AN ANALYTICAL AND NUMERICAL STUDY OF PRESSURE TRANSIENTS IN PNEUMATIC DUCTS WITH FINITE VOLUME ENDS

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Abstract: In this paper, the response of a pneumatic transmission line is analysed through two different approaches. Both the approaches, based on the same physical model, are able to simulate the dynamics of a pneumatic line, with finite volume ends. The first approach analytically provides the transients through an equation in a quasi-closed form; the second approach is based on a numerical procedure yielding the inversion of the Laplace transform by the application of a trapezoidal rule. The analysis of the mutual performances of the two approaches, in the frame of pneumatic systems normally operating in industrial automation, can be useful in terms of control of the response and could assist in the design of pneumatic systems.

1 INTRODUCTION

Pneumatic actuators are often employed in industrial automation for reasons related to their good power/ weight ratio, easy maintenance and assembly operations, clean operating conditions and low cost.

This set of advantages, however, is negatively balanced by the difficulties met during the design. Indeed, the presence of air, along with its natural compressibility, introduces further complexities to those already existing: friction forces, losses and time delays in cylinder and transmission lines (Messina, 2005), (Carducci, 2006). For these reasons, fast transients involved in wave propagations in pneumatic transmission lines deserve to be taken into account in the design of the system (Rollo, 2007).

The pneumatic transmission line, analysed in this work, consists of a tube, of a certain length, connecting two finite capacities. As far as the gas-dynamic inside the duct is concerned, in literature there are several papers dealing with such systems but only describing lines with one of the two capacities being finite. In this work, the mathematical description of a pneumatic line, with finite volume terminations, is presented. This setting complicates the mathematics of the phenomena but it is interesting because of the industrial practice (Messina, 2005), (Rollo, 2007) where finite capacities are very common. The transient in the line is described through two partial differential equations, whose solution is obtained in correspondence to suitable initial and boundary conditions. The mathematical model (Rollo, 2007) gives the pressure response for a double volume terminated pneumatic line and includes as a particular case a previous model presented by Schuder and Binder (Schuder, 1959).

The model is obtained assuming small pressure and temperature changes, such that the following assumptions are valuable (Schuder, 1959): (i) incompressible flow and (ii) laminar flow; the accuracy of the response, in correspondence of different operative conditions, has been discussed elsewhere (Rollo et al., 2007).

The assumptions of the model mainly concern the flow conditions which allow an approach based on the Laplace transform (Rollo, 2007), (Schuder, 1959). This type of model could be considered attractive in the frame of industrial automation, but a possible difficulty arises in the inverse transformation especially when an analytical description of the transient is attempted (Rollo, 2007). In this respect, a numerical method (Crump, 1976), (Duffy, 1993), that readily determines the Laplace transform inversion, could be considered attractive in order to achieve the pressure transient.

These two approaches, the analytical one (Rollo, 2007) and the numerical one (Duffy, 1993), that

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have different complexities, are taken into account herein. The first analytically provides the description of the pressure transients through an equation in a quasi-closed form. The second is numerically able to yield the inversion of the Laplace transform in a direct way, through a trapezoidal rule (Crump, 1976), (Duffy, 1993). This latter, under certain conditions, requires no manipulation on Laplace transform.

The two approaches, with the analysis of the mutual performances, can suggest, in the frame of pneumatic systems normally operating in industrial automation, the strategies in terms of design and control of the response. In this respect, interesting conclusions can be extracted.

2 SYSTEM ANALYSED

For a self comprehension of the present work, a brief description of the real system analysed is also presented. The relevant physical model which is referred to in the present work is illustrated in Fig. 1.

The system under investigation consists of two chambers having volumes Q_1, Q_2. The chambers are connected through a cylindrical tube (also termed as pneumatic transmission line) whose transversal section is constant in the range of commercial tolerances. The x-longitudinal coordinate is settled from the upstream (chamber 1: Q_1) to the downstream chamber (chamber 2: Q_2).

