RECURSIVE EXTENDED COMPENSATED LEAST SQUARES BASED ALGORITHM FOR ERRORS-IN-VARIABLES IDENTIFICATION

Tomasz Larkowski, Jens G. Linden and Keith J. Burnham
Control Theory and Applications Centre, Coventry University, Priory Street, Coventry, CV1 5FB, U.K.

Keywords: Bias compensation, Errors-in-variables, Parameter estimation, Recursive algorithms, System identification.

Abstract: An algorithm for the recursive identification of single-input single-output linear discrete-time time-invariant errors-in-variables system models in the case of white input and coloured output noise is presented. The approach is based on a bilinear parametrisation technique which allows the model parameters to be estimated together with the auto-correlation elements of the input/output noise sequences. In order to compensate for the bias in the recursively obtained least squares estimates, the extended bias compensated least squares method is used. An alternative for the online update of the associated pseudo-inverse of the extended observation covariance matrix is investigated, namely an approach based on the matrix pseudo-inverse lemma and an approach based on the recursive extended instrumental variables technique. A Monte-Carlo simulation study demonstrates the appropriateness and the robustness against noise of the proposed scheme.

1 INTRODUCTION

The errors-in-variables (EIV) approach forms an extension of the standard output error system setup in which it is postulated that only the output measurements are uncertain. In the EIV framework all measured signals, hence, including the system input, are assumed to be contaminated with noise, see (Söderström, 2007) for the recent survey on this subject. The EIV framework can offer advantages over the classical approach, mainly when the description of the internal laws governing a system is of prime interest, e.g. application areas in chemistry, image processing, fault detection etc., see (Söderström, 2007; Markovsky and Van Huffel, 2007) for further details.

One of the EIV techniques that has been shown to be robust and to yield relatively precise estimates is the extended compensated least squares (ECLS) method. The approach is based on the extended bias compensated least squares (EBCLS) and utilises separable nonlinear least squares to solve the resulting overall identification problem. The method was first proposed in (Ekman, 2005a), which considered the case of white input and output noise sequences and subsequently extended to handle the case of coloured output noise in (Ekman et al., 2006). Further analysis, considering a generalised framework, has been carried out in (Mahata, 2007).

Alternatively, by exploiting the property that the overall optimisation problem is bilinear in the unknowns, see (Ljung, 1999), which in this case corresponds to the model parameters and the input/output noise auto-correlation elements, the principle of bilinear parametrisation can be utilised. The resulting scheme, termed here the extended bilinear parametrisation method (EBPM) involves solving iteratively two ordinary least squares problems, see (Larkowski et al., 2008) for details. Although the quality of the parameters obtained by the EBPM is comparable to the quality of the estimates yielded by the ECLS, an important distinction is that the EBPM is significantly less computationally demanding than the ECLS technique.

The bilinear parametrisation method was first utilised to solve the EIV identification problem in a recursive manner in (Ekman, 2005b) for the case of white input and output noise. It has also been exploited in (Ikenoue et al., 2008) for the case of coloured input and output noise sequences and for the purpose of offline as well as online estimation. However, in both cases the term ‘bilinear parametrisation’ has not been explicitly stated. In (Ekman, 2005b) the constructed recursive algorithm is not computationally attractive, since its complexity at each iteration is actually greater than that of the corresponding batch algorithm applied in an offline manner at each recur-

Larkowski T., G. Linden J. and J. Burnham K. (2009).
RECURSIVE EXTENDED COMPENSATED LEAST SQUARES BASED ALGORITHM FOR ERRORS-IN-VARIABLES IDENTIFICATION.
In *Proceedings of the 6th International Conference on Informatics in Control, Automation and Robotics - Signal Processing, Systems Modeling and Control*, pages 142-147
DOI: 10.5220/0002211901420147

sion. Whereas, in (Ikenoue et al., 2008) due to a special choice of the instruments, the resulting algorithm is not causal, in general, hence its recursive implementation yields delayed estimates.

