

EIGENVECTOR ANALYSIS FOR OPTIMAL FILTERING UNDER DIFFERENT LIGHT SOURCES

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Keywords: Color spectrum, Eigenvector, Filtering, Illuminant, Sampling interval.

Abstract: Eigenvectors from Standard Object Colour Spectra (SOCS) set were used with several other spectra sets to find the optimal sampling intervals for optimal number of eigenvectors. The sampling intervals were calculated for each eigenvector separately. The analysis was applied not only for different sets of reflectance spectra, but also for spectra sets under different real light sources and standard illuminations. It is shown that 20 nm sampling interval for eigenvectors from SOCS set can be used for reflectance data and data under such light sources which spectrum is smooth. However, data under peaky real fluorescent light sources and standard F-illuminant require accurate 5 nm or even narrower sampling interval for the first few eigenvectors, but can be wider with some of the others. These eigenvectors from SOCS set are shown to be applicable for the other data sets. The results give guidelines for the required accuracy of eigenvectors under different light sources that can be considered e.g. in eigenvector-based filter design.

1 INTRODUCTION

Color is usually represented with three components, such as with RGB color coordinate system. In many cases, this is not enough. Trichromatic representations of color are depended on the used device and illumination, and those are affected by metameric issues (Morovic, 2002). These problems can be avoided with accurate spectral representation of color. The use of spectral color is becoming more and more popular. Spectra are needed for example in telemedicine (Nishibori, 2002), e-commerce, digital art museums (Martinez et al., 2002), art restoration, quality control (Hyvärinen et al., 1999) etc. However, accurate spectral color measurement devices are expensive and measurement may be difficult in a noisy environment. Accurate non-compressed spectral data require also a lot of memory, which will cause problems in using, storing and transferring tasks (Hauta-Kasari et al., 2006).

Spectral dimensionality has been widely studied. This consists of finding the required sampling interval of color spectra (Buchsbaum & Gottschalk, 1984; Maloney, 1986; Bonnardel & Maloney, 2000; Lehtonen et al., 2006), and transforming the spectra to another lower dimensional space (Parkkinen et al., 1989; Hyvärinen et al., 2001; Schettini, 1994; Early & Nadal, 2004). One widely used method is Principal Component Analysis (PCA) (Parkkinen et al., 1989), where the data dimensionality is reduced with the eigenvectors of the data. Several applications based on PCA, such as non-negative filters for imaging have been developed (Piché, 2002). By Hauta-Kasari et al., (1998), the eigenvector-based non-negative filters make it possible to produce the inner-product set in hardware level. From the measured data, accurate spectral information can be computed.

However, to the authors' knowledge, optimal number of eigenvectors under different illuminants

combined with the required spectral accuracy of the eigenvectors has not been studied. Here we will create such sets of eigenvectors under different light sources that can be used for several other color data sets. This includes also a study of required spectral accuracy of eigenvectors. The results can be used e.g. in filter design to create optimal number of non-negative color filters for different illuminants, which can be used generally for accurate color measurements (Piché, 2002; Hauta-Kasari et al., 1998).

2 THEORY

A widely used method for reducing the dimensions of color spectra is Principal Component Analysis (PCA) (Parkkinen et al., 1989). Let C be a correlation matrix

$$C = \frac{1}{N} \sum_{i=1}^N S_i S_i^T. \quad (1)$$

Here S_i is i th spectrum of a spectra set S and N is the number of spectra. The h first eigenvectors of the spectra set ordered by the largest eigenvalues can be calculated. The inner-product set P is then formed with equation

$$P = (\tau_1, \tau_2, \dots, \tau_h)^T S, \quad (2)$$

where $(\tau_1, \tau_2, \dots, \tau_h)$ and T denotes the eigenvectors and matrix transpose, respectively. The data can be reconstructed back to spectra with the linear combination of $(\tau_1, \tau_2, \dots, \tau_h)$ and inner-product set P .

