

# HOW TO VISUALISE ABSTRACT TOPICS IN COMPUTER AND COMMUNICATION SCIENCE

Luděk Kučera

*Dept. of Applied Mathematics, Faculty of Mathematics and Physics, Charles University  
Malostranské nám. 25, Prague, Czech Republic*

**Keywords:** Algorithm visualisation, Protocol visualisation, Algorithm animation, Program invariant, Mathematical proof, Intuition, Algorithmic idea.

**Abstract:** The paper deals with learning of algorithms and protocols using visual media and it presents experience obtained with a system *Algovision* developed at Charles University, Prague. The teaching of the paper is that learning objects and courses should attempt explaining *why* an algorithm or protocol achieves its goals rather than merely showing *what* is going on during the computation and/or communication and *how* the data change in time. This means visualising abstract topics like algorithm invariant, mathematical proof, researcher intuition, and a collection of paradigms used to achieve such task is presented, as it appeared during development of *Algovision*.

## 1 INTRODUCTION

The paper deals with learning of algorithms and protocols using visual media and it presents experience obtained with *Algovision*, a system developed at Charles University.

However, the main aim of the paper is to argue that a proper way to creating systems that make learning of algorithms and protocols easy and efficient is based on showing *why* an algorithm or protocol achieves its goal, and what is the idea which is *behind* and which lead to discovery of the algorithm or establishing a protocol, rather than merely showing *what* is going on during the computation and/or communication and *how* the data change in time.

Most of the present learning systems and course collections implement just the *what* and *how* dimension. Even though such learning objects bring all information that is necessary to learn how an algorithm or protocol works, this information is unstructured and hidden, and it is too difficult (if not impossible) for a learner to infer the laws the objects in the screen follow. This is why the present ways of visualisation of algorithms and protocols are not as successful in learning as we hoped ten or fifteen years ago.

The experience we obtained when developing and using *Algovision* is that incorporating the *why*

coordinate into interactive and/or dynamic visual objects and courses is not a straightforward task that can be described by a single paradigm or directive.

Building of why-learning objects turned out to be very case specific, but we tried to prepare a list of approaches that seem to be useful as general methods of enhancing understandability of our learning products.

There is another teaching that we get when developing *Algovision*. The system was originally conceived as a support of a teacher in the standard classroom instruction, but it turned out that the best presentations prepared for classroom teacher-students applications are in the same time the best e-learning objects and vice versa. Thus, it is not wise distinguishing between teacher support and e-learning and/or distant learning objects, because they just represent two possible applications of the same computer supported educational tools.

## 2 STATE OF THE ART

About two decades ago dynamic graphics became available and started a boom of dynamic visualisation systems aimed for learning of algorithms and protocols, for an overview see (Stasko et al., 1998), (Kerren, Stasko, 2002).

Both algorithms and protocols operate on abstract data and objects in such a way that change the value of data and the status of objects by following a fixed and predetermined set of instructions and/or rules. Even though data and objects are abstract, they are usually an abstraction of real life entities, and therefore a quite intuitive and natural way of visualisation of data and manipulated objects usually exists and doesn't bring any logical problems to users of a learning system.

A general philosophy of *animation*, which is the main way in which a dynamic interactive visual system is used to enhance learning of algorithms and protocols, is to show how the visual data representation changes to follow changes of the processed data.

The idea of animation is so natural and simple, that many researchers even try to build systems that are able to create an animation of a given algorithm or protocol automatically or using just simple directives of a user.

However our great expectations have materialized only partially and dynamic and interactive visualisation is still much less used than we would like to and we saw in our predictions in early 90's. (Catrambone, Stasko, 1996), (Hundhausen et al., 2002).

### 3 VISUALISATION OF "WHY"

#### 3.1 Why We Visualise "why"

As already pointed above, most visualisations deliver a large amount of information (where information is understood in the sense of Shannon, i.e., as the number of bits), but much less *knowledge* (where knowledge is understood in the sense of cognition theory). As an author's student said: "In order to understand what is going on in the screen I'd need to understand the algorithm that is animated" - a poor view of an animation aimed in helping students to learn the algorithm.

Such visualisations could only be used as illustrations and exemplifications for those who have already learnt the topic elsewhere, but not as tools useful for learning directly. This explains certain disappointment with a use of computer-supported learning.