In this paper a recursive realisation of the EBPM is presented for a discrete-time linear time-invariant (LTI) single-input single-output (SISO) system model in the case of the white input and coloured output noise and it is demonstrated that the above mentioned shortcomings may be avoided. The bias of the recursively calculated least squares (LS) estimator is removed at each recursion via the extended bias compensated least squares (EBCLS) technique. The online update of the pseudo-inverse of the overdetermined observation matrix is realised by considering an alternative, namely an approach based on the pseudo-inverse lemma, see (Feng et al., 2001) and an approach based on the recursive extended instrumental variables technique, see (Friedlander, 1984). The two resulting algorithms are analysed and compared with their offline counterpart via a Monte-Carlo simulation study. It is shown that the instrumental variables approach is the more preferable due to its superior robustness and improved convergence properties, in general.

2 NOTATION AND PROBLEM STATEMENT

Consider a discrete-time LTI SISO system represented by the difference equation

$$A(q^{-1})y_{0_k} = B(q^{-1})u_{0_k}, \tag{1}$$

where the polynomials $A(q^{-1})$ and $B(q^{-1})$ are given by

$$A(q^{-1}) \triangleq 1 + a_1 q^{-1} + \ldots + a_{n_a} q^{-n_a}, \tag{2a}$$
$$B(q^{-1}) \triangleq b_1 q^{-1} + \ldots + b_{n_b} q^{-n_b} \tag{2b}$$

with q^{-1} being the backward shift operator, defined by $q^{-1}x_k \triangleq x_{k-1}$. The unknown noise-free input and noise-free output signals denoted u_{0_k} and y_{0_k}, respectively, are related to the available noisy variables, denoted u_k and y_k, such that

$$u_k = u_{0_k} + \tilde{u}_k, \qquad y_k = y_{0_k} + \tilde{y}_k, \tag{3}$$

where $\tilde{u}_k$ and $\tilde{y}_k$ denote the input and output measurement noise sequences, respectively. The following standard assumptions, see e.g. (Ekman et al., 2006), are introduced:

A1 The LTI system (1) is asymptotically stable, i.e. $A(q^{-1})$ has all zeros inside the unit circle.

A2 All system modes are observable and controllable, i.e. $A(q^{-1})$ and $B(q^{-1})$ share no common factors.

A3 The system structure, i.e. n_a and n_b, is known *a priori* and $n_a \geq n_b$.

A4 The true input u_{0_k} is a zero mean, ergodic random sequence persistently exciting and of sufficiently high order, i.e. at least of order $n_a + n_b$.

A5a The additive input noise sequence $\tilde{u}_k$ of unknown variance $\sigma_{\tilde{u}}$ is an ergodic zero mean white process.

A5b The additive output noise sequence $\tilde{y}_k$ is an ergodic zero mean process characterised by an unknown auto-covariance sequence $\{r_{\tilde{y}}(0),\ r_{\tilde{y}}(1),\ \ldots\}$.

A6 The input/output noise sequences are mutually uncorrelated and uncorrelated with signals u_{0_k} and y_{0_k}.

By postulating that the output noise sequence exhibits an arbitrary degree of correlation allows for measurement sensor uncertainties to be taken into account, as well as potential disturbances in the process.

The system parameter vector is denoted

$$\theta \triangleq \begin{bmatrix} a^T & b^T \end{bmatrix}^T \in \mathcal{R}^{n_\theta} \quad , \tag{4a}$$
$$a \triangleq \begin{bmatrix} a_1 & \ldots & a_{n_a} \end{bmatrix}^T \in \mathcal{R}^{n_a}, \tag{4b}$$
$$b \triangleq \begin{bmatrix} b_1 & \ldots & b_{n_b} \end{bmatrix}^T \in \mathcal{R}^{n_b}, \tag{4c}$$

where $n_\theta = n_a + n_b$. The extended regressor vectors for the k-th measured data are defined as

$$\bar{\varphi}_k \triangleq \begin{bmatrix} -y_k & \varphi_k^T \end{bmatrix}^T \in \mathcal{R}^{n_\theta+1}, \tag{5a}$$
$$\bar{\varphi}_{y_k} \triangleq \begin{bmatrix} -y_k & \varphi_{y_k}^T \end{bmatrix}^T \in \mathcal{R}^{n_a+1}, \tag{5b}$$

