MIMO INSTANTANEOUS BLIND IDENTIFICATION BASED ON STEEPEST DESCENT METHOD

Shen Xizhong, Hu Dachao
Mechanical & Electrical department, Shanghai Institute of Technology, Shanghai, China

Meng Guang
State Key Laboratory for Mechanical System and Vibration, Shanghai Jiao Tong University, Shanghai, China

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Abstract: This paper presents a new MIMO instantaneous blind identification algorithm based on second order temporal property and steepest descent method. Second order temporal structure is reformulated in a particular way such that each column of the unknown mixing matrix satisfies a system of nonlinear multivariate homogeneous polynomial equations. The nonlinear system is solved by steepest descent method. We construct a general goal of the system and convert the nonlinear problem into an optimal problem. Our algorithm allows estimating the mixing matrix for scenarios with 4 sources and 3 sensors, etc. Finally, simulations show its effectiveness with more accurate solutions than the algorithm with homotopy method.

1 INTRODUCTION

Multiple-input multiple-output (MIMO) instantaneous blind identification (MIBI) is one of the attractive blind signal processing (BSP) problems, where a number of source signals are mixed by an unknown MIMO instantaneous mixing system and only the mixed signals are available, i.e., both the mixing system and the original source signals are unknown. The goal of MIBI is to recover the instantaneous MIMO mixing system from the observed mixtures of the source signals. In this paper, we focus on developing a new algorithm to solve the MIBI problem by using second-order temporal structure and steepest descent method.

The greater majority of the available algorithms is based on generalized eigenvalue decomposition or joint approximate diagonalization of two or more sensor correlation matrices for different lags and/or times arranged in the conventional manner (Cichocki A et al. 2002) (Hua and Tugnait 2000)(Lindgren and Veen 1996). An MIBI based on second order temporal structure (SOTS) (Laar et al. 2008) has been proposed, which arrange the available sensor correlation values in a particular fashion that allows a different and natural formulation of the problem, as well as the estimation of the more columns than sensors.

In this paper, we further develop the algorithm proposed in Laar et al. 2008 to obtain more accurate and robust solution with a new contrast function.

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2 MIBI MODEL

Let us use the usual model (Laar et al. 2008) (Cichocki and Amari 2002) (Yingbo Hua and Tugnait J K 2000) (U Lindgren and van der Veen 1996) in MIBI problem as follows

$$\mathbf{x}(t) = \mathbf{As}(t) + \mathbf{v}(t) \quad (1)$$

where $\mathbf{A} = [\mathbf{a}_1, \cdots, \mathbf{a}_m] \in \mathbb{R}^{n\times m}$ is an unknown mixing matrix with its n -dimensional array

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response vectors $\mathbf{a}_j = \left(a_{1j} \quad \cdots \quad a_{nj}\right)^{\mathrm{T}}$, $j = 1, 2, \cdots, m$, $\mathbf{s}(t) = \left[s_1(t), s_2(t), \cdots, s_m(t)\right]^{\mathrm{T}}$ is the vector of source signals, $\mathbf{v}(t) = \left[v_1(t), \cdots, v_n(t)\right]^{\mathrm{T}}$ is the vector of noises, and $\mathbf{x}(t) = \left[x_1(t), x_2(t), \cdots, x_n(t)\right]^{\mathrm{T}}$ is the vector of observations.

Without knowing the source signals and the mixing matrix, the MIBI problem is to identify the mixing matrix from the observations by estimating $\mathbf{A}$ as $\hat{\mathbf{A}}$.

The mixing matrix is identifiable in the sense of two indeterminacies, which are unknown permutation of indices of each column of the matrix and its unknown magnitude (Laar et al. 2008) (Cichocki and Amari 2002) (Yingbo Hua and Tugnait J K 2000) (U Lindgren and van der Veen 1996). Assume that each column of $\mathbf{A}$ satisfy the normalization conditions, i.e., on the unit sphere,

$$S_j\left(\mathbf{a}_j\right) = \sum_{i=1}^{n} a_{ij}{}^2 - 1 = 0; j = 1, 2, \cdots, m. \quad (2)$$

To solve the MIBI problem, we define the following concepts Def 1~2 for the derivation of the algorithm, and then make the following assumptions AS 1~4 (Laar et al. 2008).

