IFT-BASED PI-FUZZY CONTROLLERS
Signal Processing and Implementation

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Abstract: New Takagi-Sugeno PI-fuzzy controllers (PI-FCs) are suggested in this paper. The PI-FC design is based on the optimization of PI controllers in terms of the Iterative Feedback Tuning (IFT) approach. Next the parameters of the PI controllers are mapped onto the parameters of the Takagi-Sugeno PI-FCs in terms of the modal equivalence principle. An attractive design method is derived to support the implementation of low-cost PI-FCs. The design is enabled by a stability analysis theorem based on Lyapunov's theorem for time-varying systems. The theoretical approaches are validated by a case study corresponding to the position control of a servo system. Real-time experimental results are included.

1 INTRODUCTION

The design of control systems (CSs) making use of measurement data is successful in many industrial applications without models available for the controlled process. The time-consuming design of those models can be avoided. Fuzzy control is an alternative when very good steady-state and dynamic CS performance indices can be guaranteed. The systematic design of fuzzy controllers must be assisted by the analysis of the fuzzy CS structural properties i.e. stability, controllability, parametric sensitivity and robustness (Sala et al., 2005; Kovačić and Bogdan, 2006; Blažič and Škrjanc, 2007).

Iterative Feedback Tuning (IFT) is a gradient-based approach, based on input-output data recorded from the closed-loop system (Hjalmarsson et al., 1998). The performance specifications are expressed in terms of objective functions in appropriate optimization problems. Those problems can be solved by iterative gradient-based minimization implemented as IFT algorithms. IFT makes use of closed-loop experimental data to calculate the estimates of the gradients of the objective functions. Several experiments are done per iteration and the updated controller parameters are calculated based on the input-output data. So the IFT belongs to the direct data-based offline-adaptive controller designs.

The combination of IFT and fuzzy control leads

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to the convenient performance enhancement of fuzzy CSs after their initial tuning (Precup et al., 2008). The first contribution of the paper concerns the modification of the second experiment specific to IFT to be overlapped over the normal CS operation. Several useful remarks are introduced in relation with the signal processing and implementation of the IFT algorithms. The second contribution is a new design method of low-cost Takagi-Sugeno PI-FCs which is based on mapping the results from the linear case onto the fuzzy one in terms of the modal equivalence principle (Galichet and Foulloy, 1995). The third contribution is a stability analysis theorem based on Lyapunov's theorem for time-varying systems derived from (Slotine and Li, 1991) to support the PI-FC design.

The paper is organized as follows. Section 2 discusses the signal processing and implementation aspects regarding the IFT algorithm. Next, Section 3 presents the new design method for a class of Takagi-Sugeno PI-FCs. Section 4 addresses a case study associated with low-cost implementations of DC drive servo system position CSs. The conclusions are presented in Section 5.

2 SIGNAL PROCESSING AND IMPLEMENTATION ASPECTS IN IFT ALGORITHMS

The IFT-based CS structure is presented in Figure 1, where: r – the reference input, d – the disturbance input, $e = r - y$ – the control error, u – the control signal, $\boldsymbol{\rho}$ – the parameters vector having the controller tuning parameters as its components, $C(\boldsymbol{\rho})$ – the transfer function of the linear controller, a PI one, here to be replaced by the PI-FC, in order to improve the CS performance indices, F – the transfer function of the reference model that prescribes the desired behaviour to be exhibited by the closed-loop system, P – the transfer function of the controlled process, y – the controlled output, y_d – the desired output produced by the reference model, $\delta y = y - y_d$ – the model tracking error, and IFT – the Iterative Feedback Tuning algorithm, $\mathbf{i}$ – the input vector to set the performance specifications.

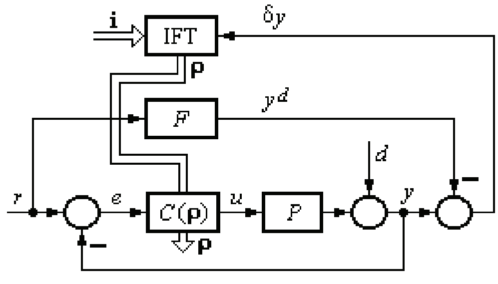

Figure 1: CS structure with IFT.