The upstream chamber consists of a five litre tank arranged with four holes in order to allow the external connections. In particular, chamber 1 is filled up through a tap air supply until an established static pressure, measured by the absolute pressure gage, is reached. An airtight adapter is screwed onto chamber 1. The adapter is made airtight through an internal membrane made of commercial sticky tape.

The test and simulated condition consists of suddenly breaking the membrane in order to allow a wave pressure travel from chamber 1 to chamber 2 and vice versa; the sudden rupture of the membrane is caused by a puncturing actuator placed at the symmetrical end with respect to the membrane; the puncturing actuator is quasi-statically activated by manually pushing its rod through orifice A.

When a step pressure signal propagates through the duct an on/off valve can be considered simulated (Rollo, 2007). Based on these motivations the approaches (analytical and numerical) have been tested with respect to the mentioned step-type signal.

The downstream volume consists of the ram chamber of a commercial double acting pneumatic actuator. The established volume Q_2 can be in practice settled by grounding the rod of the actuator at a fixed position.

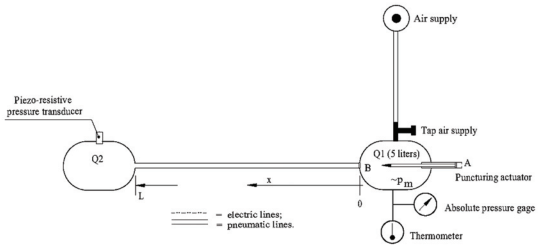

Figure 1: Scheme and nomenclature of the system.

3 SCHUDER AND BINDER EXTENDED (SBE) MODEL: ANALYTICAL APPROACH

In the SBE model (Rollo, 2007), (Schuder, 1959), the equation describing the pressure transient in the duct, obtained from an analytical solution of two partial differential equations (one-dimensional mass and momentum conservation law (Schuder, 1959)), is obtained using the Laplace transform.

This kind of procedure yields, in the Laplace domain and in correspondence of an established section of the line (x=L), the following response:

$$P(L;s)=\left(\frac{p_m-p_0}{s}\right)\cdot kQ_1\cdot \frac{1}{\left[\begin{array}{l}(kQ_1+Q_2)\cosh\left(L\sqrt{\beta}\right)+\\ \left(\frac{a}{\sqrt{\beta}}+\frac{\sqrt{\beta}}{a}kQ_1Q_2\right)\sinh\left(L\sqrt{\beta}\right)\end{array}\right]}+\frac{p_0}{s} \tag{1}$$

where a is the cross sectional area of the duct, k the ratio of specific heats at pressure and volume constant (c_p/c_v), L the length of the duct, p_m and p_0 the initial pressure in the sending volume and connecting duct respectively, Q_1 and Q_2 the sending and receiving volume respectively and β is the following parameter:

$$\beta=\frac{s(R+\rho s)}{\rho c^2} \tag{2}$$

depending on R (frictional resistance in duct in the presence of laminar flow), s the Laplace variable, ρ the density (constant) and c the sound speed (constant).

The inverse transform of (1) is not a straightforward task; in this respect, following Schuder and Binder, Jaeger's result is taken into account (Schuder, 1959): evaluating the coefficients of an exponential series, it is possible to obtain the following analytical solution showing the pressure in the time domain in the position close to chamber 2 (x=L):

$$P(L;t)_A = \frac{p_m kQ_1 + p_0(Q_2 + aL)}{kQ_1 + Q_2 + aL} - 2(p_m - p_0)e^{-\frac{Rt}{2\rho}} \cdot \sum_{n=1}^{\infty} \frac{\left[\cos\left(\frac{\vartheta_n t}{2}\right) + \frac{R}{\rho\vartheta_n}\sin\left(\frac{\vartheta_n t}{2}\right)\right]}{\alpha_n \left[\left(1 + \frac{Q_2}{kQ_1} + \frac{Q_2}{aL} + \frac{aL}{\alpha_n^2 kQ_1}\right)\sin\alpha_n + \left(\frac{Q_2\alpha_n}{aL} - \frac{aL}{\alpha_n kQ_1}\right)\cos\alpha_n\right]} \quad (3)$$