Def 1 Autocorrelation function $r_{s,ii}(t,\tau)$ of $s_i(t), \forall i \in \mathbb{N}$ at time instant t and lag τ is defined as

$$r_{s,ii}(t,\tau) \triangleq \mathrm{E}\left[s_i(t)s_i(t-\tau)\right], \forall t, \tau \in \mathbb{Z}. \quad (3)$$

Def 2 Cross-correlation function $r_{s,ij}(t,\tau)$ of $s_i(t), s_j(t), \forall i, j \in \mathbb{N}$ at time instant t and lag τ is defined as

$$r_{s,ij}(t,\tau) \triangleq \mathrm{E}\left[s_i(t)s_j(t-\tau)\right], \forall t, \tau \in \mathbb{Z}. \quad (4)$$

AS 1 the source signals have zero cross-correlation on the noise-free region of support (ROS) Ω:

$$r_{s,j_1j_2}(t,\tau) = 0, \forall 1 \le j_1 \ne j_2 \le m. \quad (5)$$

AS 2 the source autocorrelation functions are linearly independent on the noise-free ROS Ω

$$\sum_{j=1}^{m} \xi_j r_{s,jj}(t,\tau) = 0 \Rightarrow \xi_j = 0, \forall j = 1, 2, \cdots, m \quad (6)$$

AS 3 the noise signals have zero auto- and cross-correlation functions on the noise-free ROS Ω:

$$r_{n,j_1j_2}(t,\tau) = 0, \forall 1 \le j_1, j_2 \le m. \quad (7)$$

AS 4 the cross-correlation functions between the source and noise signals are zero on the noise-free ROS Ω:

$$r_{vs,ij}(t,\tau) = r_{sv,ji}(t,\tau) = 0, \quad \forall 1 \le i \le n, 1 \le j \le m. \quad (8)$$

The procedure of our proposed algorithm includes two steps, that is, step 1 is that the problem of MIBI is formulated as the problem of solving a system of homogeneous polynomial equations; and step 2 is that steepest descent method is applied to solve the system of polynomial equations. We detail these steps respectively in sections 3 and 4.

3 HOMOGENEOUS POLYNOMIAL EQUATIONS

In this section, we will review the algebraic structure of MIBI problem derived under the above assumptions, and some details can be referred to Laar et al 2008. The correlation values of the observations are stacked as

$$\mathbf{R}_{x,\Diamond} \triangleq \left[\mathbf{r}_x(t_1,\tau_1) \quad \cdots \quad \mathbf{r}_x(t_N,\tau_N)\right], \quad (9)$$

where $\mathbf{r}_x(t_N,\tau_N) = \mathrm{E}\left[\mathbf{x}(t_N) \otimes \mathbf{x}(t_N - \tau_N)\right]$, and $\otimes$ denotes Kronecker product. The homogeneous polynomial equations of degree two are expressed as

$$\mathbf{\Phi}\mathbf{A}_\Diamond = \mathbf{0}. \quad (10)$$

Here, $\mathbf{A}_\Diamond$ is the second-order Khatri-Rao product of $\mathbf{A}$, which is defined as $\mathbf{A}_\Diamond \triangleq \left[\mathbf{a}_1 \otimes \mathbf{a}_1 \quad \cdots \quad \mathbf{a}_m \otimes \mathbf{a}_m\right]$, and $\mathbf{\Phi} = \left(\varphi_{q,i_1i_2}\right)_{q=1,\cdots,Q; i_1,i_2=1,\cdots,n}$ is a matrix with $Q \times n^2$ dimensions where $Q = \frac{1}{2}n(n+1) - rank\left[\mathbf{R}_{\mathbf{x},\Diamond}\right]$, of which its rows form a basis for the nonzero left null space $\mathrm{N}\left(\mathbf{R}_{x,\Diamond}\right)$. Therefore, there are Q equations about each column of $\mathbf{A}$ in (10).