The operational variable in the transfer functions has been omitted for the sake of simplicity. However that variable will be mentioned in the sequel in the well accepted notation s for continuous-time systems and z for discrete-time ones to improve the clarity of the presentation when needed. That is also the reason for inserting or removing the argument $\boldsymbol{\rho}$.

The controller parameterization is such that the transfer function $C(\boldsymbol{\rho})$ is differentiable with respect to $\boldsymbol{\rho}$. The controller must ensure an initially stabilized CS. The initial controller tuning affects the convergence of the iterative process.

The accepted expression of the objective function $J(\boldsymbol{\rho})$ is

$$J(\boldsymbol{\rho}) = (0.5/N)\sum_{k=1}^{N}[\delta y(k,\boldsymbol{\rho})]^2, \tag{1}$$

where: N – the number of samples setting the length of each experiment. A typical objective is to find a parameters vector $\boldsymbol{\rho}^*$ to minimize $J(\boldsymbol{\rho})$ and make the error δy tend to zero by the optimization problem

$$\boldsymbol{\rho}^* = \arg\min_{\boldsymbol{\rho}\in SD} J(\boldsymbol{\rho}), \tag{2}$$

where several constraints can be imposed regarding the controlled process or the closed-loop CS. The most important constraint accepted in this paper concerns the necessity of stable CSs, and SD stands for the stability domain. Other variables including the control signal can be used. That requires additional signal processing and increased cost.

The IFT algorithms solve the optimization problem (2) by means of numerical optimization techniques. Newton's method is popular in IFT since it can be treated independently of the difficulties inherent to the model-based techniques. It evaluates repeatedly a new solution based on a point of the function and its approximate derivative. The mathematical formulation is the following update law to calculate the next set of parameters $\boldsymbol{\rho}^{i+1}$:

$$\boldsymbol{\rho}^{i+1} = \boldsymbol{\rho}^{i} - \gamma^{i}(\mathbf{R}^{i})^{-1} est[\frac{\partial J}{\partial \boldsymbol{\rho}}(\boldsymbol{\rho}^{i})], \quad (3)$$

where: i – the index of current iteration, $est[\frac{\partial J}{\partial \boldsymbol{\rho}}(\boldsymbol{\rho}^{i})]$ – the estimate of the gradient vector, γ^{i} – the step size, $\boldsymbol{\rho}^{0}$ – the initial guess of the tuning parameters, and $\mathbf{R}^{i}$ – a regular positive definite matrix. $\mathbf{R}^{i}$ can be the Hessian or the identity matrix to simplify the signal processing.

Differentiating (1), the gradient becomes

$$\frac{\partial J}{\partial \boldsymbol{\rho}}(\boldsymbol{\rho}^{i}) = (1/N)\sum_{k=1}^{N}[\delta y(k,\boldsymbol{\rho}^{i})\frac{\partial \delta y}{\partial \boldsymbol{\rho}}(k,\boldsymbol{\rho}^{i})] \cdot \quad (4)$$

To calculate the general expressions of the gradient of the output error it is necessary to make use of the information obtained from the closed-loop system. The sensitivity function S and the complementary sensitivity function T must be expressed:

$$\begin{aligned} &S(\boldsymbol{\rho}) = 1/[1 + C(\boldsymbol{\rho})P], \\ &T(\boldsymbol{\rho}) = 1 - S(\boldsymbol{\rho}) = C(\boldsymbol{\rho})P/[1 + C(\boldsymbol{\rho})P]. \end{aligned} \quad (5)$$

The differentiation of $\delta y(\boldsymbol{\rho})$ making use of (5) and Figure 1 leads to the gradient of $\delta y(\boldsymbol{\rho})$:

$$\frac{\partial \delta y}{\partial \boldsymbol{\rho}}(\boldsymbol{\rho}) = \frac{1}{C(\boldsymbol{\rho})} \cdot \frac{\partial C}{\partial \boldsymbol{\rho}}(\boldsymbol{\rho})T(\boldsymbol{\rho})(r - y) \cdot \quad (6)$$

To obtain the estimate of the gradient of the model tracking error use is made of two experiments per iteration for the PI controllers. In the first experiment, the normal one, use is made of Figure 1, the reference input r_1 is applied to the CS and the controlled output y_1 is measured. In the second experiment, the gradient one, the control error of the first experiment $e_1 = r_1 - y_1$ is applied as the reference input r_2 (Hjalmarsson et al., 1998) and the controlled output y_2 is measured. That processing is far away from the normal CS operation. Therefore a new gradient experiment is suggested here where the reference input r_2 is applied and the signal e_1 is injected after the control signal. That can be expressed as the experimental scheme for the gradient experiment illustrated in Figure 2 where the blocks F and IFT have been dropped out.

