A NEW DECONVOLUTION METHOD BASED ON MAXIMUM ENTROPY AND QUASI-MOMENT TRUNCATION TECHNIQUE

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Abstract: In this paper we present a new blind equalization method based on the quasi-moment truncation technique and on the Maximum Entropy blind equalization method presented previously in the literature. In our new proposed method, fewer moments of the source signal are needed to be known compared with the previously presented technique. Simulation results show that our new proposed algorithm has better equalization performance compared with Godard's and Lazaro's *et al.* algorithm.

1 INTRODUCTION

We consider a blind equalization problem in which we observe the output of an unknown, possibly non-minimum phase, linear system from which we want to recover its input using an adjustable linear filter (equalizer). The problem of blind equalization arises comprehensively in various applications such as digital communications, seismic signal processing, speech modeling and synthesis, ultrasonic nondestructive evaluation, and image restoration (Feng and Chi, 1999). Recently, a new blind equalization algorithm was proposed (Pinchas and Bobrovsky, 2006) with improved equalization performance compared with (Godard, 1980) and (Lazaro et al., 2005). It is valid for the real and complex (where the real and imaginary parts are independent) valued case. This new blind equalization method (Pinchas and Bobrovsky, 2006) is based on the Maximum Entropy technique and on some known moments of the source signal. The problem arises when these moments or part of them are unknown. In that case the blind equalization method (Pinchas and Bobrovsky, 2006) can not be used. Obviously, when using approximated moments instead of the real ones, the equalization performance might get worse and in some cases even lead to unacceptable performance. The quasi-moment truncation technique is related to the Hermite polynomials where the high-order central moments are approximated in terms of lower order central moments (Bover, 1978). Although the quasi-moment truncation technique (Bover, 1978) is well known in the non-linear optimal filtering theory (Bover, 1978), it is not yet been used in the field of blind equalization combined with the Maximum Entropy technique. In this paper we present a new blind equalization method based on the quasi-moment truncation technique and on the Maximum Entropy blind equalization method (Pinchas and Bobrovsky, 2006). Fewer moments of the source signal are needed to be known compared with (Pinchas and Bobrovsky, 2006). Simulation results will show that our new proposed algorithm has better equalization performance compared with Godard's (Godard, 1980) and Lazaro's *et al.* (Lazaro et al., 2005) algorithm. The paper is organized as follows: After having described the system under consideration in Section II, we describe in Section III the quasi-moment truncation technique which we use in this paper for approximating the unknown source moments. In Section IV we present our simulation results and Section V is our conclusion.

2 SYSTEM DESCRIPTION

The system under consideration is illustrated in Fig.1, where we make the following assumptions:
1. The input sequence $x(n)$ consists of zero mean

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real or complex (where the real and imaginary part of $x(n)$ are independent) random variables with an unknown even symmetric probability distribution.
2. The unknown channel $h(n)$ is a possibly nonminimum phase linear time-invariant filter in which the transfer function has no "deep zeros", namely, the zeros lie sufficiently far from the unit circle.
3. The equalizer $c(n)$ is a tap-delay line.
4. The noise $w(n)$ is an additive Gaussian white noise.
5. The function $T[\cdot]$ is a memoryless nonlinear function. It satisfies the analyticity condition:
$T(z_1 + jz_2) = T_1(z_1) + jT_2(z_2)$ where z_1, z_2, are the real and imaginary part of the equalized output respectively.

The transmitted sequence $x(n)$ is transmitted through the channel $h(n)$ and is corrupted with noise $w(n)$. Therefore, the equalizer's input sequence $y(n)$ may be written as:

$$y(n) = x(n) * h(n) + w(n) \tag{1}$$

where "$*$" denotes the convolution operation. This sequence (1) is then equalized with an equalizer $c(n)$. The equalizer's output sequence $z(n)$ may be written as:

$$\begin{aligned} z(n) = x(n) * h(n) * c(n) + w(n) * c(n) = \\ x(n) + p(n) + \tilde{w}(n) \end{aligned} \tag{2}$$

where $p(n)$ is the convolutional noise and $\tilde{w}(n) = w(n) * c(n)$. In this paper, we consider the equalizer proposed by (Pinchas and Bobrovsky, 2006) where the equalizer's taps are updated according to:

$$\begin{aligned} c_l(n+1) = c_l(n) - \mu W y^*(n-l) \qquad \text{with} \\ W = [(W_1 + W_2) - z[n]] \\ W_1 = E\left[x_1/z_1\right]\left[\frac{(z_1[n] E\left[x_1/z_1\right])}{\langle (z_1)^2 \rangle_n}\right] \\ W_2 = jE\left[x_2/z_2\right]\left[\frac{(z_2[n] E\left[x_2/z_2\right])}{\langle (z_2)^2 \rangle_n}\right] \\ \left\langle z_s^2 \right\rangle_n = (1-\beta)\left\langle z_s^2 \right\rangle_{n-1} + \beta \cdot (z_s)_n^2 \end{aligned} \tag{3}$$

where $()^*$ is the conjugate of $()$, μ is a positive stepsize parameter, l stands for the l-th tap of the equalizer, $\langle\rangle$ stands for the estimated expectation, $\left\langle z_s^2 \right\rangle_0 > 0$ $(s = 1,2)$, β is a positive stepsize parameter and $E[x_s/z_s]$ $(s = 1,2)$ is the conditional expectation derived in (Pinchas and Bobrovsky, 2006) with the use of the Maximum Entropy density approximation technique. This blind equalization algorithm (3) depends on some known moments of the source signal through the expression of the conditional expectation given in (Pinchas and Bobrovsky, 2006). The problem arises when we do not know these moments or we know only a part of them. In that case we can not use the algorithm. In the following we will show how we solve this problem and still obtain satisfying equalization performance compared with (Godard, 1980) and (Lazaro et al., 2005).

3 MOMENT APPROXIMATION

In this section we use the quasi-moment truncation technique (Bover, 1978) for approximating the unknown source moments. In the following we consider the real valued case. The quasi-moment truncation technique is related to the Hermite polynomials where the high-order central moments are approximated in terms of lower order central moments (Bover, 1978). According to (Bover, 1978), one way of achieving this is by expressing the probability density function $f_x(x)$ as an infinite series expansion in which the coefficients are known in terms of central moments. Then truncation approximations is done by assuming that high-order coefficients in this expansion are negligible. This would seem likely to occur when the basis for the expansion is an appropriate set of orthogonal polynomials (Bover, 1978). A natural choice of expansion basis is the Hermite polynomials (Bover, 1978) which was used by Kuznetsov, Stratonovich and Tikhonov (Kuznetsov et al., 1960) who introduced the name "quasi-moment" for the expansion coefficients. Thus following (Bover, 1978), the probability density function $f_x(x)$ is expressed as:

$$f_x(x) = \frac{1}{\sqrt{2\pi}\sigma_x} \exp\left(-\frac{x^2}{2\sigma_x^2}\right) \sum_{L=0}^{\infty} \frac{b_L}{L!} H_L(x) \tag{4}$$

where b_L are the quasi-moments and $H_L(x)$ are the Hermite polynomials defined by:

$$H_L(x) = \exp\left(\frac{x^2}{2\sigma_x^2}\right)\left(-\frac{d}{dx}\right)^L\left[\exp\left(-\frac{x^2}{2\sigma_x^2}\right)\right] \tag{5}$$

According to (Bover, 1978), we may deduce quite simple expressions for the quasi-moments in terms of central moments by using the property, proved by (Appel and Feriet, 1926), that the Hermite polynomials are orthogonal with their adjoint polynomials, with respect to a Gaussian weight function. By a straight forward manipulation we may find that any quasi-moment is equal to the expectation of the corresponding adjoint Hermite polynomial (Bover, 1978), namely:

$$\begin{aligned} b_L = < G_L(x) > \qquad \text{where} \\ G_L(x) = \exp\left(\frac{\widetilde{x}^2\sigma_x^2}{2}\right)\left(-\frac{d}{d\widetilde{x}}\right)^L \exp\left(-\frac{\widetilde{x}^2\sigma_x^2}{2}\right) \\ \text{with} \qquad \widetilde{x} = \frac{x}{\sigma_x^2} \end{aligned} \tag{6}$$

In the following is a list of the first six one-dimensional quasi-moments calculated by (Bover, 1978):

$$\begin{gathered} b_0 = 1; \quad b_1 = 0; \quad b_2 = 0; \quad b_3 = \left\langle x^3 \right\rangle \\ b_4 = \left\langle x^4 \right\rangle - 3\left\langle x^2 \right\rangle^2; \quad b_5 = \left\langle x^5 \right\rangle - 10\left\langle x^2 \right\rangle \left\langle x^3 \right\rangle \\ b_6 = \left\langle x^6 \right\rangle - 15\left\langle x^2 \right\rangle \left\langle x^4 \right\rangle + 30\left\langle x^2 \right\rangle^3 \end{gathered} \tag{7}$$

Now, assuming for instance that b_6 is negligible ($b_6 = 0$), an approximation for the six-th central moment in terms of lower order central moments is obtained.

