Skeleton-Based Shape Segmentation

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Abstract. Shape decomposition into meaningful parts (segmentation) problem is considered in this paper. The shape is given either as a raster object on a homogeneous background or as a polygonal figure. A new shape decomposition approach is called *skeleton-based segmentation*. The approach is based on continuous skeletons that provides an opportunity to construct visually proper segmentations reflecting the shape structure. The proposed skeleton-based segmentation method stands out against known methods because it is suitable to work correctly with pairs of shapes. For a pair of shapes it is proposed to construct isomorphic skeleton-based segmentations which can be used on shape comparison and morphing applications.

1 Introduction

Shape decomposition (segmentation) is a shape representation as a combination of components (parts). The idea is to represent complex shapes in terms of simpler components. If we have a decomposition of the shapes into parts, instead of matching the shapes globally we could proceed by matching parts.

Shape segmentation is an important problem in many document processing applications (like organizing and querying an image database, recognition and computer-vision problems, medical structure comparison etc.)

The goal of the present paper is to develop an effective *segmentation method* that may be useful when working with *pairs of shapes*, especially in recognition and morphing applications. Thus the segmentation should reflect the structure of the shape and be stable to small shape fluctuations. Moreover, two similar shapes should have similar segmentations. The segmentation method should be universal to work either with raster objects or with polygonal figures.

The method presented in the paper is based on continuous skeletons. Continuous skeleton reflects the shape structure but is not stable to boundary noise. The skeleton could be considered as a graph. Skeletal graph could be transformed to become stable to small shape variations. In case the shape is given as raster object on homogeneous background, the approximating poligonal figure and its skeleton could be constructed [1]. For a pair of shapes isomorphic skeletons and isomorphic skeleton-based segmentations are constructed that allows solving such problems as *morphing* and *shape comparison*.

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2 SHAPE SEGMENTATION

2.1 Problem Statement

Shape decomposition (segmentation) is a representation of the shape as a combination of components (parts). There are many ways to decompose a shape (pic. 1). Not all of them give a correct decomposition, choose the meaningful decomposition parts. Segmentation quality could be estimated based on different criteria, for example:

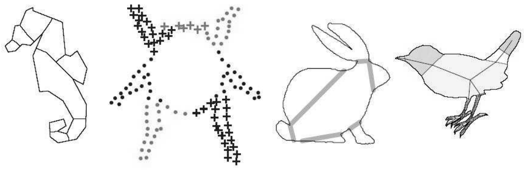

Fig. 1. Different shape decompositions.

1. Successful segmentation applications.
2. Correspondence to visual intuition.
3. Optimal number of segments in a final decomposition.
4. Computational speed of the segmentation algorithm.
5. Boundary noise stability.

The criteria 1-4 are subjective (2) or heuristic (1,3) or external a segmentation model (4). Thus such criteria can't be used to construct a correct segmentation. Noise stability requires to be correctly defined, though not sufficient to choose the best segmentation. Heuristic methods hardly fit the real applications.

In this work the segmentation problem is stated to find the best and correct decomposition for a pair of shapes. Therefore the needed segmentation should satisfy the following *requirements*:

$requirement_1$**:** shape structure reflection;
$requirement_2$**:** small shape fluctuations stability, including boundary noises;
$requirement_3$**:** similarity for a pair of similar shapes;

2.2 Previous Work in Shape Segmentation

Existing segmentation methods can be classified into two groups:

1. Those that are boundary-based, using only contour information for extracting parts.
2. Those that are region based, using information about the interior of the shape.

First are not suitable to the problems where structure analysis is required. Second often use skeletons [4, 6, 3, 5]. Most of existing approaches do not contain the correct criterion of choosing the segmentation method. The methods are hardly suitable when pairs of shapes ought to be considered. For example [4] proposes one the method that decomposes the figure into its ”significant parts” based on linear skeleton. It's not always true for two similar figures that their decomposition gives same ”significant parts” which is not correct.

2.3 Skeleton-Based Shape Segmentation

Medial axis (skeleton) of the shape [1] is a set of all maximal circles inscribed in the shape.

Let's consider the *continuous skeleton of a raster* object given on a homogeneous background as a medial axis of object's approximating figure (the figure that separates object from the background). The latter could be effectively constructed using known methods [1]. An example of the object's approximating figure is the polygonal figure of minimal perimeter [1] (fig. 2).

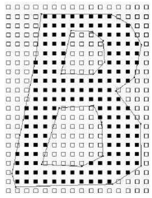

Fig. 2. Polygonal figure of minimal perimeter for a raster object.

Skeleton reflects the shape structure. Therefore using of the skeleton the first requirement of the stated problem is satisfied automaticallys.