In the remaining part of the section, we will discuss several methods to increase the "knowledge density" of a teacher-supporting and/or e-learning presentation by visualisation the underlying idea of a

computational and/or communicational method rather than its external behaviour.

#### 3.2 Changing View

There are many cases when a "natural" visualisation hides the underlying rules and it is necessary to switch to another view that makes inferring rules easy. Such a change is often based on a major discovery in the science. Let us give some examples to illustrate the paradigm.

The first example is just a metaphor, but explains well our approach. The night sky (Fig. 1A) is a magnificent tool for visualizing planet trajectories. Unfortunately, what we see looks so complex and strange that this kind of visual presentation practically prevents inferring any rule of planet movements (mankind needed centuries to do it).



Figure 1A: Natural visualisation of planet trajectories.

The proper why-learning way of presenting the subject is a drawing of the Kepler's model of the solar system (Fig. 1B).

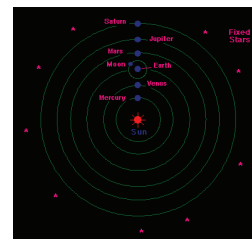


Figure 1B: Kepler's visualisation of planet trajectories.

An example from wireless communication is formally very similar to the previous celestial mechanic one. Code Division Multiple Access (CDMA) is a method how several users of an advanced cellular telephony system can share the same communication channel. A standard way of visual representation of *signatures* used in CDMA uses time-amplitude plots; in this approach the users' signals and their combination look as if they were observed on a screen of a multi-channel oscilloscope, which is the most natural visualisation

in electrical engineering. For illustration, see, e.g., [http://www.vias.org/simulations/simsoft\\_dscdma.ht.ml](http://www.vias.org/simulations/simsoft_dscdma.ht.ml).

However, a model that parallelizes Kepler's model of the preceding paragraph and explains properties and protocols of CDMA in an easier way views signatures as 0,1-vectors that form a (quasi) orthogonal basis of an  $n$ -dimensional vector space and combinations of users' bits are represented as vectors in said space, see Fig. 2 for  $n=2$ .

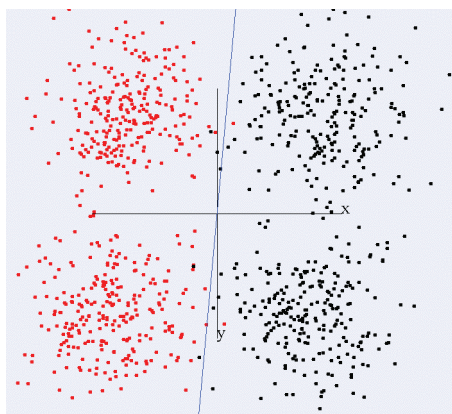


Figure 2: CDMA signatures and signals in a two-user noisy channel - a geometric intuition.

### 3.3 Program Invariants

There is a paradigm that proved to be very successful for visualisation of algorithms in a way that helps understanding the idea of the algorithm.

The paradigm comes from a field that is usually called "software verification" if developed to solve practical problems, or "theory of program correctness", if studied as a theoretical discipline.

An *invariant* of a program or an algorithm is a logical statement that represents a property of data (program variables) that remains valid (i.e., is invariant) during the whole computation and which, together with termination condition, implies a desired property of the output data (the result of a computation).

Given an invariant, it is easy (but usually very tedious) to prove that the output has always a desired property (e.g., that a shortest path algorithm really finds the shortest path).

Thus, the only problem when proving correctness of a program with respect to a given output property, is to find a proper invariant. This, however, is an extremely difficult problem; it is even proved that in general the problem of constructing an invariant for a given program and a desired output property is algorithmically insolvable.

The only known method of invariant construction is a meta-rule: you must understand, what a program is doing, and then write down all relations between variable values that you could imagine. In other words, understanding implies ability to construct an invariant.

Fortunately, this rule can be inverted: an invariant is usually a formal description of a strategy that is followed to reach the desired goal, and who knows an invariant also understands what is going on during computation.

The examples of successful use of invariants in visualization could be found in *Algovision* in the section about shortest paths (algorithms of Dijkstra and Bellman-Ford, see, e.g., Corman et al., 2001).

Since Dijkstra's shortest path algorithm is a part of Algorithm course at most CS departments over the world, many animations of the algorithm exist on the web. However, to the author's best knowledge, no one of them attempts to help a learner to see the algorithmic idea that is behind. Thus, such animations do not represent an independent learning tool, being just an illustration for those who already know the method.