4 SIMULATION

In this section we investigate the equalization performance by simulation where we use the residual ISI (intersymbol interference) as a measure of performance. Note that the ISI is often used as a measure of performance in equalizers' applications. In the following, we denote "MaxEnt" as the algorithm described by (3) with the Lagrange multipliers given in (Pinchas and Bobrovsky, 2006) where the required source moments are known. The step-size parameters for this method were denoted as μ and β and we substituted $E[z_s^2] = E[x_s^2]$ for initialization. The equalizer taps for Godard's algorithm (Godard, 1980) were updated according to:

$$\begin{gathered} c_l(n+1) = c_l(n) - \\ \mu_G \left(|z(n)|^2 - \frac{E\left[|x(n)|^4\right]}{E\left[|x(n)|^2\right]} \right) z(n) y^*(n-l) \end{gathered} \tag{8}$$

where μ_G is the step-size. The equalizer taps for algorithm (Shalvi and Weinstein, 1990) were updated according to:

$$\begin{gathered} c_i'(n+1) = c_i''(n) + \mu_{SW} \cdot \operatorname{sgn}\Upsilon(x) |z(n)|^2 z(n) \cdot \\ y^*(n-i) \quad \text{where} \quad c_i''(n) = \left(1 \Big/ \sqrt{\sum_i |c_i'|^2} \right) c_i' \end{gathered} \tag{9}$$

where $c_i''(n)$ is the vector of taps after iteration, $c_i''(0)$ is some reasonable initial guess, μ_{SW} is the step-size and $\Upsilon(x) = E\left[|x|^4\right] - 2E^2\left[|x|^2\right] - \left|E\left[x^2\right]\right|^2$ is the kurtosis associated to x. In the following, we denote algorithm (Shalvi and Weinstein, 1990) as SW. The equalizer taps for algorithm (Lazaro et al., 2005) were updated according to:

$$\begin{gathered} c_l(n+1) = c_l(n) - \\ \mu_{par} \left(\left(\frac{1}{N_{sym}} \right) \left(\sum_{k=1}^{N_{sym}} \tilde{K}_\sigma' \left(|z(n)|^2 - F(\sigma) |x_k|^2 \right) \right) \right) \cdot \\ z(n) y^*(n-l) \end{gathered} \tag{10}$$

where μ_{par} is the step-size, $\tilde{K}_\sigma'(z)$ is the derivative of $\tilde{K}_\sigma(z)$ which is the Parzen window kernel of size σ and $F(\sigma)$ is the compensation factor that depends on the kernel size. In (Lazaro et al., 2005) the Gaussian kernel with standard deviation σ was used for $\tilde{K}_\sigma(z)$: $\tilde{K}_\sigma(z) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{z^2}{2\sigma^2}\right)$. In the following, we denote algorithm (Lazaro et al., 2005) as SQD. We denote "MaxEntA" as the algorithm described by (3) with the Lagrange multipliers given in (Pinchas and Bobrovsky, 2006) where some of the required source moments are approximated according to the quasi-moment truncation technique (7). The step-size parameters for this method were denoted as μ_A and β_A and we substituted $E[z_s^2] = E[x_s^2]$ for initialization unless otherwise stated. We used in our simulation a **16QAM source** (a modulation using $\pm\{1,3\}$ levels for in-phase and quadrature components). Two channels were considered. **Channel1** (initial ISI $= 0.44$): The channel parameters were determined according to (Shalvi and Weinstein, 1990):

$$h_n = \{0 \quad \text{for} \quad n < 0; \quad -0.4 \quad \text{for} \quad n = 0 \quad 0.84 \cdot 0.4^{n-1} \quad \text{for} \quad n > 0\} \tag{11}$$

Channel2 (initial ISI $= 1.402$): The channel parameters were taken according to (Lazaro et al., 2005): $h_n = (0.2258, 0.5161, 0.6452, 0.5161)$.
For Channel1 a 13-tap equalizer was used. For Channel2 we used an equalizer with 21 taps. In our simulation, the equalizers were initialized by setting the center tap equal to one and all others to zero. The step-size parameters μ, μ_A, μ_G, β, β_A, μ_{SW}, μ_{par} were chosen for fast convergence with low steady state ISI. For the 16QAM source input propagating through Channel2, the performance of Godard's and SQD algorithm were reproduced following (Lazaro et al., 2005). For the 16QAM modulation source, two Lagrange multipliers (λ_2, λ_4) were used by the "MaxEnt" and "MaxEntA" algorithm. For the "MaxEntA" algorithm, m_6 was approximated according to the quasi-moment truncation method while the other moments m_4 and m_2 were assumed to be known. Figure 2 shows the equalization performance of "MaxEnt" and "MaxEntA" compared with (Lazaro et al., 2005) and (Shalvi and Weinstein, 1990) for the 16QAM source constellation propagating through channel1. The performance is expressed in terms of residual ISI as a function of iteration number. Figure 3 shows the equalization performance of "MaxEntA" with and without the use of initial samples for the initialization phase compared with (Lazaro et al., 2005) and (Godard, 1980) for the 16QAM source constellation propagating through channel2. According to the simulated results, our new proposed algorithm "MaxEntA" has improved equalization performance com-

pared with (Godard, 1980), (Shalvi and Weinstein, 1990) and (Lazaro et al., 2005).