where

$$\varphi_k \triangleq \begin{bmatrix} \varphi_{y_k}^T & \varphi_{u_k}^T \end{bmatrix}^T \in \mathcal{R}^{n_\theta}, \tag{5c}$$
$$\varphi_{y_k} \triangleq \begin{bmatrix} -y_{k-1} \ldots -y_{k-n_a} \end{bmatrix}^T \in \mathcal{R}^{n_a}, \tag{5d}$$
$$\varphi_{u_k} \triangleq \begin{bmatrix} u_{k-1} \ldots u_{k-n_b} \end{bmatrix}^T \in \mathcal{R}^{n_b}. \tag{5e}$$

The noise contributions in the corresponding regressor vectors are denoted by a tilde, i.e. $\tilde{[\cdot]}$, whereas the noise-free signals are denoted by a zero subscript, i.e. $[\cdot]_0$. From (3) it follows that

$$\bar{\varphi}_k = \bar{\varphi}_{0_k} + \tilde{\bar{\varphi}}_k. \tag{6}$$

The notation Σ_{gd} is used as a general notion for the covariance matrix of the vectors g_k and d_k, whereas ξ_{gf} is utilised for a covariance vector with f_k being a scalar. The corresponding estimates are denoted by a hat. In addition, $0_{g\times d}$ denotes the null matrix of arbitrary dimension $g \times d$ and a single index is used in the case of a column vector as well as in the case of a square matrix, e.g. the identity matrix I_g. The autocorrelation elements, denoted $r_{\tilde{y}}(\cdot)$ are defined as

$$r_{\tilde{y}}(\tau) \triangleq E\left[\tilde{y}_k \tilde{y}_{k-\tau}\right], \tag{7}$$

where $E[\cdot]$ is the expected value operator. Introducing

$$\rho \triangleq \begin{bmatrix}\rho_y^T & \sigma_{\tilde{u}}\end{bmatrix}^T \in \mathcal{R}^{n_a+2}, \tag{8a}$$

$$\rho_y \triangleq \begin{bmatrix}r_{\tilde{y}}(0) & \ldots & r_{\tilde{y}}(n_a)\end{bmatrix}^T \in \mathcal{R}^{n_a+1}, \tag{8b}$$

the dynamic identification problem in the EIV framework considered here is formulated as:

Problem 1. *(Dynamic EIV identification problem) Given N samples of the measured signals, i.e.* $\{u_k\}_{k=1}^N$ *and* $\{y_k\}_{k=1}^N$*, determine the vector*

$$\Theta \triangleq \begin{bmatrix}\theta^T & \rho^T\end{bmatrix}^T \in \mathcal{R}^{n_\theta+n_a+2}. \tag{9}$$

3 REVIEW OF APPROACHES

This section briefly reviews the EBCLS technique and the offline EBPM algorithm.

3.1 Extended Bias Compensated Least Squares

Denoting an estimate by $\hat{[\cdot]}$, a solution of the system (1)-(3) in the LS sense is given by

$$\hat{\theta}_{\text{LS}} = \hat{\Sigma}_{x\varphi}^{\dagger}\hat{\xi}_{xy}, \tag{10}$$

where $[\cdot]^\dagger$ is the pseudo inverse operator defined by $A^\dagger \triangleq (A^TA)^{-1}A^T$, $x_k \in \mathcal{R}^{n_x}$ denotes an arbitrary instrumental vector with $n_x \geq n_\theta$. Due to the measurement noise, unless the elements of x_k are uncorrelated with $\tilde{\varphi}_k$, the solution obtained is biased. In order to achieve an unbiased estimate of θ, a bias compensation procedure is required to be carried out (Söderström, 2007). This consideration yields the EBCLS estimator defined as

$$\hat{\theta}_{\text{EBCLS}} \triangleq \left(\hat{\Sigma}_{x\varphi} - \Sigma_{\tilde{x}\tilde{\varphi}}\right)^{\dagger}\left(\hat{\xi}_{xy} - \xi_{\tilde{x}\tilde{y}}\right). \tag{11}$$

Note that $\Sigma_{\tilde{x}\tilde{\varphi}}$ and $\xi_{\tilde{x}\tilde{y}}$, in general, are functions of ρ, which, in turn, will depend on the elements contained in the instrument vector x_k.

