

# TOWARDS A UNIFIED DOMAIN FOR FUZZY TEMPORAL DATABASES

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Keywords: Fuzzy data, Temporal database, Fuzzy interval, Data manipulation.

Abstract: Temporal Databases (TDB) have as a primary aim to offer a common framework to those DB applications that need to store or handle temporal data of different nature or source, since they allow to unify the concept of time from the point of view of its meaning, its representation and its manipulation. At first sight, it may seem that incorporation of time to a DB is a direct and even simple task, but, on the contrary, it is a quite complex aim because time may be provided by different sources, with different granularities and meaning. The situation gets more complex when the time specification is not made in precise but in fuzzy terms, where together with the inherent problems of the time domain, we have to consider the imprecision factor. To deal with this problem, the first task to perform is to unify as much as possible the representation of time in order to be able to define the range and the semantics of the necessary operators to handle data of this type.

## 1 INTRODUCTION

### 1.1 Previous Concepts

Temporal Databases, in the widest sense, offer a common framework for all database applications that involve some temporal aspects when organizing data. These databases allow to unify the time concept from several points of view: the representation, the semantics and the manipulation.

Database applications involving temporal data are not a new subject. In fact, they have been developed since the relational databases began to be used, but applications programmers were responsible for designing, representing, programming and managing the necessary temporal concepts.

Temporal Databases (from now on TDB) have partially solved the problem because they provide data types and operators for handling time.

From the point of view of the real world, there exist two basic ways for associating temporal concepts to a fact:

1. *Punctual facts*: a fact is related to an only time mark that depends on the granularity and informs

about the time when it happened. As instances, birthdays, the date for an order, an academic year, ...

2. *Time periods*: that are represented by a starting instant and an ending one, so the duration (or valid time) of the fact is implicit. Some examples are: [admission date, discharge date], [start contract date, end contract date], ...

This way of time interpretation is called **valid time**.

In the valid time relation EMP (see fig. 1) each tuple represents a version for the available information about an employee, and this version is valid only when used in the time interval [VST, VET]. The up-to-date version, also called valid tuple, is *undefined-valued* in the attribute VET (a special value).

Sometimes it is not possible for the user to give an exact but an imprecise starting/ending point for the validity period of a fact. This is the case, for example, when a patient does not exactly know when a concrete ailment or symptom started. In this case, the use of fuzzy sets theory is necessary for not missing such important information since fuzzy time values could be defined (Barro et al., 1994). This situation give

EMPNAM	EMPID	SALARY	BOSS	VST	VET
GRANT	1245	1500	9877	15-06-1997	31-05-1998
GRANT	1245	1800	9877	01-06-1998	Undefined
REDFORD	9877	1200	4588	20-08-1994	31-01-1996
REDFORD	9877	1500	4588	01-02-1996	31-03-1997
REDFORD	9877	2200	9989	01-04-1997	Undefined
BROWN	1278	2800	4588	01-05-2005	10-08-2008
STREEP	6579	4000	9877	15-06-1997	Undefined

Figure 1: Instance example of the valid time relation EMP.

rise to a large number of new problems (Bettini et al., 1998), and this paper is devoted to the definition of time domain that allows the representation of different fuzzy time specifications.

## 1.2 Previous Concepts on Fuzzy Sets

A fuzzy value is a fuzzy representation about the real value of a property (attribute) when it is not precisely known.

In this paper, according to Goguen's Fuzzification Principle (Goguen, 1967), we will call every fuzzy set of the real line *fuzzy quantity*. A *fuzzy number* is a particular case of a fuzzy quantity with the following properties:

*Definition 1.-*

The fuzzy quantity  $A$  with membership function  $\mu_A(x)$  is a **fuzzy number** (Dubois and Prade, 1987) iff:

1.  $\forall \alpha \in [0, 1], A_\alpha = \{x \in R \mid \mu_A(x) \geq \alpha\}$  ( $\alpha$ -cuts of  $A$ ) is a convex set.
2.  $\mu_A(x)$  is an upper-semicontinuous function.
3. The support set of  $A$ , defined as  $Supp(A) = \{x \in R \mid \mu_A(x) > 0\}$ , is a bounded set of  $R$ , where  $R$  is the set of real numbers.

We will use  $\tilde{R}$  to denote the set of fuzzy numbers, and  $h(A)$  to denote the height of the fuzzy number  $A$ . For the sake of simplicity, we will use capital letters at the beginning of the alphabet to represent fuzzy numbers.

The interval  $[a_\alpha, b_\alpha]$  (see figure 2) is called the  $\alpha$ -cut of  $A$ . So then, fuzzy numbers are fuzzy quantities whose  $\alpha$ -cuts are closed and bounded intervals:  $A_\alpha = [a_\alpha, b_\alpha]$  with  $\alpha \in (0, 1]$ .