If spectra data set is well defined, one might use the eigenvectors calculated from it to reduce the dimensionality of any other spectra set. In this study, we try to use the eigenvectors of a data set with different other data sets and optimize the sampling intervals of the eigenvectors. Each eigenvector was sampled to several sampling intervals straightforward and then interpolated back with Lagrange interpolation used by Fairman (1985) before reconstruction of spectra. Such number of eigenvectors and sampling intervals were chosen that the reconstructed spectra have good quality. For this, several quality and error measurements were done defined in chapter 4. This analysis was studied with the reflectance spectra and also with spectra under different real light sources and standard illuminants. However, since the eigenvectors are sampled and interpolated, those are not orthogonal after conversion, and calculating with PCA is not

straightforward. A pseudoinverse matrix is required to fix the orthogonality, and the reproduction can be calculated with

$$S^r = \tau^r \left[(\tau^r)^T \tau^r \right]^{-1} (\tau^r)^T S, \quad (3)$$

where τ^r is the sampled and interpolated eigenvectors $(\tau_1^r, \tau_2^r, \dots, \tau_h^r)$, and S^r is the reconstructed spectra set. In this study, a term *eigenvector index* is used to denote the eigenvector index number 1... h , where the eigenvectors are ordered in descending order by the eigenvalues.

3 DATA SETS

Ten different spectra sets (University of Joensuu, 2008; Japanese Standards Association, 1998; Kohonen et al., 2006; Funt & Lewis, 2000; Farnsworth, 1957; Jaaskelainen et al., 1994; Pantone, 2008) were used, listed in Table 1. The spectra originally measured with wider sampling interval than 1 nm were interpolated with Lagrange method shown by Fairman (1985) to 1 nm data and treated as original data. According to Lehtonen et al. (2006) and Sándor et al. (2005) this can be done, since interpolated reflectance spectra are very near to spectra measured with 1 nm. Spectral values outside 400...700 nm range were eliminated.

Table 1: Data sets.

Data set	Nr. of spectra	Original wavelength area [nm]	Original sampling [nm]
Dupont ¹	120	400...700	4
FM100 ²	85	400...700	5
Wood ³	1,056	390...850	5
Lumber	272	380...2700	1
Munsell Glossy ⁴	1,600	380...780	1
Munsell Matte ⁴	1,269	280...800	1
Object Spectral Reflectance Database (OSRD) ¹	170	400...700	4
Pantone ⁵	922	380...780	1
Printed Colors	2,240	380...780	1
Standard Object Colour Spectra (SOCS) ⁶	49,392	400...700	5, 10

¹Funt & Lewis, 2000. ²Farnsworth, 1957. ³Jaaskelainen et al., 1994. ⁴University of Joensuu, 2008. ⁵Pantone, 2008. ⁶Japanese Standards Association, 1998.

At first, eigenvectors from different data sets were tried to be used separately for other data sets. According to tests, suitable eigenvectors can be calculated from both Munsell sets (University of Joensuu, 2008), from Pantone set (Pantone, 2008) and from Standard Object Colour Spectra (SOCS)

set (Japanese Standards Association, 1998). The spectra of the Munsell sets and Pantone set vary a lot. For example Munsell set describes the colors of CIE $L^*a^*b^*$ coordination well, and therefore the eigenvectors are formed for the whole coordination. However, both Munsell sets and also Pantone set had problems with Wood data set (Jaaskelainen et al., 1994), requiring near 20 eigenvectors for good result, but this problem was not an issue in any set with eigenvectors of SOCS set. SOCS data set includes 49,392 spectra from photographic materials, printed colors, paints, textiles, human skin, flowers and leaves. This is a wide collection of color spectra and therefore, the eigenvectors of this set can represent several types of data well.

The sampled and interpolated eigenvectors from SOCS data set were calculated. These eigenvectors were used as a base for calculating the inner-products of other spectra sets and for reconstruction. The required number of eigenvectors and the needed sampling interval of eigenvectors were found with calculating the quality of reconstructed spectra. The number of eigenvectors was chosen as the smallest possible. This procedure was also experimented with the reflectance data, data under four different real light sources and five standard illuminants A, D65, F2, F8 and F11. Spectra of the real light sources are shown in Figure 1. In these cases, also the SOCS data was converted under the light source or illuminant before PCA calculations.

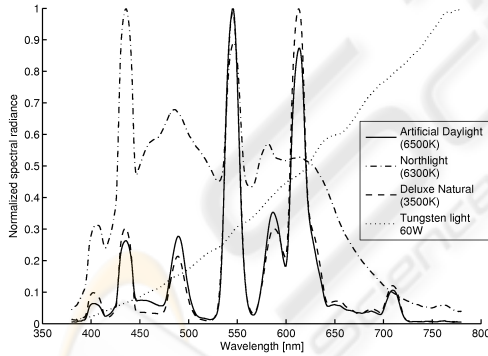


Figure 1: Spectra of the real light sources.

