Prolegomena toward Algebraic Image Analysis[1]

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Abstract. The paper is an extended abstract of analytical tutorial devoted to algebraization of image analysis.

1 Summary

Automation of image processing, analysis, estimating and understanding is one of the crucial points of theoretical computer science having decisive importance for applications, in particular, for diversification of solvable problem types and for increasing the efficiency of problem solving. The main subgoals are developing and applying of mathematical theory for constructing image models accepted by efficient pattern recognition algorithms and for standardized representation and selection of image analysis transforms. Automation of image-mining is possible by combined application techniques for image analysis, understanding and recognition.

The specificity, complexity and difficulties of image analysis and estimation (IAE) problems stem from necessity to achieve some balance between such highly contradictory factors as goals and tasks of a problem solving, the nature of visual perception, ways and means of an image acquisition, formation, reproduction and rendering, and mathematical, computational and technological means allowable for the IAE.

The mathematical theory of image analysis is not finished and is passing through a developing stage. It is only recently came understanding of the fact that only intensive creating of comprehensive mathematical theory of image analysis and recognition (in addition to the mathematical theory of pattern recognition) could bring a real opportunity to solve efficiently application problems via extracting from images the information necessary for intellectual decision making. The transition to practical, reliable and efficient automation of image-mining is directly dependent on introducing and developing of mathematical means for IAE.

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During recent years there was accepted that algebraic techniques, in particular different kinds of image algebras, is the most prospective direction of construction of the mathematical theory of image analysis and of development of an universal algebraic language for representing image analysis transforms and image models.

The purposes of this tutorial are:

- to set forth the state of the art of mathematical theory of image analysis;
- to consider the algebraic approaches and techniques acceptable for image analysis;
- to present a methodology, mathematical and computational techniques for automation of image mining on the base of Descriptive Approach to Image Analysis (DAIA) and to consider an example (automated diagnosis of hematological deceases).

The program of the tutorial is following:

- Introduction «On a way to a unified theory»

The introduction presents a history of image analysis algebraization (M.Duff, G.Matheron, J.Serra, J.von Neumann, S.Sternberg, S.Unger, and others).

- Chapter 1 "State of the art of mathematical theory of image analysis".

In the chapter a current state of the mathematical theory of image analysis is analyzed.

- Chapter 2 "The Algebraic Approaches and Techniques in Image Analysis. Algebraization of Pattern Recognition (1970 – till now)".

The chapter considers leading approaches in the mathematical theory of image analysis oriented for automation of image analysis and understanding.

Chapter 2 consists of the two following sections: 2.1 «The Basic Theories»; 2.2 «Image Algebras».

Section 2.1 «The Basic Theories» describes three basic theoretical approaches in the field of pattern recognition: 1) "Pattern Theory" (U.Grenander) – techniques for data representation and transformation on the base of regular combinatorial structures and algebraic and probabilistic means in pattern recognition; 2) "Theory of Categories Techniques in Pattern Recognition" (M.Pavel) – formal descriptions of pattern recognition algorithms via transforms of initial data preserving its class membership; 3) "The Algebraic Approach to Recognition, Classification and Forecasting Problems" (Yu.Zhuravlev) – mathematical set-up of a pattern recognition problem, correctness and regularity conditions, multiple classifiers.

Section 2.2 «Image Algebras» contains a description of two known image algebras: 1) Standard Image Algebra by G.Ritter – a unified algebraic representation of image processing and analysis operations; 2) Descriptive Image Algebra by I.Gurevich – a unified algebraic language for describing, performance estimating and standardizing algorithms for image analysis, recognition and understanding.

- Chapter 3 "Descriptive Approach to Image Analysis and Understanding (DAIA) and its main tools"

The chapter describes basic concepts and mathematical tools of DAIA of I.B.Gurevich and his school (conceptualization of notions for characterizing images in pattern recognition problems; basic model of image recognition process; descriptive image models).

Chapter consists of the three following sections: 3.1 “Descriptive Approaches – basic steps”; 3.2 “DAIA”; 3.3 “Descriptive Image Models”.