where t is the time; α_n and θ_n are functions of the geometry of the line and initial flow conditions; in particular, the poles in Equation (1) are function of α_n too (Rollo, 2007), (Schuder, 1959). In fact, by substituing in Equation (1)

$$L\sqrt{\beta} = L\left(\frac{s(R + \rho s)}{\rho c^2}\right)^{\frac{1}{2}} = i\alpha \quad (4)$$

and equating to zero the common denominator of Equation (1), an implicit equation in α is obtained; once it is solved (numerically, with the Newton-Raphson method), after some simple mathematical manipulations, it is possible to obtain, along with $s_0=0$ (Rollo, 2007), (Schuder, 1959):

$$s_n = -\frac{R}{2\rho} \pm \frac{i}{2}\sqrt{\left(\frac{2\alpha_n c}{L}\right)^2 - \left(\frac{R}{\rho}\right)^2} \quad (5)$$

in which the real part of the poles is negative or zero.

The quasi-closed form solution (3) depends on the number of the terms in the series; the only non-serious drawback is the necessity of resorting to a numerical method in order to assess α_n. Equation (3) yields p(L;t) for a double volume terminated pneumatic line and it is an extension of a previous model presented by Schuder and Binder (Schuder, 1959). The analytical approach to the SBE model is one of the most complete treatments among those presented about transients in pneumatic lines within relevant literature, in which the influence of pressure waves propagating in ducts is, sometimes, neglected or poorly described. Furthermore, even if it is obtained using the following assumptions: (i) incompressible flow and (ii) laminar flow, it can be adopted, without significantly reducing the accuracy of the response, in correspondence to certain operative conditions (Rollo et al., 2007), where the relevant investigations showed its ability to describe pressure transients including reflecting waves.

4 NUMERICAL APPROACH

The Jaeger's results, in the SBE model, are related to the fact that the solution (3) depends i) on the numerical evaluation of α_n and ii) on a certain amount of labor for the mathematical procedure leading to Equation (3) in the inverse Laplace transform (Rollo, 2007), (Schuder, 1959). A kind of resolution allowing to obtain the inverse transform of Equation (1) in a direct way, could be considered attractive in the frame of systems normally operating in industrial automation. In this respect the authors suggest, in this work, to solve the problem of readily determining the inverse Laplace transform using a numerical approach. Within relevant literature, a large number of different methods for numerically inverting the Laplace transform have been introduced and tested: one of these uses a Fourier series approximation (Crump, 1976), (Duffy, 1993). In fact, in (Duffy, 1993) the following straightforward application of a trapezoidal rule in order to provide the numerical inversion of Laplace transform (here referred to Eq. (1)) is proposed:

$$P(L;t) = \frac{e^{\mu t}}{2t}\left\{\frac{1}{2}\mathrm{Re}[P(L;\mu)] + \sum_{z=1}^{\infty}(-1)^z\left[\mathrm{Re}\left[P\left(L;\mu + \frac{z\pi i}{t}\right)\right] + \mathrm{Im}\left[P\left(L;\mu + \frac{(2z-1)\pi i}{2t}\right)\right]\right]\right\} \quad (6)$$

where t is the time, i the imaginary unit, Im and Re indicate the imaginary and real part of the quantity in the brackets respectively and μ must be greater than the real part of any singularity (poles) in P(L;s) (Duffy, 1993).

