

# DIFFERENTIAL-DRIVE STEERING SYSTEM USING PLANETARY GEARING FOR OMNIDIRECTIONAL MOBILE ROBOT

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**Abstract:** Holonomic omnidirectional mobile robot is useful with its high mobility in narrow or crowded area, and omnidirectional robot equipped with normal tires is desired for difference excess, vibration suppression and ride comfort. Caster-drive mechanism using normal tire has been developed to realize a holonomic omnidirectional robot, however, there remains some problems. This paper presents effective systems to control the caster-drive wheels of omnidirectional mobile robot. Differential-Drive Steering System (DDSS) using planetary gearing is proposed to improve the operation ratio of motors. DDSS generates driving and steering torque effectively from two motors. Simulation results show the proposed system is effective for holonomic omnidirectional mobile robots.

## 1 INTRODUCTION

An omnidirectional robot is highly maneuverable in narrow or crowded area including residences, offices, warehouses and hospitals. It can be applied to an autonomous mobile robot in a factory, a wheelchair and so on. Several kinds of omnidirectional mobile robots and their applications have been developed by (West, 1992), (Pin, 1994), (Damoto, 2002) and (Kitagawa, 2008). However, these robots realized their omnidirectional motion by using special wheels such as mechatronic wheels, ball wheels, omni-disks and omni-wheels.

To improve the ride comfort, vibration suppression, slippage reduction and ability of difference excess, omnidirectional robots equipped with normal tires have been strongly required. (Arai, 1981) proposed an omnidirectional vehicle equipped with normal tires. However, it was a non-holonomic vehicle which has to adjust the direction of wheels before changing the moving direction of vehicle.

Holonomic omnidirectional vehicles, which can move in any direction without changing the direction of tires beforehand, equipped with normal tires include dual-wheel type by (Wada, 2000) and caster-drive(active-caster) type by (Wada, 1996).

The dual-wheel type has problems as follows. Number of wheels is limited to two, and it is impossible to get high friction or to adapt a rough terrain by synchronous drive of many wheels. Moreover, a passive wheel is needed to stabilize the posture of vehicle.

The caster-drive wheel has offset between the steering axis and the center of wheel. The wheel can move in any direction by controlling the steering axis and the driving wheel independently by using two motors. A holonomic omnidirectional motion of a robot can be realized by using two or more caster-drive wheels.

However, the caster-drive wheel has a problem as follows. When the vehicle is in steady motion including straight motion and rotation with constant curvature, only the driving motor works and the steering motor becomes idle. When the vehicle changes its moving direction, high load is applied to the steering motor. Therefore, high power is required both for the driving and steering motor. It causes increase of mass.

The aim of our research is to develop a holonomic omnidirectional mobile robot with caster-drive wheel minimizing the motor power by using the interference of output of two motors. New gearing mechanism is proposed to realize the interference.

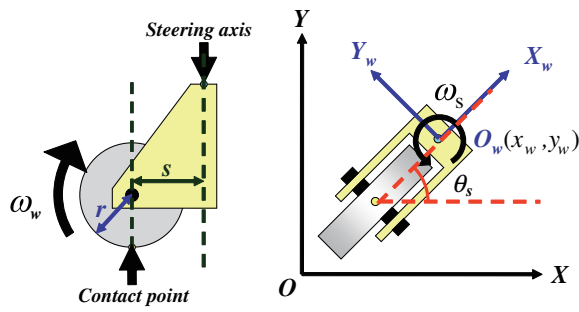


Figure 1: Caster-drive wheel.

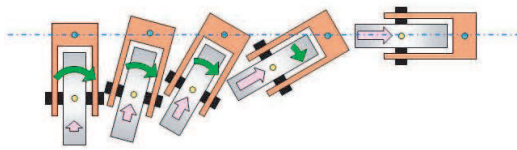


Figure 2: Lateral motion to right.

## 2 PRINCIPLE

### 2.1 Omnidirectional Motion using Caster-Drive Wheel

The principle of holonomic omnidirectional motion using caster-drive wheels is described in this section. Figure 1 shows the caster-drive wheel. The position and orientation of wheel can be represented by the position  $O_w(x_w, y_w)$  of steering axis and the orientation  $\theta_s$  from the contact point between the wheel and the ground to the steering axis with reference to the fixed frame  $O - XY$ .