Section 3.1 «Descriptive Approaches – basic steps» contains a description of papers in the field of pattern and image recognition in 1960’s, which gives main attention to a formal description of initial data, and to a formalization of description procedures of their transforms (F.Ambler, G.Barrow, R.Burstall, T.Evans, S.Kaneff, R.Kirsh, R.Narasimhan, A.Rosenfeld, A.Shaw).

Section 3.2 «DAIA» contains a description of basic concepts of DAIA. The main intention of DAIA is to structure different techniques, operations and representations being applied in image analysis and recognition. The axiomatic and formal constructions of DAIA establishes conceptual and mathematical base for representing and describing images and its analysis and estimation. The DAIA provides an opportunity to solve the problems connected with the development of formal descriptions for an image as a recognition object as well as the synthesis of procedures for an image recognition and understanding. The analysis of the problems is based on the investigation of inner structure and content of an image as a result of the procedures “constructing” it from its primitives, objects, descriptors, features and tokens.

Section 3.3 «Descriptive Image Models (DIM)» contains a description of mathematical objects providing representation of information carried by an image and by an image legend (context) in a form acceptable for a recognition algorithm. This section places high emphasis on multiple DIM and multi-aspect image representations.

- Chapter 4 “Example”

Chapter “Example” demonstrates application of the descriptive techniques in an application problem - automating of morphologic analysis of cytological specimens (lymphatic system tumors).

This chapter consists of the three following sections: 4.1 “Problem set-up”, 4.2 “Mathematical means used for formal representation of a descriptive model of an information technology for early diagnostic analysis of cytological specimens”; 4.3 “Discussion of the results’.

- Conclusions

The last chapter “Conclusions” discusses open questions of the mathematical theory of image analysis. Future researches for development of this field are outlined.

The program of the tutorial is briefly presented at Table 1.

Table 1. Tutorial summary.

Name of a Part	**Content**
Introduction	
State of the art of mathematical theory of image analysis.	
The Algebraic Approaches and Techniques in Image Analysis. Algebraization of Pattern Recognition (1970 – till now).	2.1 The Basic Theories
	2.2 Image Algebras
Descriptive Approach to Image Analysis and Understanding (DAIA) and its main tools	Descriptive Approaches
	DAIA
	Descriptive Image Models
Example	Problem set-up
	Mathematical means used for formal representation of a descriptive model of an information technology for early diagnostic analysis of cytological specimens
	Discussion of the results
Conclusion	

Figure 1 presents a classification reflecting the authors' point of view on the contemporary hierarchy of algebras and the place of DIA in this hierarchy.

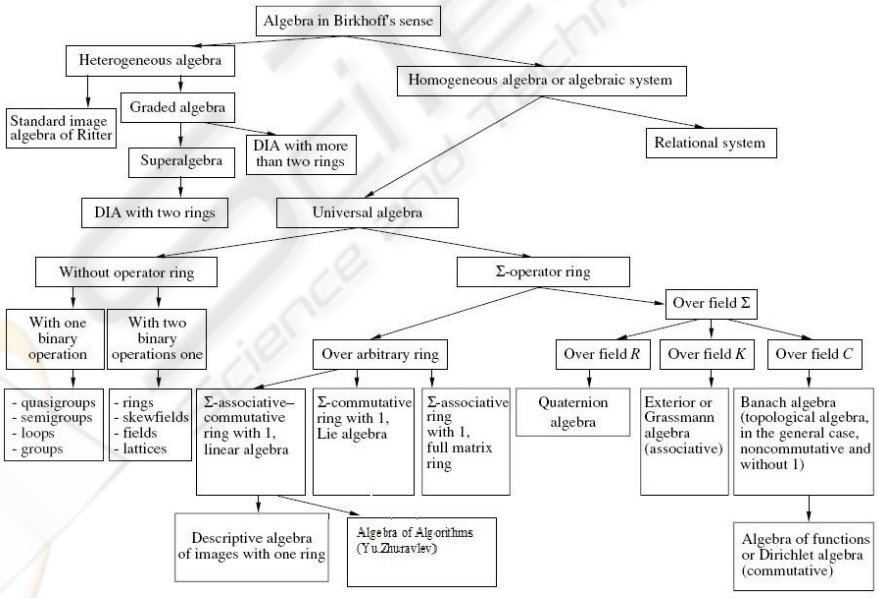

Fig. 1. Algebraic links.

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