The *skeleton-based segmentation* is considered as a special decomposition (fig. 3) on a finite number of skeleton edges areas (segment areas). The skeleton-based segmentation of a shape can be constructed by definition as described in algorithm 1.

To simplify the algorithm several remarks should be made. By definition the segment area $SubArea(v_iv_{i+1})$ of an edge v_iv_{i+1} is the set of all perpendiculars each point of an edge v_iv_{i+1} to $B(R)$. However there is no need ro find the continuum number of perpendiculars. The segment area $SubArea(v_iv_{i+1})$ is bounded by two (for a terminal edge) or four (for an in internal edge) as well as the boundary $B(R)$.

The skeleton-based segmentation shown on a figure 3 has the follwong elements:

(a) a polygonal approximating border (or a polygon itself in case the shape is given as a polygon);
(b) continuous skeleton;
(c) perpendiculars from skeletons vertices;
(d) segment areas (skeleton edges areas).

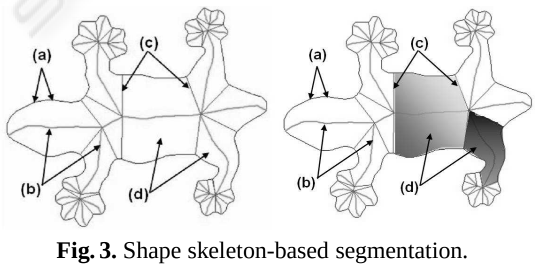

Fig. 3. Shape skeleton-based segmentation.

Algorithm 1: Skeleton-based segmentation construction: $\mathfrak{M} : Shape \rightarrow Seg(Shape)$.

Require: $Shape \in \{R, P\}$, Shape is a raster object or a polygonal figure P;
Ensure: Skeleton-based segmentation $Seg(Shape)$;
1: **if** figure is raster object $(Shape = R)$ **then**
2: construction of a polygonal boundary approximation $Appr(R)$ of a raster object R;
3: let the boundary of a shape is a constructed boundary approximation $Border(Shape) := Appr(R)$;
4: **else**
5: let the boundary of a shape is a given polygon $Border(Shape) := P$;
6: construction of the shape's skeleton $sk = MA(Shape)$;
7: n — the number of skeleton sk edges;
8: $v_1, ..., v_m$ — all skeletal graph sk vertices;
9: **for** all vertices v_k: $k = 1, ..., m; deg(v_k) \geq 3$ **do**
10: construction of perpendiculars from v_k to $Border(Shape)$;
11: the number of perpendiculars from v_k equals to v_k: $deg(v_k)$;
12: **for** $i = 0, \ldots, n$ **do**
13: fix the edge $e_i = v_i v_{i+1}$;
14: find an edge area $SubArea(e_i)$;
15: $Seg(Shape) = \bigcup_{i=0}^{n-1} SubArea(v_i v_{i+1})$ — needed skeleton-based segmentation.

The medial axis is not stable to small shape changes. Therefore segmentations based on such skeleton are not stable either. Therefore applications with pairs of objects involved can't use such segmentations (fig. 4). The segmentation method should be adapted to be more stable and to satisfy the 3rd problem statement requirement.

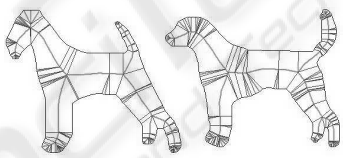

Fig. 4. Skeleton-based segmentations for a pair of shapes.

3 Skeleton-based Segmentation for a Pair of Shapes

The proposed segmentation method may use a modified continuous skeleton (step 6). A subgraph of the medial axis could be taken, for example. When choosing the best skeleton we have to eliminate several problems.

- A skeleton may often have *noise branches* that have nothing in common with general shape's structure (odd noisy branches affect odd not meaningful segments appear).
- A skeleton may have serious structure changes (branches reversing) affected by small *boundary variations* (therefore segments may be reversed in similar figures as well).

The first problem could be solved as an example by noisy branching cutting with a fixed accuracy [8]. Thus skeletons and skeleton-based segmentation become more stable (fig. 5). However the second problem remains unsolved. Moreover as per stated problem a pair of similar shapes should have similar segmentations (the third requirement of 2.1). This requirement should be mathematically reformulated. Therefore the definition of *"segmentation isomorphism"* should be given.

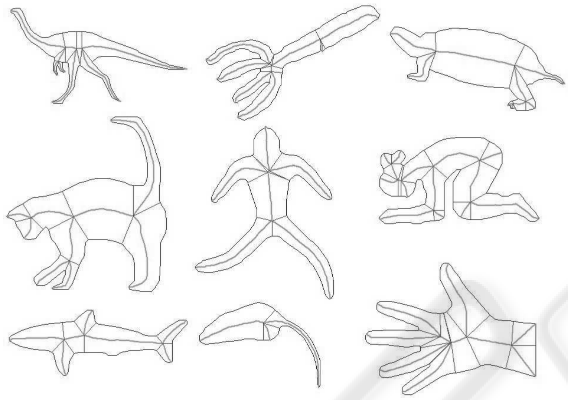

Fig. 5. Segmentations based on a skeleton with a fixed accuracy.