By rotating the driving wheel with the angular velocity  $\omega_w$ , velocity  $\dot{x}_w = r\omega_w$  generates in the direction of  $X_w$  axis. Here,  $r$  is the radius of driving wheel. By rotating the steering axis with the angular velocity  $\omega_s$ , velocity  $\dot{y}_w = -s\omega_s$  would generate at the center of wheel in the direction of  $Y_w$  axis. Here,  $s$  is the offset between the steering axis and the center of driving wheel in the direction of  $X_w$ . However, reacting velocity  $\dot{y}_w = s\omega_s$  generates at the steering axis in the direction of  $Y_w$  axis, because the position of the driving wheel is fixed by the friction with the ground. Therefore, the velocity  $(\dot{x}_w, \dot{y}_w)$  of caster-drive wheel can be controlled by changing  $\omega_w$  and  $\omega_s$ .

Figure 2 shows an example of motion. The initial orientation  $\theta_s$  of the wheel is set to be  $\theta_s = \pi/2$  in the frame  $O - XY$ . The motion as shown in Fig. 2 can be given by changing  $\omega_w$  and  $\omega_s$  appropriately. Even though the rotating wheel itself can not generate

lateral motion to the right, the lateral motion of the robot, which is fixed to the steering axis, is realized.

Each wheel does not have to control the orientation of the robot by itself.

The direct kinematic equation is denoted by the state vector  $x_w = [x_w, y_w]^T$  and the input vector  $u_w = [\omega_w, \omega_s]^T$  as

$$\dot{x}_w = B_w u_w, \quad (1)$$

where

$$B_w = \begin{bmatrix} r \cos \theta_s & -s \sin \theta_s \\ r \sin \theta_s & s \cos \theta_s \end{bmatrix}. \quad (2)$$

The inverse kinematic equation becomes

$$u_w = B_w^{-1} \dot{x}_w, \quad (3)$$

where

$$B_w^{-1} = \begin{bmatrix} \frac{1}{r} \cos \theta_s & \frac{1}{s} \sin \theta_s \\ -\frac{1}{s} \sin \theta_s & \frac{1}{r} \cos \theta_s \end{bmatrix}. \quad (4)$$

Holonomic omnidirectional motion  $(\dot{x}, \dot{y}, \dot{\theta})$  of a mobile robot can be achieved by using two caster-drive wheels or more. Furthermore, synchronous drive with arbitrary number of wheels and rotation mechanism yields three dimensional holonomic omnidirectional motion by three motors.

### 2.2 Differential-Drive Steering System (DDSS)

In this section, we develop a useful method for constructing a caster-drive wheel using Differential-Drive Steering System (DDSS). The DDSS outputs driving and steering velocities from two motors using differential-drive gearing.

Figure 3 shows the principle of the DDSS. Just like a usual planetary gearing, it is composed of sun gear(A), outer ring gear(B), planet gear(C) and planet carrier(D). The planet carrier(D) holds the planet gear(C) and rotates relative to the sun gear(A) and the outer ring gear(B). However, unlike usual planetary gearing, the DDSS is 2-input 2-output system without fixing any component. A and B are independently driven by two motors. C and D provide output torque.

Figure 4 shows the mechanism of the DDSS. D, which is fixed to the chassis(E), provides the steering torque, and C, which leads to the driving wheel via the bevel gear, provides the driving torque.

Let  $\omega_A, \omega_B, \omega_C$  and  $\omega_D$  be the angular velocity of A, B, C and D in Fig. 3, and  $Z_A, Z_B$  and  $Z_C$  be the number of teeth of A, B and C, respectively.

When  $\omega_D = 0$ , the steering angular velocity  $\omega_s$  becomes zero, and we obtain

$$\omega_A = -\frac{Z_B}{Z_A} \omega_B = -\frac{Z_C}{Z_A} \omega_C, \quad (5)$$

$$\omega_D = 0. \quad (6)$$

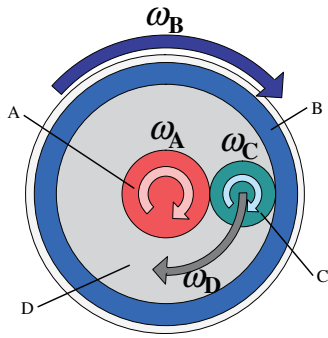


Figure 3: Principle of Differential-Drive Steering System (DDSS).