The accuracy and efficiency of such a numerical approach depend on a suitable choice of some parameters. In particular, μ can be evaluated through certain considerations about the discretization error (related to the step size π/t for the trapezoidal rule) deriving from the application of Eq. (6). Following the approach proposed in (Crump, 1976), introducing the hypothesis that the function of interest is bounded by:

$$|P(L;t)| \leq Me^{\lambda t} \tag{7}$$

(with M and λ real numbers) it is possible to choose μ through the following relation:

$$\mu = \lambda - \frac{\ln(Err)}{2t} \tag{8}$$

where Err is the error parameter within the numerical accuracy desired and the parameter λ can be chosen slightly larger than the maximum of the real part of all the poles (Crump, 1976). Once μ is known, the series (6) can be summed until it converges to the desired number of significant figures (Crump, 1976). Usually the series in (6) can converge slowly (this has been observed in some tests not reported here); furthermore, (Crump, 1976) the use of a sequence accelerator in conjunction with the numerical inversion is recommended, also in order to obtain a reduction of the truncation error (indeed the series in Eq. (6) is not summed to infinity). In this case, following the considerations in (Crump, 1976), (Duffy, 1993), here Wynn's epsilon-algorithm is adopted. More specifically, to accelerate the convergence of the sequence of partial sums in (6) using the epsilon-algorithm, it is possible to calculate them as in (9):

$$\begin{aligned} S_0 &= \frac{1}{2}\mathrm{Re}[P(L;\mu)], \\ S_z &= S_{z-1} + (-1)^z \begin{bmatrix} \mathrm{Re}\left[P\left(L;\mu + \frac{z\pi i}{t}\right)\right] + \\ \mathrm{Im}\left[P\left(L;\mu + \frac{(2z-1)\pi i}{2t}\right)\right] \end{bmatrix}, \\ z &= 1,...,2N+1. \end{aligned} \tag{9}$$

It is possible, then, to define $\varepsilon_{-1}^{(m)}=0$, $\varepsilon_0^{(m)}=S_m$, m=0, 1,..,2N and then put:

$$\varepsilon_{p+1}^{(m)} = \varepsilon_{p-1}^{(m+1)} + \left[\varepsilon_p^{(m+1)} - \varepsilon_p^{(m)}\right]^{-1}, \quad p = 0,...,2N \tag{10}$$

In this way, the sequence $\varepsilon_0^{(0)}$, $\varepsilon_2^{(0)}$, $\varepsilon_4^{(0)}$,.., $\varepsilon_{2N}^{(0)}$ gives better successive approximation to the sum of the series (Crump, 1976). So, Equation (6), including the sequence accelerator, becomes Equation (11):

$$P(L;t)_{NUM} = \frac{e^{\mu t}}{2t} \cdot \varepsilon_{2N}^{(0)} \tag{11}$$

5 ANALYTICAL VS NUMERICAL APPROACH: CONDITIONS, RESULTS AND DISCUSSIONS

The analytical approach presented in Section 3 (Rollo, 2007) was used for an interesting comparison, with respect to a more refined model NLC (for Non Laminar Compressible flow) (Rollo, 2007), (Rollo et al., 2007). This latter model, whose behaviour was validated through experimental investigations (Rollo, 2007) on the physical model of Fig. 1, takes into account i) flow not necessarily laminar and ii) compressible flow. Through a suitable error parameter, the discrepancy on the response of a pneumatic line with established geometrical characteristics (L=2.53m, D=3 or 6 mm, Q_1= 5dm^3), for polyurethane ducts with an assumed internal roughness of 3μm (Rollo, 2007) was estimated for various flow conditions. In both models, the relevant solution can be obtained and displayed, in space (at a fixed time) and in time (at a fixed position); however, the major relevance of the performance in industrial applications (Messina, 2005), (Rollo, 2007) is related to the behaviour of the pressure in the ram chamber of the actuator (x=L). Therefore, only p(L;t) was discussed, in correspondence of the various receiving volumes Q_2 (obtained fixing the stroke of the pneumatic actuator employed as downstream volume, in correspondence of different positions).