3.2 Extended Bilinear Parametrisation Method

The bilinear parametrisation method is applicable for problems that are bilinear in the parameters, see (Ljung, 1999) for details, and it is presented here in accordance with the development proposed in (Larkowski et al., 2008).

Based on the EBCLS rule given by (11) a bilinear (in the parameters) cost function can be formulated, i.e.

$$\hat{\Theta} = \arg\min_{\Theta} V(\Theta), \tag{12}$$

where

$$V(\Theta) \triangleq \left\|\hat{\xi}_{xy} - \xi_{\tilde{x}\tilde{y}} - \left(\hat{\Sigma}_{x\varphi} - \Sigma_{\tilde{x}\tilde{\varphi}}\right)\theta\right\|_2^2. \tag{13}$$

Note that the instruments x_k must be chosen such that the resulting problem is soluble, i.e. the total number of unknowns is less than or equal to the total number of equations, see (Larkowski et al., 2008) for a detailed treatment. Alternatively, utilising the bilinearity property, (13) can be re-expressed as

$$V(\Theta) = \left\|\hat{\xi}_{xy} - \hat{\Sigma}_{x\varphi}\theta - W\rho\right\|_2^2, \tag{14}$$

where $W \triangleq S_1 - S_2(\theta) \in \mathcal{R}^{n_x \times (n_a+2)}$ such that $S_1\rho \triangleq \xi_{\tilde{x}\tilde{y}}$ and $S_2(\theta)\rho \triangleq \Sigma_{\tilde{x}\tilde{\varphi}}\theta$.

It is observed that for fixed ρ (i.e. the expressions $\Sigma_{\tilde{x}\tilde{\varphi}}$ and $\xi_{\tilde{x}\tilde{y}}$) the cost function (13) is linear in θ. Analogously, for fixed θ (i.e. the matrix W) the cost function (14) is linear in ρ. Consequently, a natural approach is to treat (13) and (14) as separate LS problems, cf. (Ljung, 1999). This leads to a two-step algorithm where the LS solutions of the sub-problems defined by (13) and (14) are obtained at each iteration. Furthermore, local convergence of such algorithm is guaranteed, see (Ljung, 1999).

4 RECURSIVE EXTENDED BILINEAR PARAMETRISATION METHOD

This section presents the proposed recursive realisation of the EBPM technique, denoted REBPM. First the problem of an online update of the parameter vector is addressed. Subsequently, two approaches for updating the pseudo-inverse of the extended observation matrix are considered. Finally, the problem of calculating the input noise variance and the autocorrelation elements of the output noise is discussed.

4.1 Recursive Update of Parameter Vector

Considering (11) and by making use of (10) it follows that

$$\theta = \hat{\theta}_{\text{LS}} + \hat{\Sigma}_{x\varphi}^{\dagger}\left(\Sigma_{\tilde{x}\tilde{\varphi}}\theta - \xi_{\tilde{x}\tilde{y}}\right). \tag{15}$$

It is remarked that the expression $\hat{\Sigma}^{\dagger}_{x\varphi}\left(\Sigma_{\tilde{x}\tilde{\varphi}}\theta - \xi_{\tilde{x}\tilde{y}}\right)$ represents the bias of the LS estimator. Since the true value of θ on the right hand side of (15) is unknown, a natural approach is to utilise the most recent estimate, i.e. the previous value. This leads to the following recursive EBCLS scheme

$$\hat{\theta}^k_{\mathrm{EBCLS}} = \hat{\theta}^k_{\mathrm{LS}} + \left(\hat{\Sigma}^k_{x\varphi}\right)^{\dagger}\left(\Sigma^k_{\tilde{x}\tilde{\varphi}}\hat{\theta}^{k-1}_{\mathrm{EBCLS}} - \xi^k_{\tilde{x}\tilde{y}}\right). \quad (16)$$

Despite that an inevitable error is introduced by assuming $\hat{\theta}^k_{\mathrm{EBCLS}} \approx \hat{\theta}^{k-1}_{\mathrm{EBCLS}}$, the above approach also known as the stationary iterative LS principle (Björck, 1996), has been successfully employed in several recursive as well as iterative algorithms.