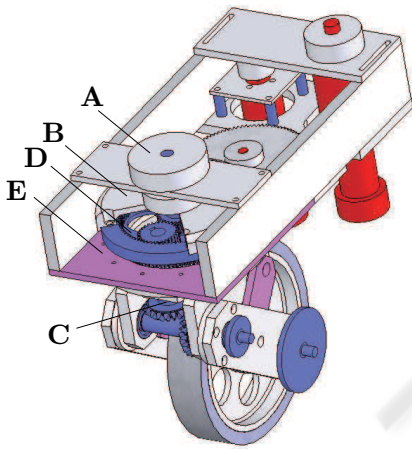


Figure 4: Mechanism of DDSS.

When  $\omega_C - \omega_D = 0$ , the driving angular velocity  $\omega_w$  becomes zero because C does not rotate between A and B, and we obtain

$$\omega_A = \omega_B = \omega_C = \omega_D. \quad (7)$$

The direct kinematic equation, which derives driving and steering output  $u_w = [\omega_w, \omega_s]^T$  from motor input  $u_P = [\omega_A, \omega_B]^T$ , can be described as

$$u_w = \begin{bmatrix} \omega_C - \omega_D \\ \omega_D \end{bmatrix} = B_P u_P, \quad (8)$$

where

$$B_P = \begin{bmatrix} -\frac{Z_A Z_B}{Z_C(Z_A + Z_B)} & \frac{Z_A Z_B}{Z_C(Z_A + Z_B)} \\ \frac{Z_A}{Z_A + Z_B} & \frac{Z_B}{Z_A + Z_B} \end{bmatrix}. \quad (9)$$

The inverse kinematic equation becomes

$$u_P = B_P^{-1} u_w, \quad (10)$$

where

$$B_P^{-1} = \begin{bmatrix} -\frac{Z_C}{Z_A} & 1 \\ \frac{Z_C}{Z_B} & 1 \end{bmatrix}. \quad (11)$$

Next, we derive the motor power ratio of the DDSS. Joint torques  $T_A$ ,  $T_B$ ,  $T_C$  and  $T_D$  of A, B, C and D, respectively, are given by

$$\begin{bmatrix} T_C \\ T_D \end{bmatrix} = \begin{bmatrix} -\frac{Z_C}{Z_A} & \frac{Z_C}{Z_B} \\ \frac{Z_A + Z_C}{Z_A} & \frac{Z_B + Z_C}{Z_B} \end{bmatrix} \begin{bmatrix} T_A \\ T_B \end{bmatrix}, \quad (12)$$

where positive direction of each torque is same as that of angular velocity in Fig. 3.

For an omnidirectional mobile robot with the DDSS, steady motion including straight motion and rotation with constant curvature is achieved by  $\omega_s (= \omega_D) = 0$ . When  $\omega_s = 0$  ( $T_D = 0$ ), the joint torques are given from (12) by

$$T_A = -\frac{Z_A}{Z_B} T_B, \quad (13)$$

$$T_C = -\frac{2Z_C}{Z_A} T_A, \quad (14)$$

$$T_D = 0. \quad (15)$$

The power ratio of two motors is given from (5) and (13) by

$$\begin{aligned} P_A : P_B &= T_A \omega_A : T_B \omega_B \\ &= \left( -\frac{Z_A}{Z_B} T_B \right) \left( -\frac{Z_C}{Z_A} \omega_C \right) : T_B \frac{Z_C}{Z_B} \omega_C \\ &= 1 : 1 \end{aligned} \quad (16)$$

On the other hand, when  $\omega_w = 0$  ( $T_C = 0$ ), the joint torques are given from (12) by

$$T_A = \frac{Z_A}{Z_B} T_B, \quad (17)$$

$$T_C = 0, \quad (18)$$

$$T_D = \frac{2(Z_A + Z_C)}{Z_A} T_A. \quad (19)$$

The power ratio is given from (7) and (17) by

$$\begin{aligned} P_A : P_B &= T_A \omega_A : T_B \omega_B \\ &= \frac{Z_A}{Z_B} T_B \omega_B : T_B \omega_B \\ &= Z_A : Z_B. \end{aligned} \quad (20)$$

Letting the diameter of sun gear and planet gear be the same in Fig. 1 yields

$$P_A : P_B = 1 : 3. \quad (21)$$

### 2.3 Operation Ratio of Motors

In this section, we discuss the operation ratio of motors by comparing the DDSS to a conventional caster-drive wheel. We define the operation ratio  $\delta$  of motors as

$$\delta = \frac{(\text{Sum of motor power in motion})}{(\text{Sum of rated power of motors})}. \quad (22)$$