If there is, at least, one point  $x$  verifying  $\mu_A(x) = 1$  we say that  $A$  is a *normalized* fuzzy number.

Sometimes, a trapezoidal shape is used to represent fuzzy values. This representation is very useful as the fuzzy number is completely characterized by the four parameters  $(m_1, m_2, a, b)$  as shows figure 3 and the height  $h(A)$  when the fuzzy value is not normalized. We will call *modal set* all values in the interval

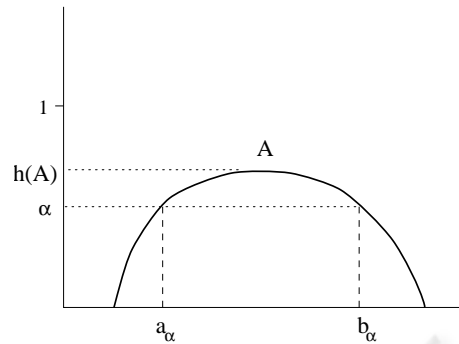


Figure 2: Fuzzy number.

$[m_1, m_2]$ , i.e., the set  $\{x \in Supp(A) \mid \forall y \in R, \mu_A(x) \geq \mu_A(y)\}$ . The values  $a$  and  $b$  are called left and right *spreads*, respectively.

In our approach, we will use trapezoidal and normalized fuzzy values.

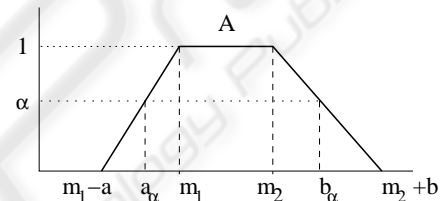


Figure 3: Trapezoidal fuzzy number.

## 2 FUZZY TIME REPRESENTATION

### 2.1 Imprecision Measure on Fuzzy Values

As pointed out in the previous section, we are going to translate fuzzy uncertainty into imprecision under certain conditions. The most important of these conditions is that the amount of information provided by the fuzzy number remains equal before and after the transformation. So then, the first step is to define an information function for fuzzy numbers.

In (González et al., 1999) we propose an axiomatic definition of information, partially inspired in the theory of generalized information given by Kampé de Fériet (de Fériet, 1973) and that can be related to the precision indexes (Dubois and Prade, 1987) and the specificity concept, introduced by Yager in (Yager, 1981).

*Definition 1.-*

Let  $\mathcal{D} \subseteq \tilde{R} \mid R \subseteq \mathcal{D}$ ; we say that the application  $I$  defined as:

$$I: \mathcal{D} \longrightarrow [0, 1]$$

is an **information** on  $\mathcal{D}$  if it verifies:

1.  $I(A) = 1, \forall A \in R$
2.  $\forall A, B \in \mathcal{D} \mid h(A) = h(B) \text{ and } A \subseteq B \implies I(B) \leq I(A)$ .

The information about fuzzy numbers may depend on different factors, in particular, on imprecision and certainty (Chountas and Petrounias, 2000). We focus on general types of information related only to these two factors.

To compute a measure of the imprecision contained in a fuzzy number, we will consider a measure of the imprecision of its  $\alpha$ -cuts, which are closed intervals on which the following function is defined:

$$\forall A \in \tilde{R}, f_A(\alpha) = \begin{cases} b_\alpha - a_\alpha & \text{if } \alpha \leq h(A) \\ 0, & \text{otherwise} \end{cases}$$

From this imprecision function on the  $\alpha$ -cuts, we define the total imprecision of a fuzzy value as a combination of the imprecision in every level  $\alpha$ . When  $\alpha = 0$ , we will consider that  $f_A(0)$  is the length of the support set.

*Definition 2.-*

The **imprecision** of a fuzzy number is defined as follows:

$$f: \tilde{R} \longrightarrow R_0^+$$

$$\forall A \in \tilde{R}, f(A) = \int_0^{h(A)} f_A(\alpha) d\alpha$$

That is, the imprecision function  $f$  coincides with the area below the membership function of the fuzzy value. Since we are considering that fuzzy time values are always normalized, then  $h(A) = 1$ .

## 2.2 Unified Domain for Temporal Data

In the introduction we have seen that, in classical TDB, the valid time is managed thanks to the extension of the tables schemata by adding two new attributes (Clifford and Rao, 1987) (Elmasri and Wu, 1990), the valid start time -VST- and the valid end time -VET- to determine the period of validity of the fact expressed by a tuple.

In this paper we are going to consider that the information provided by the VST and VET for the classical TDB is fuzzy, in the sense that we are not completely sure about when the current values of the tuple began to be valid.