4.2 Recursive Update of Pseudo-inverse

Considering equation (16), it is observed that a recursive update of the pseudo-inverse of $\hat{\Sigma}^k_{x\varphi}$ as well as of the LS estimate, i.e. $\hat{\theta}^k_{\mathrm{LS}}$, is required. This problem can be tackled by two approaches described below.

Approach based on the Matrix Pseudo-inverse Lemma - REBPM$_1$. The first, i.e. direct approach is to utilise an extension of the matrix inverse lemma, namely the matrix pseudo-inverse lemma, see (Feng et al., 2001). This allows the recursive computation of the expression $\hat{\Sigma}^{\dagger}_{x\varphi}$ as well as the corresponding $\hat{\theta}_{\mathrm{LS}}$. The algorithm can be summarised as follows:

$$\hat{\theta}^k_{\mathrm{LS}} = \hat{\theta}^{k-1}_{\mathrm{LS}} + L_k\left(y_k - \varphi^T_k\hat{\theta}^{k-1}_{\mathrm{LS}}\right), \quad (17a)$$

$$L_k = \frac{\left(\hat{\Sigma}^{k-1}_{x\varphi}\right)^{\dagger}x_k}{k-1+\varphi^T_k\left(\hat{\Sigma}^{k-1}_{x\varphi}\right)^{\dagger}x_k}, \quad (17b)$$

$$\left(\hat{\Sigma}^k_{x\varphi}\right)^{\dagger} = \frac{k}{k-1}\left[\left(\hat{\Sigma}^{k-1}_{x\varphi}\right)^{\dagger} - L_k\varphi^T_k\left(\hat{\Sigma}^{k-1}_{x\varphi}\right)^{\dagger}\right], \quad (17c)$$

$$\hat{\Sigma}^k_{x\varphi} = \hat{\Sigma}^{k-1}_{x\varphi} + \frac{1}{k}\left(x_k\varphi^T_k - \hat{\Sigma}^{k-1}_{x\varphi}\right), \quad (17d)$$

$$\hat{\xi}^k_{xy} = \hat{\xi}^{k-1}_{xy} + \frac{1}{k}\left(x_k y_k - \hat{\xi}^{k-1}_{xy}\right). \quad (17e)$$

The main shortcoming of the pseudo-inverse approach when dealing with practical applications results from its relatively high sensitivity with respect to the initialisation of the pseudo-inverse of the matrix $\hat{\Sigma}^k_{x\varphi}$. This issue is not trivial and can lead to a divergence of the overall algorithm. In order to appropriately initialise the expression $\left(\hat{\Sigma}^k_{x\varphi}\right)^{\dagger}$, it is required that the pseudo-inverse is computed offline after an arbitrary number, denoted α, of measurements is taken and before the recursive algorithm commences operation.

Remark 1. *It is noted that the uniqueness of* $\left(\hat{\Sigma}_{x\varphi}\right)^{\dagger}$ *in the case of recursive approaches is not always guaranteed when utilising equations* (17)*, see (Linden, 2008) for further details. As a consequence, the corresponding estimate of* θ^k_{LS} *may not represent the optimal, in terms of the minimum variance, solution to the overdetermined set of equations given by* (10)*.*

Approach based on Extended Instrumental Variables - REBPM$_2$. An alternative to employing the matrix pseudo-inverse lemma, an approach based on the recursive extended instrumental variables technique, see (Friedlander, 1984), can be utilised in order to obtain, albeit indirectly, a recursive update of $\left(\hat{\Sigma}^k_{x\varphi}\right)^{\dagger}$. Define

$$P_k = \left[\left(\hat{\Sigma}^k_{x\varphi}\right)^T\hat{\Sigma}^k_{x\varphi}\right]^{-1}. \quad (18)$$