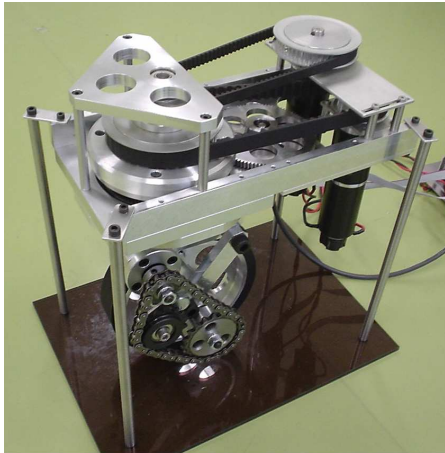


Figure 5: Prototype DDSS.

The ratio ( $P_{A0} : P_{B0}$ ) of rated power of two motors used in the DDSS is set to be 1:1. The ratio of rated power used in conventional method is also set to be 1:1 as denoted in (Wada, 1996).

We calculate the operation ratio  $\delta$  in case of driving motion ( $T_D = 0$ ). Let  $P$  be the sum of motor output power needed to achieve the motion. The result of the conventional method is  $\delta = \frac{P}{P_{A0} + P_{B0}} = 0.5$  from  $P_{A0} = P_{B0} = P$ . The result of the DDSS is  $\delta = \frac{P}{P_{A0} + P_{B0}} = 1$  from  $P_{A0} = P_{B0} = \frac{P}{2}$ .

Next, we calculate  $\delta$  in case of steering motion ( $T_C = 0$ ). The result of conventional method is  $P_{A0} = P_{B0} = P$  and  $\delta = 0.5$ . The result of the DDSS is  $P_{A0} = P_{B0} = \frac{3}{4}P$  and  $\delta = 0.67$ , because  $P_{B0} = \frac{3}{4}P$  from (21).

The output power of motors can be decreased by using the DDSS as a caster-drive wheel because of its high operation ratio of motors. It means that the size of robot become smaller by using the DDSS.

### 3 CONSTRUCTION OF OMNIDIRECTIONAL MOBILE ROBOT

We constructed a prototype model of the DDSS to check the availability of proposed mechanism as shown in Fig. 5. Torques of two motors are transmitted to the sun gear(A) and the outer ring gear(B) by the timing belts, and driving and steering torques are generated. Effectiveness of the proposed DDSS was confirmed by this apparatus.

Figure 6 and Table 1 show picture and specification of an omnidirectional mobile robot with four DDSS wheels, respectively. The proposed omnidirectional robot has capability of climbing slope of 10

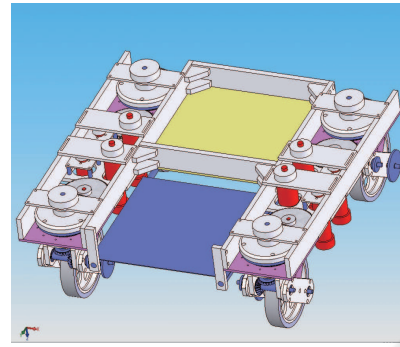


Figure 6: Omnidirectional Mobile Robot.

Table 1: Specification of Omnidirectional Mobile Robot.

|                      |                         |
|----------------------|-------------------------|
| Size (D × W)         | 600 × 600 [mm]          |
| Weight               | 70 [kg]                 |
| Motor power          | 150 [W] × 8             |
| Max. velocity        | 6 [km/h]                |
| Max. acceleration    | 0.5 [m/s <sup>2</sup> ] |
| Max. slope angle     | 10 [deg]                |
| Max. step difference | 60 [mm]                 |
| Max. loading weight  | 100 [kg]                |

[deg], accelerating 0.5 [m/s<sup>2</sup>] and exceeding difference of 60 [mm] with carrying load of 100 [kg].

## 4 SIMULATION

### 4.1 Simulation Method

To show the performance of the DDSS, simulations are conducted. The simulation model is constructed by SolidWorks and DADS. The radius  $r$  of the wheel and the offset  $s$  shown in Fig. 1 were given as  $r = 80$  [mm] and  $s = 50$  [mm], respectively.