In general, graph algorithms are typical examples where an animation is useless as a learning tool unless it displays in a proper way the algorithm invariant.

### 3.4 Animated Constructions

In certain cases, both as a part of instruction in computer hardware and software, an algorithm is represented as a combination circuit, which can be seen both as a layout of an asynchronous chip without feedback and a logical scheme of a program without loops.

A standard animation that can be found in the web for many problems shows how data propagate through the circuit from inputs to outputs.

Animating a circuit function is good, but much better learning results can be obtained when using (as it is in *Algovision*), *two* orthogonal animations. The first one is the function animation mentioned above; the other one shows in a step-by-step way how the circuit is constructed.

An animation of construction begins by showing the circuit as a single black box with inputs and outputs, but without any internal structure displayed.

The animation continues by refining building blocks in a way that illustrates well the logical structure of a circuit. At each refinement stage a circuit is fully functional, i.e., it is possible to animate its function.

In Algovision the multi-animated approach is used in a pure form in the course on Bitonic Sorting (Batcher, 1990), and in a slightly modified way in a course on Carry Look-Ahead binary adder.

### 3.5 Animated Proofs

In some cases it turned out to be difficult to integrate directly explanation of the underlying idea or the proof of correctness into animation. In such a case it is necessary to demonstrate properties of a computing method in a way that might be formulated as a mathematical proof. Such a proof is often a proof of the existence of certain object.

E.g., the correctness of a bitonic splitter follows immediately from the existence of certain bisection of the input sequence, and in Algovision, the correctness proof can be viewed as a construction of such bisection.

In this way, the essence of a mathematical proof is often a construction algorithm, which can be animated in the same way as any other algorithm, the only difference being that it operates with properly visualised abstract notions rather than with objects that are more or less straightforward generalization of real-world entities.

### 3.6 Visualised Intuition

The most general way of explaining the approach of the present paper is the following: in order to achieve a new discovery, a researcher is guided by his or her intuition. It is difficult or perhaps impossible to explain in more specific terms what is an intuition, but creative researchers understand surprisingly well the term.

Often, when speaking about their initial intuition, researchers use visual terms and images, and it is just sufficient to put such visual ideas to the screen.

We are sure that our present view of the solar system was originally a vague intuition in Kepler's head, and similar is the case with many other human intellectual achievements.

One example, perhaps too simple to illustrate well the idea of this subsection, but relatively easy and short, is the Voronoi diagram in the plane and how we can see it in a way that has proved to be quite fruitful and which is used in Algovision.

Many facts about Voronoi diagrams, including the background intuition for an important algorithm that is included in Algovision (Fortune 1987), follow directly from the following view of the situation, see Fig. 4.

The plane containing the sites is viewed as a horizontal plane embedded into the 3D space. Each site is the top of a corresponding cone that has a vertical axis. In such a way one obtains a mountain range, which, viewed in vertical direction from above looks exactly like the Voronoi diagram of the sites (more precisely, visible intersections of cones, projected to the site plane, give the diagram).

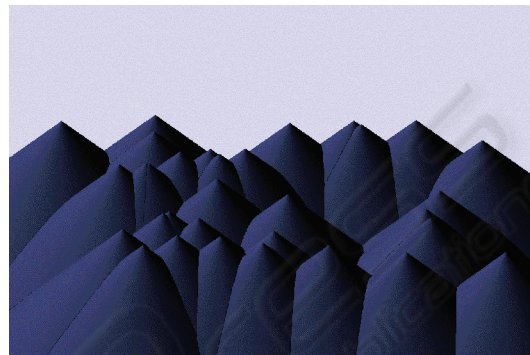


Figure 4: Voronoi diagram - cone mountains.

Although this intuition is again a folklore and highly simplifies understanding certain Voronoi diagram algorithms, it is not used in standard learning courses, because it doesn't help too much if displayed as a static figure similar to Fig. 4.

However, Algovision uses standard methods of 3D graphics (mouse controlled rotations, etc.) which allows observing the mountains horizontally or under a general angle to see how the mountains are built, vertically to see the Voronoi diagram, and under 45° to visualise original Fortune's intuition.

### 3.7 Virtual Tools, Devices and Gadgets

In certain cases it is very useful to build virtual tools or devices that look and function like physically existing or abstract measurement or visualising devices. When playing with them, or using them to solve given problems, a learner gets new knowledge in a much more efficient way, compared to standard and widely used learning procedures.