4 QUALITY AND ERROR MEASURES

Two quality measures and one error measure were used to define the quality of reconstructed spectra. The error measure is ΔE , which measures the visual color difference in CIE $L^*a^*b^*$ color coordination as

$$\Delta E = \left(\Delta L^*{}^2 + \Delta a^*{}^2 + \Delta b^*{}^2 \right)^{1/2}, \quad (4)$$

where ΔL^* , Δa^* and Δb^* are the component differences between the original and reconstructed color values in CIE $L^*a^*b^*$ color space. According to Ohta and Robertson (2005), $\Delta E = \sim 1.0$ is usually discriminable. Parkkinen et al. (1989) use color limit of average $\Delta E < 0.5$. Equal-energy spectrum was used as illuminant with calculating the tristimulus values.

Goodness-of-Fit Coefficient (GFC) (Hernández-Andrés et al., 2001) is a correlation based quality measure between two spectra, measuring the similarity of two spectra. It is defined as

$$\mathcal{E}_{GFC} = \frac{\sum_{k=1}^n s_k^o s_k^r}{\left[\sum_{k=1}^n (s_k^o)^2 \right]^{1/2} \left[\sum_{k=1}^n (s_k^r)^2 \right]^{1/2}}, \quad (5)$$

where n is number of channels in spectrum. Terms s_k^o and s_k^r are the wavelength channel values of original and reconstructed spectra, respectively. According to Hernández-Andrés et al. (2001), good limit for this quality measure is 0.999 and accurate limit 0.995.

Peak Signal-to-Noise Ratio PSNR is widely used quality measure in image compression, defined as

$$\mathcal{E}_{PSNR} = 10 \log_{10} \frac{\hat{s}^2}{\mathcal{E}_{MSE}}, \quad (6)$$

where \mathcal{E}_{MSE} is Mean Square Error and \hat{s} is the theoretical maximum of a channel value in spectrum.

Based on Parkkinen et al. (1989), Ohta & Robertson (2005) and Hernández-Andrés et al. (2001), the quality and error limits were chosen as average $\Delta E < 0.5$, average GFC > 0.999 and average PSNR > 40 dB. To obtain accurate results, all of these limits must be satisfied in spectra reconstruction. Also, when selecting a suitable number of eigenvectors and sampling interval, it is required that all narrower sampling intervals and higher number of PCA components must satisfy with the limits.

The quality and error measures and the selected limits were compared with each other with the data sets under Artificial Daylight source, illuminant F11 and illuminant D65. With each light source, the GFC limit 0.999 and PSNR limit 40 dB correspond well with each other, and both accept and reject same sampling intervals. Similar result is found with ΔE compared to GFC or PSNR for the data under Artificial Daylight source. However, the ΔE limit is

more unforgiving with data under F11 illuminant, accepting 1...4 nm sampling intervals, whereas GFC and PSNR accept only 1...2 nm sampling intervals. Also, ΔE error does not always correspond with the sampling interval. With some cases the average visual error is smaller but the average spectral error is higher with wide sampling interval than with narrow sampling interval. Some small variations can also be found with the quality measures, e.g. with F11 illuminant GFC and PSNR values are better with 5 nm sampling interval than 4 nm interval. For these reasons, also all more accurate sampling intervals than the selected one must satisfy with the limits.

5 PARAMETER SELECTIONS

The results of the required number of PCA components and sampling intervals of different eigenvectors with different reflectance data sets and data sets under Artificial Daylight source are shown in Table 2. The eigenvectors of SOCS data is used. For all reflectance data sets, 20 nm interval is enough for the eigenvectors. The required number of eigenvectors varies between 6...11, depending on the variety of colors in data set and data set difference from SOCS set. However, majority of the data sets can be represented with eight eigenvectors. For data under Artificial Daylight source, 4...6 nm interval is needed for the first eigenvector, but can be wider with most of the data sets with higher index eigenvectors. Also, 4...8 eigenvectors are required depending on the data set.