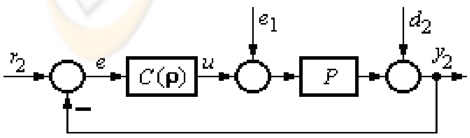

Figure 2: Experimental scheme for gradient experiment.

Accepting the lower subscript pointing out the index of the current experiment, the reference input and controlled output in the normal experiment are

$$r_1 = r, \; y_1(\boldsymbol{\rho}) = T(\boldsymbol{\rho})r + S(\boldsymbol{\rho})d_1 \cdot \quad (7)$$

For the gradient experiment, making use of the Figure 2, the results are

$$\begin{aligned} &r_2 = K\, r, \; y_2(\boldsymbol{\rho}) = T(\boldsymbol{\rho})r_2 + \\ &+ \{P/[1 + C(\boldsymbol{\rho})P]\}(r_1 - y_1(\boldsymbol{\rho})] + S(\boldsymbol{\rho})d_2, \end{aligned} \quad (8)$$

where the gain K has been inserted to show the proportional reference inputs in the two experiments. Next (7) is multiplied by K, extracted from (8), the relationship (6) is used and the result becomes

$$\begin{aligned} &\frac{\partial \delta y}{\partial \boldsymbol{\rho}}(k,\boldsymbol{\rho}^{i}) = \frac{\partial C}{\partial \boldsymbol{\rho}}(q^{-1},\boldsymbol{\rho}^{i})[y_2(k,\boldsymbol{\rho}^{i}) - \\ &- K\, y_1(k,\boldsymbol{\rho}^{i})] - S(\boldsymbol{\rho}^{i})[d_2(k) - K\, d_1(k)]\, / \\ &C(q^{-1},\boldsymbol{\rho}^{i}). \end{aligned} \quad (9)$$

The second term in the right-hand side of (9) depends on the disturbance inputs, it affects the gradient, so it should be alleviated. Neglecting that term the estimate of the gradient of δy is evaluated:

$$\begin{aligned} &est[\frac{\partial \delta y}{\partial \boldsymbol{\rho}}(k,\boldsymbol{\rho}^{i})] = \frac{\partial C}{\partial \boldsymbol{\rho}}(q^{-1},\boldsymbol{\rho}^{i})[y_2(k,\boldsymbol{\rho}^{i}) - \\ &- K\, y_1(k,\boldsymbol{\rho}^{i})]. \end{aligned} \quad (10)$$

The alleviation of the second term in the right-hand side of (9) can be done by the proper initial tuning of the controller parameters because $S(\boldsymbol{\rho}^{i})$ plays the role of filter. That term can lead to shifted estimates with negative effects on the convergence. A similar approach (Hildebrand et al., 2005) is characterized by an additional prefilter designed as solution to optimization problems. That filter is not introduced here to simplify the signal processing accepting that K=1. The role of $\boldsymbol{\rho}^{0}$ is highlighted from that point of view.

Summarizing all signal processing aspects mentioned before in the linear case, one iteration in the IFT algorithm consists of the following steps.

Step 0. Set $\boldsymbol{\rho}^{0}$.

Step 1. The two experiments are done making use of the CS structures presented in Figure 1 and Figure 2 and the outputs y_1 and y_2 are measured.

Step 2. The output of the reference model is generated, y_d, and the output error δy is calculated.

Step 3. The estimate of gradient of J is calculated according to (4) and (10).

Step 4. The next set of parameters $\boldsymbol{\rho}^{i+1}$ is calculated in terms of the update law (3).

3 DESIGN OF TAKAGI-SUGENO PI-FUZZY CONTROLLERS

The Takagi-Sugeno PI-FC is a discrete-time controller built around the two inputs-single output fuzzy controller (TISO-FC), Figure 3, where $\Delta e(k)=e(k)-e(k-1)$ and $\Delta u(k)=u(k)-u(k-1)$ is the increment of control error and signal, respectively. The fuzzification is done by the membership functions presented in Figure 4, the inference engine employs the MAX and MIN operators assisted by the rule base presented in Table 1, and the weighted average defuzzification method is employed.