5 CONCLUSIONS

We have derived in this paper a new blind equalization method based on the quasi-moment truncation technique and on the Maximum Entropy blind equalization method (Pinchas and Bobrovsky, 2006). In our proposed algorithm, fewer moments of the source signal are needed to be known compared with (Pinchas and Bobrovsky, 2006). Simulation results indicate that the new proposed algorithm has improved equalization performance compared with (Godard, 1980) and (Lazaro et al., 2005).

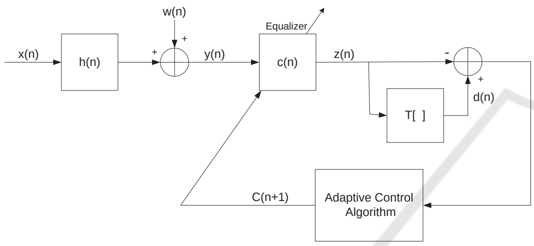

Figure 1: Baseband communication system.

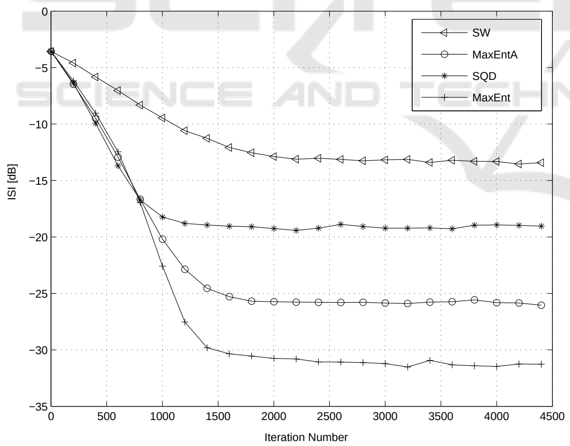

Figure 2: Performance comparison between equalization algorithms for a 16QAM source input going through channel1. The averaged results were obtained in 100 Monte Carlo trials for a SNR of 30 (dB). The step-size parameters were set to: $\mu_{SW} = 2.5e\text{-}5$, $\mu = 3e\text{-}4$, $\beta = 2e\text{-}4$, $\mu_A = 3.5e\text{-}4$, $\beta_A = 4e\text{-}4$ and $\mu_{par} = 2.5e\text{-}4$. In addition we set $F(\sigma) = 1$, $\sigma = 15$ and ε to 0.5, 0 for MaxEnt and MaxEntA respectively.

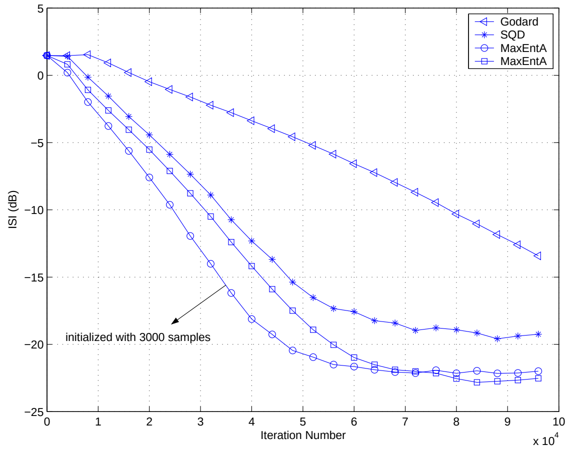

Figure 3: Performance comparison between equalization algorithms for a 16QAM source input going through channel2. The averaged results were obtained in 50 Monte Carlo trials for a SNR of 30 (dB). The step-size parameters were set to: $\mu_A = 2e\text{-}4$, $\beta_A = 2e\text{-}6$, $\mu_A = 2.5e\text{-}4$ for “o” , $\beta_A = 2e\text{-}6$ for “o”, $\mu_{par} = 1e\text{-}4$ and $\mu_G = 1e\text{-}5$. In addition we set $\varepsilon = 0.5$, $F(\sigma) = 1$ and $\sigma = 15$.

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