$\mathbf{\Phi}$ can be calculated by SVD of $\mathbf{R}_{x,\Diamond}$, and split into signal and noise subspace parts as $\mathbf{R}_{x,\Diamond} = \mathbf{U}_s\mathbf{\Sigma}_s\mathbf{V}_s^{\mathrm{T}} + \mathbf{U}_v\mathbf{\Sigma}_v\mathbf{V}_v^{\mathrm{T}}$. The left null space of $\mathbf{R}_{x,\Diamond}$ is $\mathbf{\Phi} = \mathbf{U}_v^{\mathrm{T}}$.

By eq.(10), the maximum number $M_{\max}$ of columns that can be identified with n sensors equals

$$M_{\max} = \frac{1}{2}n(n+1) - (n-1). \quad (11)$$

4 STEEPEST DESCENT METHOD

In this section, we summarize the main ideas behind the so-called steepest descent method (Richard and Faires 2001) that provides a deterministic means for solving a system of nonlinear equations, and then we employ the steepest descent method to solve the equations in (10) to form our algorithm.

We expand the expression in (10) as

$$f_q\left(\mathbf{a}_j\right)=\sum_{i_1\le i_2;i_1i_2=1,\cdots,n}\varphi_{q;i_1i_2}a_{i_1j}a_{i_2j}=0; \quad q=1,\cdots,Q;\forall j=1,\cdots,m \tag{12}$$

and then define our optimal goal function when combining the constraint in (2) as

$$g\left(\mathbf{a}_j\right)\triangleq\sum_{q=1}^{Q}f_q^{\,2}\left(\mathbf{a}_j\right)+\gamma^2S_j^{\,2}\left(\mathbf{a}_j\right), \quad \forall j=1,\cdots,m \tag{13}$$

Here, γ is added as a homogeneous factor, which is applied to make the different square items in (13) well-proportioned, and in our algorithm we set $\gamma=0.1$. Notice that we don't think it as a penalty term for imposing the constraint for it just adjust the constraint in (2) and (12) to have the same level of function values. To satisfy the constraint (2) we normalize $\mathbf{a}_j$ in each iterative step to unit vector.

The direction of greatest decrease in the value of $g\left(\mathbf{a}_j\right)$ at $\mathbf{a}_j^{(k)}$ with k -th iteration is the direction given by its minus gradient $-\nabla g\left(\mathbf{a}_j\right)$ of $g\left(\mathbf{a}_j\right)$. The gradient is expressed as

$$\nabla g\left(\mathbf{a}_j\right)=2\mathbf{J}\left(\mathbf{a}_j\right)^{\mathrm{T}}\mathbf{F}\left(\mathbf{x}\right). \tag{14}$$

Here, $\mathbf{F}\left(\mathbf{x}\right)=\left(f_1\left(\mathbf{x}\right),\cdots,f_Q\left(\mathbf{x}\right),\gamma S\left(\mathbf{x}\right)\right)^{\mathrm{T}}$, and $\mathbf{J}\left(\mathbf{a}_j\right)$ is its Jacobian matrix. The objective is to reduce $g\left(\mathbf{a}_j\right)$ to its minimal value of zero, and an appropriate choice for $g\left(\mathbf{a}_j\right)$ is

$$\mathbf{a}_j^{(k+1)}=\mathbf{a}_j^{(k)}-\alpha\nabla g\left(\mathbf{a}_j^{(k)}\right), \tag{15}$$

where $\alpha_0=\arg\min_{\alpha}g\left(\mathbf{a}_j^{(k+1)}\right)$ is the critical point. We can apply any single-variable function optimal method to find the minimum value of $g\left(\mathbf{a}_j^{(k+1)}\right)$ by an appropriate choice for the value α. In our algorithm, we use Newton's forward divided-difference interpolating polynomial, detailed in Richard and Faires 2001.

We employ the initial solutions as equal distributed vectors in the super space of $\mathbf{a}_j$. To guarantee that all the local minimums of the proposed algorithm are obtained, we can use 8 or more initial solutions equal distributed in the super space, and then find the correct solutions by clustering method. For simplicity, we decide the four correct solutions by their minimum distances between each other.