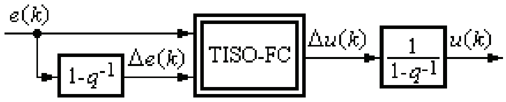

Figure 3: Structure of Takagi-Sugeno PI-fuzzy controller.

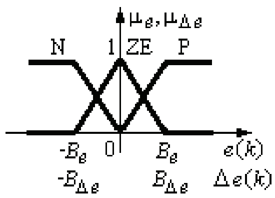

Figure 4: Input membership functions of TISO-FC.

Table 1: Rule base as decision table of TISO-FC.

$\Delta e(k)$	$e(k)$		
	N	ZE	P
P	$\Delta u_k = f_k$	$\Delta u_k = f_k$	$\Delta u_k = \eta f_k$
ZE	$\Delta u_k = f_k$	$\Delta u_k = f_k$	$\Delta u_k = f_k$
N	$\Delta u_k = \eta f_k$	$\Delta u_k = f_k$	$\Delta u_k = f_k$

The rule base of the PI-FC can be reduced to two rules (Johanyák and Kovács, 2007). The rule consequents (Table 1) point out the term $f(k)$:

$$f(k)= K_P[\Delta e(k)+\alpha e(k)]. \qquad (11)$$

Eq. (11) corresponds to the recurrent equation of an incremental digital PI controller. The Takagi-Sugeno PI-FCs will exhibit as bumpless interpolators between two linear PI controllers. The additional parameter η with typical values within $0<\eta<1$ reduces the overshoot.

The parameters K_P and α in (11) can be obtained either directly in the discrete-time form or by the continuous-time form of the PI controller

$$C(s) = k_c(1+sT_i)/s = k_C[1+1/(sT_i)], \qquad (12)$$

followed by the discretization in terms of the sampling period T_s (in terms of quasi-continuous control), where T_i is the integral time constant and k_C, $k_C = T_i k_c$, is the controller gain. In case of Tustin's discretization method applied here the parameters K_P and α obtain the expressions

$$K_P = k_C[1-T_s/(2T_i)], \alpha = 2T_s/(2T_i - T_s). \qquad (13)$$

Accepting the approximations specific to the quasi-continuous digital control (Precup et al., 2008) the Takagi-Sugeno PI-FCs can be considered as continuous-time fuzzy controllers. However the calculation of the maximum T_s such that the stability is also ensured is of interest. The IFT-based design method dedicated to the accepted class of Takagi-Sugeno PI-FCs consists of the following steps.

Step 1. T_s is set and an initial linear tuning method is applied to calculate the initial controller parameters, K_P and α. They can be obtained also by an initial guess based on the designer's experience.

Step 2. The initial data of the IFT algorithm and the reference model parameters are set.

Step 3. The IFT algorithm presented in the previous Section is applied resulting in the optimal controller parameters.

Step 4. B_e and η are chosen according to the performance specifications. The stability analysis to be presented as follows is taken into account. Next the modal equivalence principle is applied:

$$B_{\Delta e} = \alpha B_e. \qquad (14)$$

The current trends in the stability analysis of fuzzy CSs employ Lyapunov's (Wang et al., 2007), Krasovskii's and La Salle's approaches (Tian and Peng, 2006), the describing function method or algebraic approaches (Michels et al., 2006; Jantzen, 2007). The stability analysis to be presented as follows employs the formalism applied in (Lam and Leung, 2008; Lam and Ling, 2008). The state-space equation of the controlled process is

$$\dot{\mathbf{x}}(t) = \mathbf{f}(\mathbf{x},t) + \mathbf{b}(\mathbf{x},t)u(t), \mathbf{x}(t_0) = \mathbf{x}_0, \qquad (15)$$

where $\mathbf{x} = [x_1 \quad x_2 \quad ... \quad x_n]^T \in D$ is the state vector, $n \in N^*$, $\dot{\mathbf{x}} = [\dot{x}_1 \quad \dot{x}_2 \quad ... \quad \dot{x}_n]^T$ is the derivative of $\mathbf{x}$ with respect to the independent time variable t, $\mathbf{f}, \mathbf{b} : D \times [0,\infty) \to R^n$ are continuous functions of t:

$$\mathbf{f}(\mathbf{x},t) = [f_1(\mathbf{x},t) \quad f_2(\mathbf{x},t) \quad ... \quad f_n(\mathbf{x},t)]^T, \qquad (16)$$
$$\mathbf{b}(\mathbf{x},t) = [b_1(\mathbf{x},t) \quad b_2(\mathbf{x},t) \quad ... \quad b_n(\mathbf{x},t)]^T,$$