The overall results of the reflectance data sets and data sets under different light sources are shown in Table 3. Here, the total averages of different quality and error measures were used, weighted equally between different sets. With reflectance data and data under smooth light sources the required sampling interval for SOCS eigenvectors is 20 nm. In average for all data sets, ten PCA components are required with the reflectance data and data under D65 illuminant. Eight PCA components are enough for data under illuminant A and Tungsten light source. The required sampling interval for the first few eigenvectors of SOCS are 4...5 nm with data sets under real fluorescent light sources, but interval can be wider for higher index eigenvectors. Depending on the light source, 5...8 PCA components are required. For data under F-illuminants, 1...3 nm interval is needed, but with F2 and F11, the interval can be wider for higher index eigenvectors. In average for all data sets, 5...10 PCA components are required depending on the light source. It was also found that 0...2 less eigenvectors

from SOCS set are enough for the use with SOCS spectra set alone, compared to use with other data sets. Some average and bad examples of spectra reconstruction are shown in Figure 2 when the eigenvectors of SOCS data with sampling intervals shown in Table 3 are used.

Table 2: Required sampling intervals of eigenvectors of SOCS data when used with different data sets.

Reflectance data set	Eigenvector index										
	1	2	3	4	5	6	7	8	9	10	11
Dupont	20	20	20	20	20	20	20	20	20	-	-
FM100	20	20	20	20	20	20	20	20	-	-	-
Forest	20	20	20	20	20	20	20	20	20	20	20
Lumber	20	20	20	20	20	20	-	-	-	-	-
Munsell G.	20	20	20	20	20	20	20	20	-	-	-
Munsell M.	20	20	20	20	20	20	20	20	-	-	-
OSRD	20	20	20	20	20	20	20	20	20	20	-
Pantone	20	20	20	20	20	20	20	20	-	-	-
Printed Col.	20	20	20	20	20	20	-	-	-	-	-
SOCS	20	20	20	20	20	20	20	20	-	-	-

Data under	Eigenvector index										
Artif. Dayl.	1	2	3	4	5	6	7	8			
Dupont	5	4	5	11	9	11	20	10			
FM100	4	7	6	11	15	-	-	-			
Forest	5	9	9	20	13	10	8	-			
Lumber	6	9	8	6	-	-	-	-			
Munsell G.	5	7	6	5	15	-	-	-			
Munsell M.	5	7	5	11	15	-	-	-			
OSRD	5	5	5	15	6	13	10	20			
Pantone	4	7	6	9	11	14	-	-			
Printed Col.	5	5	9	13	14	-	-	-			
SOCS	5	7	9	10	11	-	-	-			

Table 3: Required average sampling intervals of eigenvectors of SOCS data when used with all data sets and under different light sources.

All data sets	Eigenvector index										
Light source	1	2	3	4	5	6	7	8	9	10	
Reflectance	20	20	20	20	20	20	20	20	20	20	
Artif. Daylight	5	7	5	5	10	-	-	-	-	-	
Northlight	5	12	12	13	8	6	8	20	-	-	
Deluxe Natural	4	4	5	9	15	-	-	-	-	-	
Tungsten lamp	20	20	20	20	20	20	20	20	-	-	
A	20	20	20	20	20	20	20	20	-	-	
D65	20	20	20	20	20	20	20	20	20	20	
F2	1	3	5	9	13	6	20	-	-	-	
F8	2	1	3	3	1	2	3	1	3	-	
F11	2	6	3	7	6	3	7	7	-	-	

The spectra from each group and each light source were divided in four groups based on the resulted average quality and error calculations. The average relative numbers of spectra in different error groups for different light sources are shown in Table 4, when quality and error measures are weighted

equally between the test sets. SOCS data was not included in the test sets, it was only used to form the eigenvectors. In general, over 90% of the spectra sets are located in highest quality groups a) and b). Only exceptions are found with Artificial Daylight source and F11 illuminant, where the relative number is 80%. Only about 1% of the spectra give high error, see group d). A small exception is found with F11 illuminant, where the number of bad spectra is 3.8%. However, most of these spectra are selected as bad only because PSNR values very near 30 dB is achieved, but not quite. If this quality limit was changed to PSNR = 28 dB, the relative number of bad spectra would be 1.6%.