In this approach the expression P_k is updated recursively, rather than the total pseudo-inverse $\left(\hat{\Sigma}^k_{x\varphi}\right)^{\dagger}$. The algorithm can be summarised as:

$$\hat{\theta}^k_{\mathrm{LS}} = \hat{\theta}^{k-1}_{\mathrm{LS}} + K_k\left(v_k - \phi^T_k\hat{\theta}^{k-1}_{\mathrm{LS}}\right), \quad (19a)$$

$$K_k = P_{k-1}\phi_k\left[\Lambda_k + \phi^T_k P_{k-1}\phi_k\right]^{-1}, \quad (19b)$$

$$\Lambda_k = \begin{bmatrix} -x^T_k x_k & 1 \\ 1 & 0 \end{bmatrix}, \quad (19c)$$

$$\phi_k = \begin{bmatrix} w_k & \frac{1}{k}\varphi_k \end{bmatrix}, \quad (19d)$$

$$w_k = \frac{k-1}{k}\left(\hat{\Sigma}^{k-1}_{x\varphi}\right)^T x_k, \quad (19e)$$

$$v_k = \frac{1}{k}\begin{bmatrix} (k-1)x^T_k\hat{\xi}^{k-1}_{xy} \\ y_k \end{bmatrix}, \quad (19f)$$

$$P_k = P_{k-1} - K_k\phi^T_k P_{k-1} \quad (19g)$$

with $\hat{\Sigma}^k_{x\varphi}$ and $\hat{\xi}^k_{xy}$ updated as in equations (17d) and (17e), respectively. Since it is the expression P_k which is obtained recursively, hence, in order to calculate $\left(\hat{\Sigma}^k_{x\varphi}\right)^{\dagger}$, for the recursive bias compensation equation (16), an additional matrix product has to be computed, i.e.

$$\left(\hat{\Sigma}^k_{x\varphi}\right)^{\dagger} = P_k\left(\hat{\Sigma}^k_{x\varphi}\right)^T. \quad (20)$$

Consequently, the pseudo-inverse of $\hat{\Sigma}^k_{x\varphi}$ is obtained in an indirect manner. Moreover, note that the recursive algorithm (19a) requires an inverse of the matrix of dimension 2×2 at each recursion. This, however, does not significantly increase the associated computational burden. On the other hand, the important advantages of this algorithm are that, firstly, it can be easily initialised and, secondly, it is relatively insensitive to the quality of the initial values. With reference to (Friedlander, 1984), in the case of no *a priori* information the initialisation can be performed as

$$\Sigma_{z\phi}^{0} = \mu \begin{bmatrix} I_{n_\theta} \\ 0_{(n_x - n_\theta)\times n_\theta} \end{bmatrix}, \; P_k^0 = \frac{1}{\mu^2} I_{n_\theta}, \; \theta_{\text{LS}}^0 = 0_{n_\theta \times 1}. \quad (21)$$

The scalar parameter μ allows the speed of convergence to be adjusted, hence affects the 'smoothness' of $\hat{\Theta}$ (i.e. large value of μ corresponds to the slow convergence and smooth parameters). Further algorithmic details ensuring that the update of the matrix P_k, given by (19g), is (semi-) positive definite are addressed in (Friedlander, 1984).

4.3 Determination of Noise Auto-correlation Elements

Since the matrix W^k is sparse, in general, the computational effort involved in its pseudo-inverse is negligible when compared to that of $\hat{\Sigma}_{x\varphi}^k$. Therefore, it is the pseudo-inverse of $\hat{\Sigma}_{x\varphi}^k$ which forms a crucial bottleneck of the overall algorithm. Consequently, a recursive computation of $\hat{\rho}^k$ is not considered here and its estimate is determined offline at each recursion by solving (14) in the LS sense, i.e.

$$\hat{\rho}^k = \left(W^k\right)^{\dagger} \left(\hat{\xi}_{xy}^k - \hat{\Sigma}_{x\varphi}^k \hat{\theta}_{\text{EBCLS}}^k\right). \quad (22)$$