T stands for matrix transposition, and the disturbance is absent. The PI-FC inputs are (n=2):

$$x_1(t) = e(t) = r - y(t) = e(k),$$
$$x_2(t) = \dot{x}_1(t) = \Delta e(k) / T_s. \quad (17)$$

The expression of the control signal is

$$u = (\sum_{i=1}^{r_B} \alpha_i u_i) / (\sum_{i=1}^{r_B} \alpha_i), \quad (18)$$

where u_i is the control signal produced in the consequent of the *i*-th rule, $i = \overline{1, r_B}$, r_B is the number of fuzzy control rules, and α_i is the firing strength (Precup et al., 2008).

The Lyapunov function candidate is

$$V : D \times [0, \infty) \to R, V(\mathbf{x}, t) = g(t)\mathbf{x}^T \mathbf{P}\, \mathbf{x}, \quad (19)$$

where $\mathbf{P} \in R^{n \times n}$ is a constant positive definite matrix and $g : [0, \infty) \to [0, \infty)$ is a continuously differentiable function. The derivative of V with respect to time with the system constrained to (15) is

$$\begin{aligned}\dot{V}(\mathbf{x}, t) &= F(\mathbf{x}, t) + B(\mathbf{x}, t)u, F(\mathbf{x}, t) = \\ &= g(t)[\mathbf{f}(\mathbf{x}, t)]^T \mathbf{P}\, \mathbf{x} + g(t)\mathbf{x}^T \mathbf{P}\, \mathbf{f}(\mathbf{x}, t) + \\ &+ \dot{g}(t)\mathbf{x}^T \mathbf{P}\, \mathbf{x}, B(\mathbf{x}, t) = g(t)[\mathbf{b}(\mathbf{x}, t)]^T \mathbf{P}\, \mathbf{x} + \\ &+ g(t)\mathbf{x}^T \mathbf{P}\, \mathbf{b}(\mathbf{x}, t),\end{aligned} \quad (20)$$

and its expression calculated for $u = u_k(\mathbf{x})$ is $\dot{V}_k(\mathbf{x}, t)$.

Theorem 1. Let $\mathbf{x} = \mathbf{0} \in D \subset R^n$ be an equilibrium point for (15) controlled by the accepted PI-FC and V the Lyapunov function candidate (19) such that the following two conditions are fulfilled:

$$\begin{aligned}&V(\mathbf{x}, t) \le W^1(\mathbf{x}), \\ &\dot{V}_k(\mathbf{x}, t) \le -W_k^2(\mathbf{x}), k = \overline{1, r_B}, \forall t \ge 0, \forall \mathbf{x} \in D,\end{aligned} \quad (21)$$

where W^1 and W_k^2 are continuous positive definite functions on D. Then $\mathbf{x} = \mathbf{0}$ will be uniformly asymptotically stable.

Proof: Use is made of (20) and (21) leading to

$$\dot{V}(\mathbf{x}, t) \le -[\sum_{k=1}^{r_B} (W_k^2(\mathbf{x})\alpha_k(\mathbf{x}))] / [\sum_{k=1}^{r_B} \alpha_k(\mathbf{x})] \cdot \quad (22)$$

Therefore Lyapunov's theorem for time-varying systems is fulfilled due to the conditions (21) and (22), and the equilibrium point $\mathbf{x} = \mathbf{0}$ will be uniformly asymptotically stable.

Theorem 1 offers sufficient stability conditions in the choice of the parameters B_e and η. Its application has been implemented for a real-world process in the Intelligent Systems Laboratory with the "Politehnica" University of Timisoara (PUT).

4 CASE STUDY

The experimental setup is built around the INTECO DC servo system with backlash laboratory equipment, Figure 5, with rated amplitude of 24 V, current of 3.1 A, torque of 15 N cm and speed of 3000 rpm. The inertial load weighs 2.03 kg.