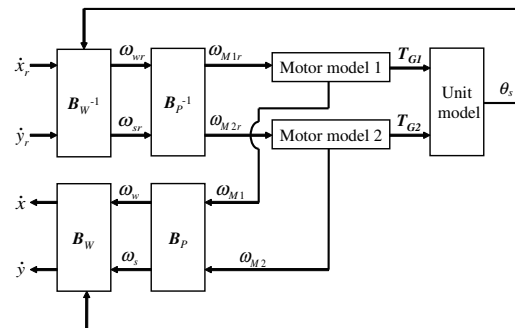


Figure 7: Control system of DDSS.



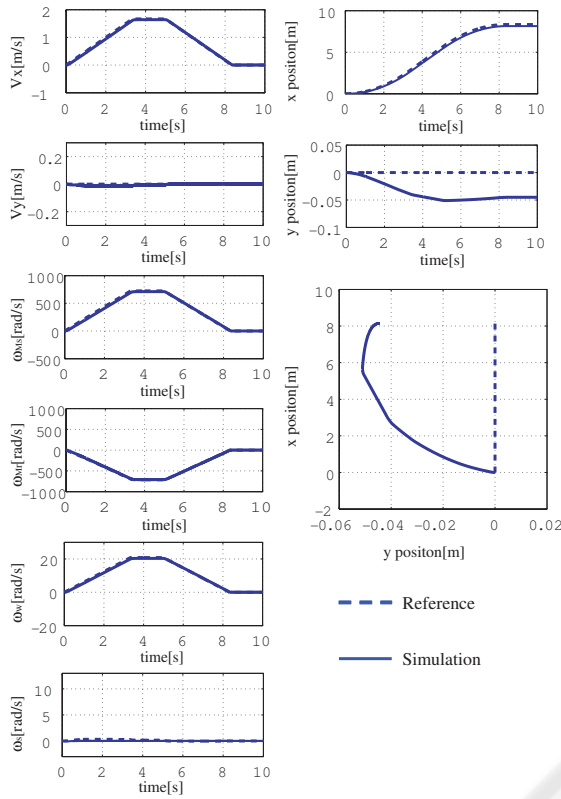


Figure 8: Simulation results ( $V_x = 6$  km/h,  $V_y = 0$  km/h).

Figure 7 shows the control system of a wheel used in this simulation. The reference angular velocities  $\omega_{M1r}$  and  $\omega_{M2r}$  are given by (3) and (10). The velocity  $(\dot{x}, \dot{y})$  of the DDSS is given from the angular velocities  $\omega_{M1}$  and  $\omega_{M2}$  and the steering angle  $\theta_s$ .

Four translational motions toward +X, -X, +Y and -Y direction with maximum velocity of 6 [km/h] (= 1.67 [m/s]) and maximum acceleration of 0.5 [m/s<sup>2</sup>] were examined. The initial value of  $\theta_s$  was set to be zero in any case.

## 4.2 Simulation Results and Discussion

Figures 8 through 11 show the simulation results of motions toward +X, -X, +Y and -Y direction, respectively, from the initial state  $\theta_s = 0$ . Here,  $V_x$ ,  $V_y$ ,  $\omega_{Ms}$ ,  $\omega_{Mr}$ ,  $\omega_w$  and  $\omega_s$  indicate the velocity in X-direction, the velocity in Y-direction, the angular velocity of sun gear motor, the angular velocity of outer ring gear motor, the angular velocity of wheel and the angular velocity of steering axis, respectively.

As seen from these graphs, error from the reference trajectory is within 6 [cm] in any case. We can also see the feature of caster-drive wheel. In Fig. 9, the angular velocity  $\omega_s$  of steering axis becomes large