5 SIMULATIONS

We adopt three mixtures of four speech signals the same example as in Laar et al. 2008. For convenience, we name our algorithm as MIBI Steepest Descent and the algorithm in Laar et al. 2008 as MIBI Homotopy.

The speech signals are sampled as 8kHz, consist of 10,000 samples with 1,250ms length, and are normalized to unit variance $\sigma_s=1$. The signal sequences are partitioned into five disjoint blocks consisting of 2000 samples, and for each block, the one-dimensional sensor correlation functions are computed for lags 1, 2, 3, 4 and 5. Hence, in total for each sensor correlation functions 25 values are estimated and employed, i.e., the employed noise-free ROS in the domain of block-lag pairs is given by

$$\Omega=\left\{(1,1),\cdots,(1,5),(2,1),\cdots,(5,5)\right\},$$

where the first index in each pair represents the block index and the second the lag index. The sensor signals are obtained from (1) with 3×4 mixing matrix,

$$\mathbf{A}=\begin{bmatrix}0.6749 & 0.4082 & 0.8083 & -0.1690\\ 0.5808 & -0.8165 & 0.1155 & -0.5071\\ 0.4552 & -0.4082 & -0.5774 & 0.8452\end{bmatrix}.$$

The noise signals are mutually statistically independent white Gaussian noise sequences with variances $\sigma_v^{\,2}=1$. The signal-to-noise ratio (SNR) is -1.23dB, which is quite bad. We set the maximum iterative number is 30, and stop the iteration if the correction of the estimated is smaller than a certain tolerance 10^{-3}.

Let θ_i be the included angle between the j -th column of $\mathbf{A}$ and its estimate. The estimated mixing matrix is

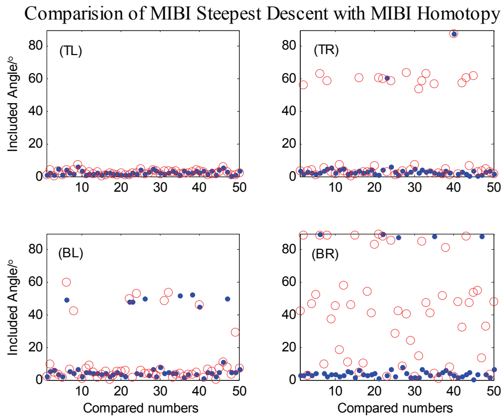

Figure 1: Comparisons of MIBI Steepest Descent with MIBI Homotopy.

$$\hat{\mathbf{A}} = \begin{bmatrix} 0.6542 & 0.4007 & 0.8239 & -0.1235 \\ 0.5984 & -0.8152 & 0.0884 & -0.5343 \\ 0.4625 & -0.4183 & -0.5597 & 0.8362 \end{bmatrix},$$

and the included angles are 1.6120, 0.7214, 2.0583 and 3.0781. We see that the estimated columns approximately equal the ideal ones.

Figure 1 shows the Comparisons of MIBI Steepest Descent with MIBI Homotopy. TL, TR, BL and BR in Figure 1 are respectively the estimated included angles along different running times between the first, second, third and fourth columns and their estimates. Blue dot indicates the result of MIBI_SD algorithm; and Red circle indicates the results of MIBI_Homotopy algorithm. We see that in TL and BL figure, the included angles are almost the same with each other, but in TR and BR figure, the estimates by MIBI Steepest Descent are better than the ones by MIBI Homotopy. Therefore, we conclude that the algorithm with steepest descent has better performance than MIBI Homotopy.

6 CONCLUSIONS

In this paper, we further develop the algorithm proposed in Laar et al. 2008 to obtain more accurate and robust solution with a new contrast function in (13). SOTS is considered only on a noise-free region of support. We project the MIBI problem in (1) on the system of homogeneous polynomial equations in (10) of degree two. Steepest descent method is used for estimating the columns of the mixing matrix, which is quite different from the algorithm in Laar et al. 2008 which applied homotopy method. This MIBI method presented in this paper allows estimating the mixing matrix for several underdetermined mixing scenarios with 4 sources and 3 sensors. Simulations show its effectiveness with more accurate solutions.

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