This comparison highlighted that, for a fixed geometrical configuration, the error between the SBE and NLC models is as small as the pressure ratio p_m/p_o is close to 1 (Rollo, 2007); in this case the match of SBE response with the NLC curve is very satisfactory. This comparison was made after appropriate convergence tests, that suggested to use n=0,…..,30 in the Equation (3) for the SBE model.

In this work, the comparison between the analytical (3) and numerical (11) approach is discussed, in correspondence of some settings yielding a satisfactory agreement of the SBE with the NLC model. In particular, a line with L=2.53m, D=3 mm and Q_1= 5dm^3 will be considered. The case of interest concerns the transient caused by an upstream initial pressure in Q_1 of 1.1 times higher than initial atmospheric pressure. This setting is completed arranging the actuator employed as downstream volume to establish, in a first case, a volume Q_2 = 1.5 cm^3 (capacity corresponding to the dead space of the ram chamber) and, in a second case, Q_2 = 26.5 cm^3 (Rollo, 2007).

A suitable error parameter is introduced (12) for the detection of discrepancy of the two approaches (3) and (11):

$$\mathrm{ERROR} = \left| \frac{p(L;t)_A - p(L;t)_{NUM}}{p(L;t)_A} \right| \qquad (12)$$

in correspondence to various values of the time (10 ms≤t≤150 ms). In applying (11), it is assumed $Err=10^{-6}$ and, for the considerations about Eqs. (5) and (8), it can be assumed $\lambda=0$. In this way, the μ value is known in all the time instants considered. The comparison will be made firstly assuming, as far as the numerical approach (11) is concerned, N=25. In this respect, for the first case (Q_2=1.5 cm^3) the following Table 1 was produced:

Table 1: Simulated Pressure with L=2.53 m, D=3 mm, p_m/p_0=1.1, Q_2=1.5 cm^3 (n=30, N=25).

Time Instants (ms)	Analytical Pressure (bar)	Numerical Pressure (bar)	ERROR
10	1.092289	1.092282	6.8E-06
20	1.116303	1.116265	3.4E-05
30	1.101538	1.101535	2.1E-06
40	1.097992	1.097992	3.5E-08
50	1.099557	1.099561	3.7E-06
60	1.099927	1.099927	1.5E-07
70	1.099681	1.099659	1.9E-05
80	1.099693	1.099693	1.7E-07
90	1.099725	1.099726	2.8E-07
100	1.099725	1.099726	1.1E-07
110	1.099723	1.099723	1.1E-07
120	1.099724	1.099724	2.9E-09
130	1.099724	1.099724	1.5E-08
140	1.099724	1.099724	4.5E-09
150	1.099724	1.099724	4.4E-10
Pressure computational time (ms)	~ 5	~ 290	

In Table 1 the first column shows the time instants taken into account, the second and third columns show the pressure values obtained with the two approaches, the analytical one and the numerical one respectively, the fourth column shows the ERROR values. Table 1 shows an agreement of the analytical and numerical results (second and third column) that can be considered very satisfactory; furthermore, it is possible to notice that the computational times (evaluated on a 2.8 GHz Pentium IV using a Matlab routine) show that the analytical approach needs about 5 ms to provide the pressure values in the second column, whilst the numerical computational time is about 290 ms.

As far as the second case (Q_2=26.5 cm^3) is concerned, the following Table 2 was produced:

Table 2: Simulated Pressure with L=2.53 m, D=3 mm, p_m/p_0=1.1, Q_2=26.5 cm^3 (n=30, N=25).