The more immediate solution to this problem is to soften the VST and the VET in such a way that they

may contain fuzzy dates represented by means of a fuzzy number. This means that, if we use the parametrical representation for fuzzy numbers, we need to store four values for the VST and four values for the VET, as shown in figure 4. Since the meaning of the attributes VST and VET is the period of time during which the values of a tuple are valid, it is more convenient to summarize the information given by the two fuzzy attributes in an only but fuzzy interval (from now on FVP or fuzzy validity period). This situation can be represented by the trapezoidal fuzzy set shown in figure 5 which incorporates the semantics of our problem. As can be seen in such figure, the left and right sides of the interval is the part that reflects the imprecision about the starting and ending time point of the validity time of the facts associated.

EMPNAM	EMPID	SALARY	BOSS	EXPERTISE	VST	VET
GRANT	1245	1500	9877	TRAINEE	~15-06-1997	~31-05-1998
GRANT	1245	1500	9877	JUNIOR	~01-06-1998	~undefined

$(31-12-2050, 31-12-2050, 0, 0)$   
 $(01-06-1998, 01-06-1998, 2, 2)$

Figure 4: Internal representation of a fuzzy date.

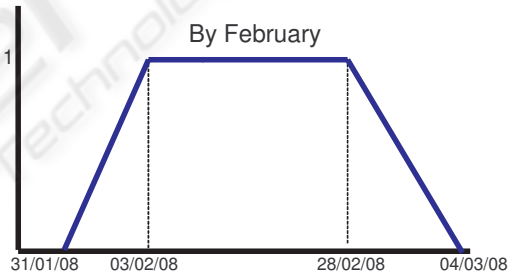


Figure 5: Fuzzy Period of Time for a Valid Tuple.

This representation has the advantage that, not only periods of time, but fuzzy dates can also be represented in a unified way. Think that a parametrical representation as (m,m,a,b) represents a central time point with some imprecision at both sides, what is interpreted as a fuzzy date.

The problem now is that the imprecision provided by the two fuzzy dates must be translated to the interval that summarizes the considered period of time. That is, all the imprecision of the starting date must be converted in the imprecision of the left side of the interval and, in the same way, all the imprecision of the ending date must be converted in the imprecision of the right side of the interval.

If we consider that a way to measure the imprecision of a fuzzy set is to compute its area, the problem we have in hands is a matter of geometrical computation.

The posed problem is shown in a graphical way in figure 6.

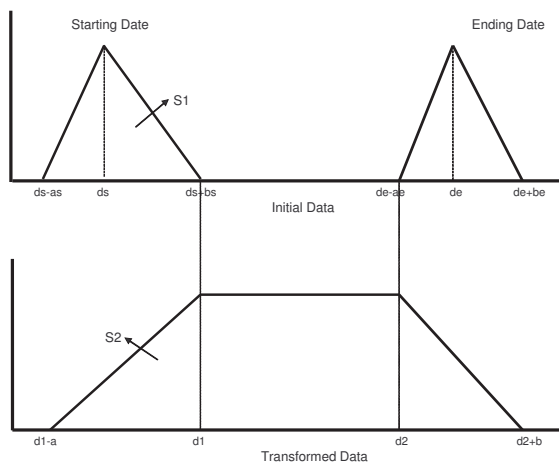


Figure 6: Transformation of two fuzzy dates into a fuzzy period preserving imprecision.

The resulting fuzzy interval is obtained by means of the equality  $S_1 = S_2$  that obliges to maintain the same amount of imprecision after the transformation is performed.

$$S_1 = S_2 \Rightarrow \frac{(d_s + b_s) - (d_s - a_s)}{2} = \frac{d_1 - (d_1 - a)}{2}$$

If we assume that the data associated to this time specification are precisely known from  $(d_s + b_s)$  to  $(d_e - a_e)$ , then  $d_1 = d_s + b_s$  and both terms become equal and  $d_1 - a = d_s - a_s$ , as shown in figure 6. The same substitution should be made to obtain the right part of the interval.

As it was explained in section 1.2, it is quite easy to represent a fuzzy interval with this characteristics since only four parameters need to be stored in order to specify it. In (Medina et al., 1994) (Medina et al., 1995) is presented a generalized model of fuzzy DB that supports this representation for fuzzy data and the corresponding implementation in a classical relational DB system (Oracle).

### 3 CONCLUSIONS AND FUTURE WORK

In this paper we have shown how to represent different time specifications in a unified way. The representation considered is the fuzzy interval, which results very suitable for both precise and imprecise time points and periods when the time is interpreted as

*valid time*. For the case that two fuzzy dates are provided by the user, it is necessary to perform a transformation to convert this original time information into a fuzzy interval that preserves the imprecision involved.

### ACKNOWLEDGEMENTS

This work has been partially supported by research projects TIN2008-02066/TIC and P07-TIC-03175.

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