5 SIMULATION STUDIES

This section addresses a numerical analysis of the two proposed recursive realisations of the EBPM approach, namely REBPM$_1$ and REBPM$_2$, when applied for the purpose of identifying a SISO discrete-time LTI second order system within the EIV framework. The system to be identified is described by

$$\theta = \begin{bmatrix} -1.5 & 0.7 & 1.0 & 0.5 \end{bmatrix}^T \quad (23)$$

with the input generated by

$$u_{0_k} = 0.5 u_{0_{k-1}} + \beta_k, \quad (24)$$

where β_k is a white, zero mean sequence of unity variance. The input noise sequence is zero mean, white of variance $\sigma_{\tilde{u}}$ and the coloured output noise sequence is generated by

$$\tilde{y}_k = 0.7\tilde{y}_{k-1} + \gamma_k, \quad (25)$$

where γ_k is zero mean, white and of variance σ_γ. In the case of both algorithms the instrumental vector is based on the instruments proposed in (Ekman et al., 2006), i.e. built from delayed inputs and delayed outputs, and utilised with $n_x = 10$.

Table 1: Results of the estimation of model parameters and auto-correlation elements of the noise sequences.

	true	EBPM	REBPM$_1$	REBPM$_2$
SNR $\approx$ 11dB				
a_1	-1.500	-1.501 ± 0.041	-1.504 ± 0.051	-1.494 ± 0.023
a_2	0.700	0.701 ± 0.045	0.705 ± 0.056	0.694 ± 0.024
b_1	1.000	0.998 ± 0.039	0.996 ± 0.045	1.001 ± 0.038
b_2	0.500	0.500 ± 0.072	0.495 ± 0.083	0.508 ± 0.051
$\sigma_{\tilde{u}}$	0.100	0.100 ± 0.054	0.095 ± 0.065	0.124 ± 0.052
$r_{\tilde{y}}(0)$	3.922	3.273 ± 2.349	2.647 ± 3.902	3.834 ± 1.376
$r_{\tilde{y}}(1)$	2.745	2.168 ± 1.938	1.631 ± 3.275	2.618 ± 1.174
$r_{\tilde{y}}(2)$	1.922	1.540 ± 0.949	1.250 ± 1.612	1.721 ± 0.715
e_1	–	0.001 ± 0.001	0.004 ± 0.005	0.001 ± 0.001
e_2	–	0.097 ± 0.120	1.187 ± 3.464	0.143 ± 0.197
Λ	–	0	2	0
T	–	–	1.381 ± 0.102	1.663 ± 0.134

The robustness of the two algorithms is examined via a Monte-Carlo simulation study comprising of 100 runs. The mean values of the estimates obtained at the last recursion, i.e. for $k = N$ are recorded and compared with the corresponding results produced by the offline EBPM. The the overall quality of the estimators is assessed via the following two performance criteria:

$$e_1 \triangleq \left\|\hat{\theta}_\lambda^{\text{N}} - \theta\right\|_2^2, \qquad e_2 \triangleq \left\|\hat{\rho}_\lambda^{\text{N}} - \rho\right\|_2^2, \quad (26)$$

where λ denotes the λ-th Monte-Carlo run. Prior to the calculation of the performance indeces e_1 and e_2 the possible outliers are removed from the data. An estimate is classified as an outlier if $\left\|\hat{\theta}_\lambda^{\text{N}}\right\|_2 > 10$. The number of outliers is denoted by Λ. Additionally, a computation time in seconds, denoted T, is recorded.