Time Instants (ms)	Analytical Pressure (bar)	Numerical Pressure (bar)	ERROR
10	1.018830	1.018831	3.6E-07
20	1.053451	1.053449	2.1E-06
30	1.076250	1.076250	1.6E-08
40	1.087726	1.087726	1.9E-08
50	1.093537	1.093539	1.6E-06
60	1.096557	1.096557	9.1E-09
70	1.098006	1.098006	3.3E-08
80	1.098703	1.098703	2.0E-07
90	1.099047	1.099047	5.3E-09
100	1.099214	1.099214	1.6E-07
110	1.099294	1.099293	1.1E-06
120	1.099333	1.099333	5.3E-12
130	1.099352	1.099352	5.9E-09
140	1.099361	1.099361	1.4E-09
150	1.099366	1.099366	1.9E-10
Pressure computational time (ms)	~ 5	~ 290	

Table 2 is also able to show a satisfactory agreement of the results of the two approaches (3) and (11) and, as in Table 1, it is possible to notice that the numerical computational time is about 290 ms, whilst the analytical approach needs about 5 ms to provide the pressure values in all the time instants considered.

Table 1 and Table 2 can suggest that the numerical approach (11) is not always advisable in engineering applications in which the performance is required in terms of design and fast control of the response. A possible way to highlight this drawback can be based on the study of the discrepancies with the analytical approach, decreasing N in such a way as to reduce the computational time of the numerical approach (like in Table 3). In Table 3, the first column shows the operative conditions taken into account, the second column the mean of ERROR values and the third the numerical computational times. As can be seen, the computational times decrease if N decreases, but this behaviour still seems far from the more attractive computational times of the analytical approach. Furthermore decreasing the value of N, the mean ERROR value slightly increases.

Table 3: ERROR for a pneumatic line with L=2.53 m, D=3 mm, p_m/p_0=1.1, Q_2=1.5 cm^3 and Q_2=26.5 cm^3, n=30.

	Mean of ERROR values	Numerical computational times (ms)
N=25 Q_2=1.5 cm^3	4.4E-6	290
N=25 Q_2=26.5 cm^3	3.8E-7	290
N=20 Q_2=1.5 cm^3	2.4E-5	204
N=20 Q_2=26.5 cm^3	7.2E-7	204
N=10 Q_2=1.5 cm^3	6.4E-5	77
N=10 Q_2=26.5 cm^3	7.6E-6	77

An estimation of the behaviour of both approaches concerning the transient in the aforementioned pneumatic lines, can be obtained through the following Fig. 2 and Fig. 3, useful also in order to estimate the pressure transients of Eq. (3) and Eq. (11) in terms of design of the line.

Finally, also Figure 4 has been produced. This latter figure has been introduced with the motivation of showing the satisfactory agreement of both the proposed approaches, also in correspondence of an intermediate receiving volume (Q_2=16.5 cm^3).

6 CONCLUSIONS

In this paper, two approaches of different complexities, concerning the dynamics of a pneumatic transmission line with finite volume ends, have been analysed: one analytical and another one numerical. The first one provides the description of the pressure transients through an equation in a quasi-closed form and gives the pressure response for a double volume terminated pneumatic line.

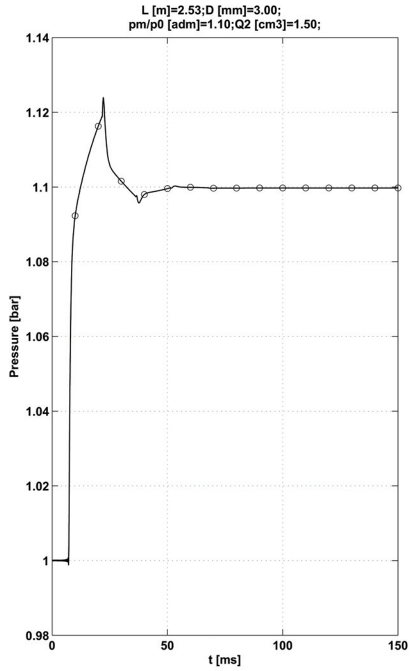

Figure 2: Simulated pressure at the receiving volume through analytical (__) and numerical (°) approach with D=3mm, L=2.53m, p_m/p_0=1.1 and Q_2=1.5cm^3 (n=30, N=25).