A typical example of the paradigm that appears in Algovision is a Discrete Fourier Transform (DFT) gadget. A learned draws a function in the upper window (or selects a function from a predefined list prepared to cover all interesting cases and features of the problem) and the lower window shows the corresponding Fourier spectrum.

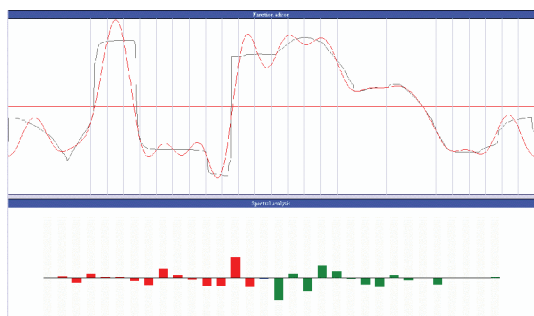


Figure 5: Discrete Fourier Transform gadget.

However, the main use of the gadget in learning DFT goes in the opposite direction. Given an input function (drawn in black), a learner should find the spectrum by himself or herself; a red function that corresponds to the actual spectrum appears in the upper window and should match the black one in all sampling points represented by the vertical lines.

The device provides several levels of hints to make this difficult task easier (e.g., sets the correct value of certain spectrum item, or at least indicates whether the present value is too small or large).

It takes typically several hours of hard work in a trial-and-error style to find ways to match at least roughly the black and the red functions, because the correspondence between a digital signal and its Fourier image is conceptually rather complex, but learners find the task challenging and even funny and eventually get surprisingly high level of understanding of the essence of DFT.

### 3.8 Visual Hints

Visual presentations can quite often be enhanced by visual hints. One source of such hints is a use of colours when visualising algorithms and protocols with temporal features. Any displayed object has typically a particular status; it can be processed and/or exhausted (dead), new, fresh, pending, active, etc. In our cultural range such terms are associated with colors (fresh=green, dead=black, attention or stop=red, etc.).

Another kind of visual hints uses shapes or forms. An example of a shape visual hint is used in the Algovision implementation of AVL-tree course, where balance of a node is shown using ideas of Calder's mobiles.

## 4 CONCLUSIONS

We found that visual learning objects and courses directed to algorithms and communication protocols should be constructed to explain the underlying idea, in other words, *why* it works in a presented way. Doing so is more art than science or methodology, but we listed several paradigms and approaches that proved to be useful and give good results.

## ACKNOWLEDGEMENTS

Work on Algovision was partially supported by Czech Ministry of Education, Youth and Sports.

## REFERENCES

- Batcher, K., 1990. On bitonic sorting networks, *ICPP 1*, pp. 376-379.
- Cormen, T., Leiserson, C., Rivest, R., Stein, C., 2001. *Introduction to Algorithms*, The MIT Press, Cambridge, MA, 2<sup>nd</sup> edition.
- Catrambone, R., Stasko, J., Do Algorithm Animations Aid Learning?, 1996. *TR GIT-GVU96-18*, Georgia Institute of Technology.
- Dijkstra, E.V., 1959. A note on two problems in connection with graphs. In *Numerische Mathematik*, 1 (1959), pp. 269–271.
- Fleischer, R., Kučera, L. 2001. Algorithm Animation for Teaching. *Software Visualization* pp. 113-128
- Fortune, R., Lopes, J., 1999. Paper templates. In *TEMPLATE'06, 1st International Conference on Template Production*. INSTICC Press.
- Fortune, S., 1987. A sweepline algorithm for Voronoi diagrams. *Algorithmica* 2, pp. 153-174.
- Hundhausen, C., Douglas S., Stasko, J., 2002. A Meta-Study of Algorithm Visualization Effectiveness, *J. of Visual Languages & Computing*, 13, pp. 259-290.
- Kerren, A., Stasko, J., 2002. *Software Visualization*. Springer-Verlag Berlin Heidelberg.
- Kučera, L., 2005. Visualisation of algorithms, *Int. J. of Continuing Engineering Education and Life Long Learning* 15 (3-6) pp. 212 – 224
- Kučera, L., Algovision, <http://kam.mff.cuni.cz/~ludek>
- Stasko, J., Domingue, J., Brown, M., Price, B., 1998. *Software Visualization*, The MIT Press, Cambridge, MA.