3.1 Segmentation Isomorphism

Two *graphs are isomorphic* $G \cong H$ if a mapping between their vertices keeps their edges adjacency.

Segmentation dual graph is a plain graph that has

– *vertices* corresponding to segments and two vertices;
– two vertices make an *edge* if corresponding segments are adjacent in segmentation (i.e. has the common border).

Two segmentations are isomorphic if their dual graphs are isomorphic.

Skeleton-based segmentations seem to reflect the visual meaningfulness of the shape (fig. 3,5). However they are hardly isomorphic for two similar shapes (an example on fig. 3). It means that two similar figures have different meaningful parts which is incorrect. When we work with pairs of shapes it is useful to find their isomorphic segmentations for several reasons:

1. If shapes are similar, their corresponding parts (by isomorphism) are often those "significant parts" reflecting general shape structure. Therefore it is easy to
   (a) make shape analysis;
   (b) compare shapes;
   (c) solve a morphing problem by map corresponding parts.
2. When shapes are not really similar we still may use their isomorphic segmentations as follows:
   (a) estimate the difficulty (or impossibility) of isomorphic segmentations construction, therefore compare shapes;
   (b) solve the morphing problem even for not similar shapes.

3.2 Skeletal Graphs

Medial axis can be represented as a planar graph [2], i.e. *a skeletal graph*. *Vertices* of the skeletal graph are the centers of maximal inscribed circles that touch the shape's boundary $\partial\Omega$ in three or more points. *Edges* of the skeletal graph touch the shape's boundary $\partial\Omega$ in two or more points. A skeleton vertex that has only one incident edge is called *a terminal vertex*, more than one edge — *a knot*. An edge that is incident to a terminal vertex is called *a terminal edge*.

Skeletal graphs provide a good opportunity to work effectively with shape structure.

3.3 Skeleton Isomorphism

Two skeletons are isomorphic if their skeletal graphs are isomorphic and the traversal order of terminal vertices is the same in both graphs.

Theorem 1: *Segmentations based on isomorphic skeletons are isomorphic as well.*

Basing on the theorem 1 obtaining isomorphic skeleton-based segmentations could be reduced to obtaining isomorphic skeletons for a pair of shapes. Here is the problem: how could we obtain isomorphic skeletons of two shapes if medial axis is so unstable to small shape fluctuations and noise? The solution is given in [7]. In that work the followong problem has been solved: *For two given raster objects I_1 and I_2 construct their approximating shapes $F_1' \in \Phi_{\varepsilon I}$ and $F_2' \in \Phi_{\varepsilon I}$ that have isomorphic skeletons $ma(F_1) \cong ma(F_2)$.* $\Phi_{\varepsilon I}$ denotes the class of shape approximations with an ε accuracy, i.e. all shapes f that has not more than ε distance to the fixed shape I are in class $\Phi_{\varepsilon I}$, i.e. $\Phi_{\varepsilon I} = \{f : \mu(f, I) < \varepsilon\}$. The distance between raster shape and its approximating shape $\mu(f, I)$ is defined as a maximum of two Hausdorff distances: between black pixels and the approximating shape and between white pixels and the approximating shape supplement.

As a stated problem solution an algorithm is given in [7]. In the algorithm the skeleton and the figure are changed simultaneously at each step. Two types of operations are handled: terminal branches cutting and internal knots merging (branch deletion). Shape is changed a bit at each step. The algorithm stops when two isomorphic skeletons are obtained. Therefore as an output of an algorithm two changed (not more than for a fixed accuracy ε) shapes are obtained. Let's denote an algorithm described in [7] as follows: $\mathfrak{L}(Shape_1, Shape_2) \rightarrow \{Shape_1\prime, Shape_2\prime\}$, where $MA(Shape_1\prime) \cong MA(Shape_2\prime)$.

Therefore, skeleton-based segmentation construction for a pair of shapes is based on the theorem 1 and could be done using the algorithm 2. An example of isomorphic segmentations obtained by this algorithm is shown on figure 6.

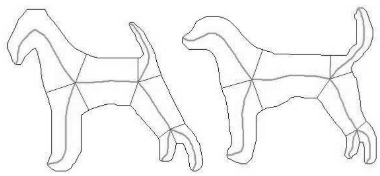

Fig. 6. Isomorphic skeleton-based segmentations.

