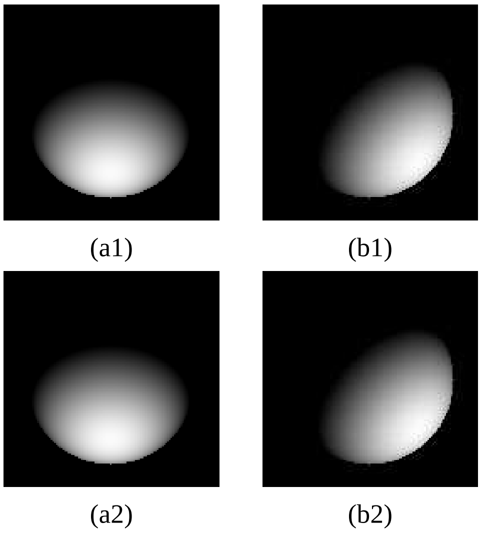


Figure 5: The arbitrary light source images generated by using the proposed method. (a1) and (b1) show images with two different light sources, which are generated by using the intensity projection matrix. (a2) and (b2) show ground truth images.

images illuminated by two different light sources, which are linearly generated by using the intensity projection matrix. (a2) and (b2) show ground truth images. As shown in this figure, arbitrary light source images can be generated properly by using the proposed linear representation.

4.3 Accuracy Evaluation under the Noise in Light Source Positions

We next evaluated the error of the proposed method caused by the noise in light source positions. In this evaluation, arbitrary light source images are generated by using the intensity projection matrix estimated by adding Gaussian noise with the standard deviation of 1.0 to all the light source positions. The computation is iterated 100 times changing the light source positions, and the RMS errors of generated images are measured. The same evaluation has been done changing the number of images used for computing the intensity projection matrix. The results of the evaluation is shown in Fig. 6. As shown in this figure, the RMS error of generated arbitrary light source images is going to be small if we use more images for computing the intensity projection matrix.

Figure 6: Accuracy of generated arbitrary light source images under the noise in light source positions. The horizontal axis is the number of images used for estimating the intensity projection matrix, and the vertical axis is the RMS error of generated images.

4.4 Accuracy Evaluation under the Noise in Images

We finally evaluated the error of the proposed method caused by the image noise. In this evaluation, we added Gaussian noises with the standard deviation of 1.0 to the image intensity, and generated arbitrary light source images 100 times by using the estimated intensity projection matrix as before. The RMS errors of generated images are measured changing the number of images used for computing the intensity projection matrix. Fig. 7 shows the results of the evaluation. As shown in this figure, the proposed method provides us better results if we use more images for estimating the intensity projection matrix.

Figure 7: Accuracy of generated arbitrary light source images under the noise in image intensity. The horizontal axis is the number of images used for estimating the intensity projection matrix, and the vertical axis is the RMS error of generated images.

5 CONCLUSIONS

In this paper, we proposed a method for representing near light source images linearly. For deriving the linear representation, we introduced quadratic terms of a light source position, and the homogeneous representation of image intensity is employed. We showed that the image intensity can be represented linearly by using the quadratic terms of the light source position. By using the proposed linear representation, the 3D shape of objects can be recovered linearly from images taken under near light sources.

The existing methods of shape from shading can recover the 3D shape only if the 3D shape is smooth and differentiable. However, the proposed method recovers the 3D shape directly without using surface normals, and thus it can recover the 3D shape even if the shape is not smooth. The efficiency of the proposed method was shown in the experiments. The accuracy of the proposed method was also evaluated.

The proposed linear representation of image intensity can be considered as a projective camera projection from 9D space to 1D space, and thus the proposed method can be extended to the case where both the position of light sources and the 3D shape of objects are unknown by employing the multiple view geometry in the ordinary cameras.

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