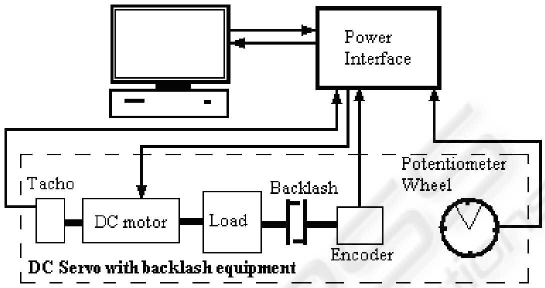

Figure 5: Structure of experimental setup.

The transfer functions of the simplified controlled process and reference model are

$$P(s) = \frac{k_P}{s(1 + sT_\Sigma)}, \; F(s) = \frac{1}{s^2 + 1.5s + 1}, \quad (23)$$

$k_P = 139.88$, $T_\Sigma = 0.9198\,\text{s}$. The continuous-time PI controller has been obtained by frequency domain design which yields the parameters $k_C = 0.01036$ and $T_i = 3.1043\,\text{s}$. Discretizing with $T_s = 0.01\,\text{s}$, the initial digital PI controller parameters are $\boldsymbol{\rho}^0 = [K_P = 0.01035 \quad \alpha = 0.0029]^T$. The parameters obtained after 10 iterations for the step size $\gamma^i = 10^{-6}$ are $\boldsymbol{\rho}^{10} = [0.010346 \quad 0.003226]^T$. Setting $B_e = 20$ and $\eta = 0.5$, (14) results in $B_{\Delta e} = 0.06452$.

The constant reference input $r = 40\,\text{rad}$ has been applied. The behaviour of the CS with linear controller before IFT is illustrated in Figure 6.

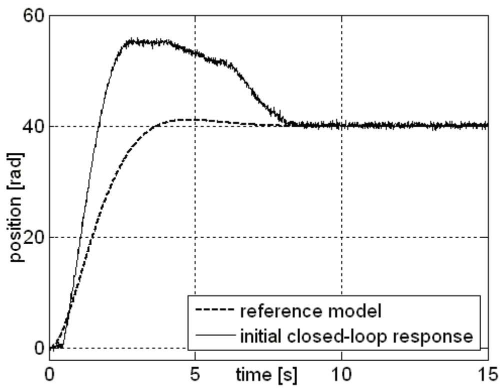

Figure 6: Reference model output and controlled output (position) versus time for linear CS before IFT.

The behaviour of the CS with PI-FC after IFT is presented in Figure 7. The performance indices (overshoot and settling time) of the CS have been improved. A band-limited white noise of variance 0.01 has been applied as the disturbance input *d*.

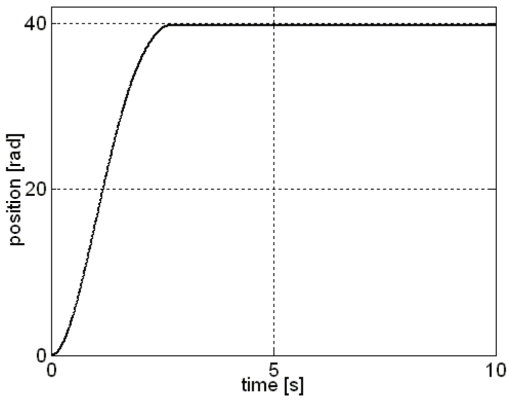

Figure 7: *y* versus time for fuzzy CS after IFT.

5 CONCLUSIONS

The paper has proposed a stable design method dedicated to a class of Takagi-Sugeno PI-FCs. It is based on mapping the IFT-based linear case results onto the fuzzy control results.

Several signal processing aspects regarding the simplification of the implementation have been discussed. They involve an original gradient experiment. A single gradient experiment is needed. However an additional one can be employed in other CS structures.

The stability analysis can be applied to the fuzzy control of time-varying systems. It is valid because of the quasi-continuous digital implementation of the controllers that enables the controller design.

One future research topic concerns the convergence analysis. Although the stability analysis suggested is attractive, the convergence is not guaranteed. The future research will be dedicated to the application of the approaches to other fuzzy controller structures (Valente de Oliveira and Gomide, 2001; Vaščák, 2007; Pedrycz, 2009).

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