The initial values of the parameters are set as follows: $\alpha = 50$ for the REBPM$_1$ and $\mu = 100$ for the REBPM$_2$. In order to provide a fair comparison, in the case of the REBPM$_2$, the bias compensation phase is enabled from sample 50 onwards, although the expressions $\hat{\theta}_{\text{LS}}^k$ and $\left(\hat{\Sigma}_{x\varphi}^k\right)^{\dagger}$ are recursively calculated from the commencement of the algorithm. The values of the noise parameters are chosen as $\sigma_{\tilde{u}} = 0.1$ and $\sigma_\gamma = 2.0$. Consequently, the noise auto-correlation vector is given by

$$\rho = \begin{bmatrix} 3.922 & 2.745 & 1.922 & 0.100 \end{bmatrix}^T, \quad (27)$$

which yields an approximately equal signal-to-noise ratio (SNR) of around 11dB on both the input and the output signals. The results expressed obtained in terms of mean value $\pm$ standard deviation are presented in Table 1. It is observed that the mean values of the model parameters, obtained by the algorithms, see e_1, are relatively accurate and close to the true values and are also characterised by acceptable standard deviations. In the case of e_2 the estimates $\hat{\rho}$ are relatively less precise, especially those produced by the REBPM$_1$.

In general, comparison of the two recursive realisations of the EBPM reveals that it is the $REBPM_2$ which produces the more accurate results overall. Moreover, it is noted that in the case of the $REBPM_1$ the algorithm diverged twice, producing two outliers. In terms of the computational burden, the time required by the $REBPM_2$ is slightly greater when compared to that of $REBPM_1$, i.e. the former technique is faster by approximately 17% with respect to the latter method.

In general, the experiments carried out seem to suggest that the $REBPM_2$ is more advantageous than the $REBPM_1$ due to a simpler initialisation, greater robustness and an absence of convergence problems, at least under the conditions considered here.

6 CONCLUSIONS

A recursive realisation of the extended bilinear parametrisation method for the identification of dynamical linear discrete-time time-invariant single-input single-output errors-in-variables models has been proposed. Two alternative approaches for the online update of the pseudo-inverse of the extended observation covariance matrix have been considered. The first approach is based on the pseudo-inverse matrix lemma, whereas the second is constructed within the framework of the extended instrumental variables technique. For the cases considered, the two resulting algorithms appear to be relatively robust and they are also found to yield precise estimates of the model parameters. Results suggest that the instrumental variables based approach would appear to be the superior of the two developed algorithms.

REFERENCES

Björck, Å. (1996). *Numerical Methods for Least Squares Problems*. SIAM, Philadelphia.

Ekman, M. (2005a). Identification of linear systems with errors in variables using separable nonlinear least squares. In *Proc. of 16th IFAC World Congress*, Prague, Czech Republic.

Ekman, M. (2005b). *Modeling and Control of Bilinear Systems: Applications to the Activated Sludge Process*. PhD thesis, Uppsala University, Sweden.

Ekman, M., Hong, M., and Söderström, T. (2006). A separable nonlinear least-squares approach for identification of linear systems with errors in variables. In *14th IFAC Symp. on System Identification*, Newcastle, Australia.

Feng, D., Zhang, H., Zhang, X., and Bao, Z. (2001). An extended recursive least-squares algorithm. *Signal Proc.*, 81(5):1075–1081.

Friedlander, B. (1984). The overdetermined recursive instrumental variable method. *IEEE Trans. on Automatic Control*, 29(4):353–356.

Ikenoue, M., Kanae, S., Yang, Z., and Wada, K. (2008). Bias-compensation based method for errors-in-variables model identification. In *Proc. of 17th IFAC World Congress*, pages 1360–1365, Seul, South Korea.

Larkowski, T., Linden, J. G., Vinsonneau, B., and Burnham, K. J. (2008). Identification of errors-in-variables systems via extended compensated least squares for the case of coloured output noise. In *The 19th Int. Conf. on Systems Engineering*, pages 71–76, Las Vegas, USA.

Linden, J. G. (2008). *Algorithms for recursive Frisch scheme identification and errors-in-variables filtering*. PhD thesis, Coventry University, UK.

Ljung, L. (1999). *System Identification - Theory for the User*. Prentice Hall PTR, New Jersey, USA, 2nd edition.

Mahata, K. (2007). An improved bias-compensation approach for errors-in-variables model identification. *Automatica*, 43(8):1339–1354.

Markovsky, I. and Van Huffel, S. (2007). Overview of total least-squares methods. *Signal Proc.*, 87(10):2283–2302.

Söderström, T. (2007). Errors-in-variables methods in system identification. *Automatica*, 43(6):939–958.