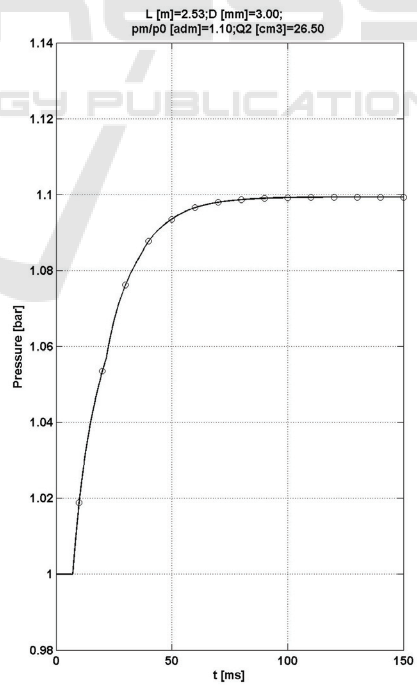

Figure 3: Simulated pressure at the receiving volume through analytical (__) and numerical (°) approach with D=3mm, L=2.53m, p_m/p_0=1.1 and Q_2=26.5cm^3 (n=30, N=25).

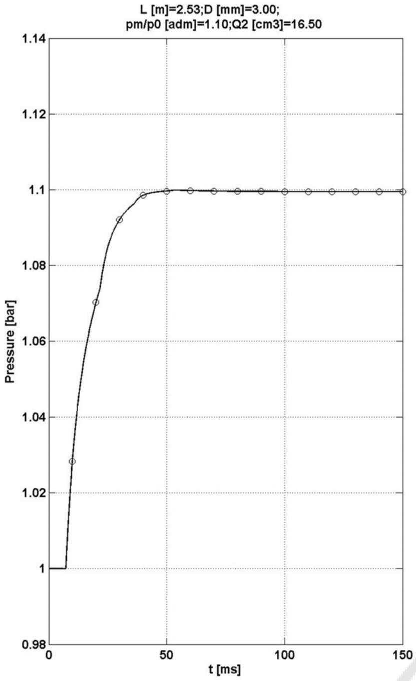

Figure 4: Simulated pressure at the receiving volume through analytical (__) and numerical (°) approach with D=3mm, L=2.53m, p_m/p_0=1.1 and Q_2=16.5cm^3 (n=30, N=25).

The second, numerically, is able to yield the inversion of the Laplace transform through a trapezoidal rule (in conjunction with a sequence accelerator). Using certain geometrical configurations and flow conditions, for which it was shown that the SBE model can be used without significantly reducing the accuracy of the response, the two kinds of resolution can be compared and analysed. The introduction of a suitable error parameter, able to provide the discrepancies of the two approaches, allows interesting discussions.

The trapezoidal rule, in its numerical simplicity, could avoid the long mathematical procedures yielding the inversion of the Laplace transform. The advantage related to the application of this direct rule seems, however, negatively balanced by the extra computational efforts required to achieve a satisfactory convergence (the series approximation can converge slowly and, usually the use of a sequence accelerator in conjunction with the numerical inversion is highly recommended). For these reasons, the numerical approach could not always be advisable in engineering applications in which performances are required in terms of design and control of the response. This, indeed, has been showed by the comparison with an analytical solution in a quasi-closed form, in terms of computational times. The possible drawback of this latter approach is a verbose procedure giving the final relation and the need to resort to a numerical method in order to assess all the poles involved in the Laplace transform. However, the relevant simulations carried out highlight an excellent behaviour of the analytical approach: these properties can be considered attractive also considering the satisfactory overlap of the curves provided by the analytical approach with a more performing numerical model (NLC), whose excellence has been confirmed with experimental validations.

The study presented in this paper gives the possibility of investigating relevant dynamic behaviours, suggesting an efficient estimation of control and design parameters, in the frame of systems normally operating in industrial automation.

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