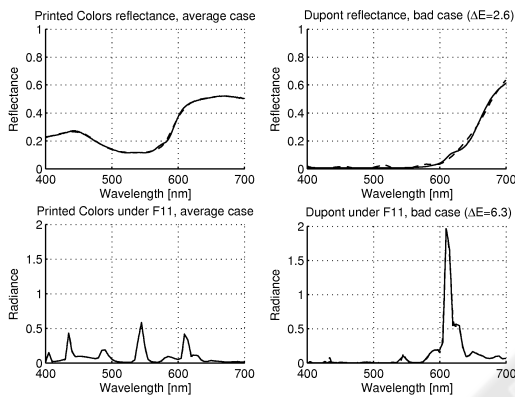


Figure 2: Some average and bad examples of spectrum reconstruction. Original spectrum is shown as solid line and reconstructed one as dashed line.

Table 4: Relative number of spectra distributed with quality and error measures when eigenvectors of SOCS data with sampling interval listed in Table 3 is used.

All data sets,
not SOCS

Light source	a)	b)	c)	d)
Reflectance	83.7%	14.8%	1.5%	0.0%
Artificial Daylight	40.1%	42.8%	15.4%	1.7%
Northlight	57.4%	34.9%	6.8%	0.9%
Deluxe Natural	56.5%	35.7%	6.7%	1.2%
Tungsten lamp	68.9%	20.9%	9.0%	1.2%
A	74.3%	21.5%	3.8%	0.4%
D65	64.4%	33.8%	1.8%	0.0%
F2	56.4%	34.1%	8.3%	1.2%
F8	62.8%	26.4%	9.5%	1.3%
F11	30.5%	47.7%	18.0%	3.8%
Average	59.5%	31.3%	8.1%	1.2%

a) ($\Delta E < 0.5$) AND ($GFC > 0.999$) AND ($PSNR > 40dB$)

b) ($0.5 \leq \Delta E < 1.0$) AND ($0.995 < GFC \leq 0.999$) AND ($34dB < PSNR \leq 40dB$)

c) ($1.0 \leq \Delta E < 3.0$) OR ($0.990 < GFC \leq 0.995$) OR ($30dB < PSNR \leq 34dB$)

d) ($\Delta E \geq 3.0$) OR ($GFC \leq 0.990$) OR ($PSNR \leq 30dB$)

6 CONCLUSIONS

For reflectance data and data under those light sources, which spectrum is smooth, the eigenvectors are also smooth, and wide 20 nm sampling interval is enough. For data under real fluorescent light sources, the peaky dominating shape caused by the light source is located in the eigenvectors, and the required sampling interval is accurate in lower indexes, near 5 nm, but can be a bit wider with some higher indexes, near 10 nm. F-illuminants require more accurate sampling interval compared to real fluorescent light sources, between 1...7 nm depending on the eigenvector index. Since the higher index inner-products do not contain much overall information, the corresponding eigenvectors can have some more errors than lower index eigenvectors. Few examples of erroneous eigenvectors are shown in Figure 3.

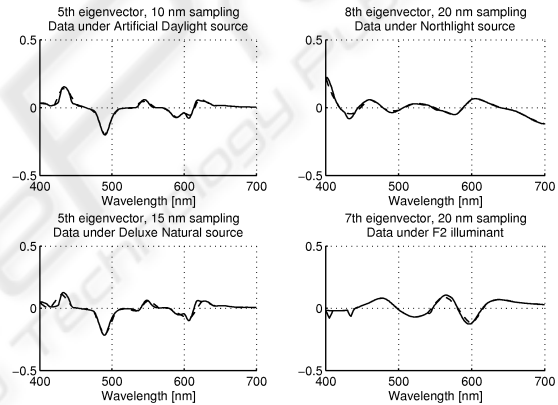


Figure 3: Some bad examples of eigenvectors with wide sampling. Original eigenvector is shown as solid line and reconstructed one as dashed line.

The data set under a light source, which spectrum is peaky, require also few less PCA components compared to reflectance data or data under a smooth light source. The aggressive shape of peaky light source limits the spectra more similar to each other and therefore less PCA components are required.

The results show also that eigenvectors defined from large variety of data, such as from SOCS data set, work very well generally with other data sets. The required sampling interval of eigenvectors depends on the eigenvector index and the light source. All data sets under a light source give similar sampling intervals, but the required number of eigenvectors is different with different data sets and light sources. However, a general required number of eigenvectors was found for different light sources

separately, which can be used to generate low errors. The results can be useful in applications based on eigenvectors, such as in designing optimal non-negative filters for different light sources.

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