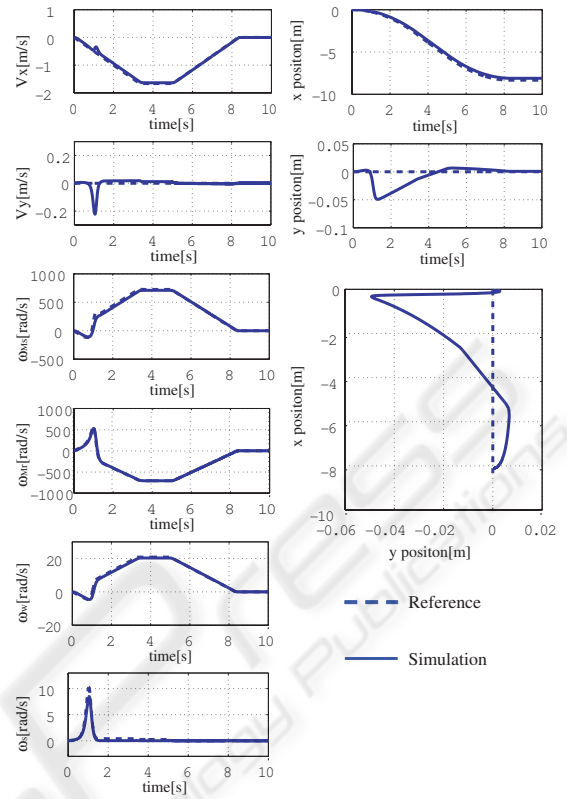


Figure 9: Simulation results ( $V_x = -6$  km/h,  $V_y = 0$  km/h).

in the beginning of motion because the moving direction is opposite to the initial direction  $\theta_s = 0$ .

## 5 CONCLUSIONS

We proposed Differential-Drive Steering System (DDSS) for caster-drive wheel of holonomic omnidirectional mobile robot. The DDSS can provide high operation ratio of motors rather than conventional caster-drive wheel. Numerical analysis, examination by prototype model and simulation results showed effectiveness of the DDSS.

Future works include

- construction of an omnidirectional mobile robot and experiments,
- posture control on rough terrain,
- application to an omnidirectional wheelchair.

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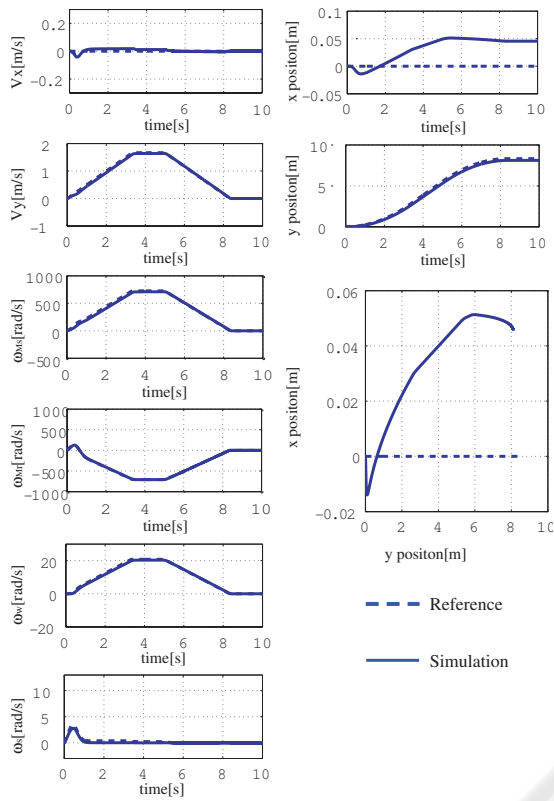


Figure 10: Simulation results ( $V_x = 0$  km/h,  $V_y = 6$  km/h).

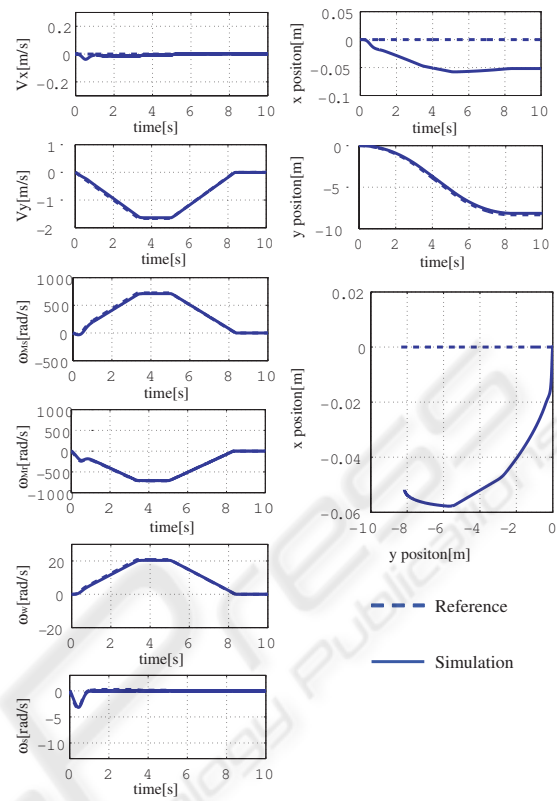


Figure 11: Simulation results ( $V_x = 0$  km/h,  $V_y = -6$  km/h).

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