Algorithm 2: Skeleton-Based Segmentation Construction for a Pair of Shapes: $\mathfrak{A}(Shape_1, Shape_2) \rightarrow \{Seg(Shape_1), Seg(Shape_2)\} : Seg(Shape_1) \cong Seg(Shape_1)$

Require: $Shape_1 \in \{R, P\}$,$Shape_2 \in \{R, P\}$. Shapes are raster objects or polygonal figures;
Ensure: $Seg(Shape_1), Seg(Shape_2)$ — isomorphic skeleton-based segmentations;
1: construction of the close shapes having isomorphic skeletons using the algorithm

$$\mathfrak{L}(Shape_1, Shape_2) \rightarrow \{Shape_1\prime, Shape_2\prime\};$$

2: 1st skeleton-based segmentation construction: $\mathfrak{M}(Shape_1\prime)) \rightarrow Seg(Shape_1)$.
3: 2nd skeleton-based segmentation construction: $\mathfrak{M}(Shape_2\prime)) \rightarrow Seg(Shape_2)$.

4 Shape Comparison Using Skeleton-based Segmentation

Shape comparison is the problem of definition the similarity measure for two given shapes. Skeleton-based segmentation can be used to solve this problem in the following ways:

1. Let two shapes are fixed and two isomorphic skeleton-based segmentations are constructed for them. The cost should be defined that means how difficult is to obtain the isomorphic skeleton-based segmentations (fig. 7). This cost is considered to be a similarity measure for two shapes.
2. The process of isomorphic skeleton-based segmentations is not taken into account. Let's consider the final isomorphic skeleton-based segmentations. Corresponding significant parts could be matched and thus a very good and correct metrics between two segmentations could be defined.

5 Shape Morphing Using Skeleton-Based Segmentation

Shape morphing problems can be stated as follows:

5.1 Problem 1: Moving Object Painting

Given: two shapes D_1 and D_2. The first shape D_1 has the color function $f(x), x \in D_1$

The task: paint the second shape D_2 according to the colouration of the first one D_1. (fig. 6)

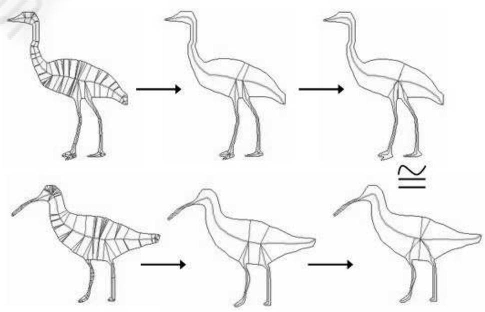

Fig. 7. Shape Comparison Basing on Cost.

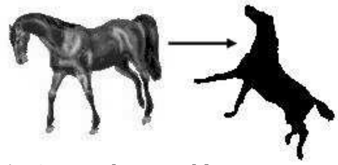

Fig. 8. Morphing problem statement.

5.2 Problem 2: Shapes Mapping

<u>Given</u>: two shapes D_1 and D_2.

<u>The task</u>: find the homeomorphic mapping of the first shape D_1 into the second one D_2.

5.3 The Stated Problems Solution Using Skeleton-Based Segmentation

The second problem could be solved by construction of two isomorphic skeleton-based segmentations of shapes D_1 and D_2. If two skeleton-based segmentations are isomorphic then each pair of segments can be mapped, i.e. a homeomorphism $\varphi : D_1 \to D_2$ could be constructed.

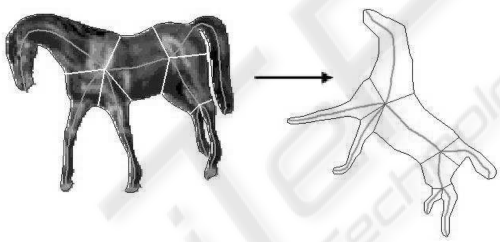

Fig. 9. Morphing Problem Solution.

The first problem may use the solution of the second problem, i.e. the homeomorphism $\varphi : D_1 \to D_2$. Once the homeomorphism is constructed the second shape D_2 could be painted as follows: $\forall x \in D_2$ the color is $f(\varphi^{-1}(x))$.

6 CONCLUSIONS

In this paper a new approach to shape decomposition is presented. An approach involves skeleton-based segmentation construction. The proposed decomposition reflects the shape structure, stable to small shape fluctuations and to boundary noise. The central idea lays in using continuous skeletons. An effective algorithm to construct such segmentations is presented for a shape given either as a raster object on homogeneous background or as a polygon.

For a pair of shapes it is proposed to construct isomorphic skeleton-based segmentations. Isomorphic decompositions are easily applied to shape comparison and morphing problems. Further development and implementation of methods are in progress.

Acknowledgements

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