AN APPROACH OF ROBUST QUADRATIC STABILIZATION OF NONLINEAR POLYNOMIAL SYSTEMS

Application to Turbine Governor Control

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Abstract: In this paper, robust quadratic stabilization of nonlinear polynomial systems within the frame work of Linear Matrix Inequalities (LMIs) is investigated. The studied systems are composed of a vectoriel polynomial function of state variable, perturbed by an additive nonlinearity which depends discontinuously on both time and state. Our main objective is to show, by employing the Lyapunov stability direct method and the Kronecker product properties, how a polynomial state feedback control law can be formulated to stabilize a nonlinear polynomial systems and, at the same time, maximize the bounds on the perturbation which the system can tolerate without going unstable. The efficiency of the proposed control strategy is illustrated on the Turbine - Governor system.

1 INTRODUCTION

The problem of robust quadratic stabilization for nonlinear uncertain systems has attracted a considerable attention and several methods have been proposed in the literature (Siljak, 1989) (Leitmann, 1993) (Kokotovic and Arcak, 1999). In some very interesting works, Lyapunov stability theory has been used to design control laws for systems with structured or unstructured parametric uncertainties and state perturbations.

The basic principle of quadratic stabilization is to find a feedback controller such that the closed-loop system is stable with a fixed Lyapunov function. This problem was initially proposed in (Barmish, 1985) to study the control of uncertain systems satisfying the so-called matching conditions. Since then, various results have been reported, including a necessary and sufficient condition given in (Barmish, 1985) and the Riccati equation method established in (Petersen and Hollot, 1986). Particularly, they consider the class of linear continuous systems subject to additive perturbations which are nonlinear and discontinuous functions in time and state of the system. The perturbations are uncertain and all we know about them is that they are contained within quadratic bounds.

Recently, the Linear Matrix Inequality (LMI) method has been widely used in quadratic stabilization since it can be solved efficiently using the interior-point method (Boyd et al., 1994). In this fact, the quadratic feedback stabilization of this type of systems using LMI approach has received a great deal of attention in the Siljak-Stipanovic'works (Siljak and Stipanovic, 2000) (Stipanovic and Siljak, 2001) (Siljak and Zecevic, 2005), in which sufficient conditions for quadratic stabilizability are developed. Latter, a new method which gives a less conservative result compared to that of (Siljak and Stipanovic, 2000), by using a descriptor model transformation of the considered system, where an improved sufficient condition for robust quadratic stabilization is given in terms of Linear Matrix Inequality (LMI) (Zuo and Wang, 2005). However, the proposed results remain restrictives to the systems represented by a linear nominal part and these conditions are rather difficult to check and, in general, a nonlinear control law is required.

The main contribution of the present paper consists in the replacement of the linear constant part by a nonlinear polynomial function based on the Kronecker power of the state vector (Rotella and Tanguy, 1988) (Benhadj Braiek et al., 1995) (Brewer, 1978), which has the advantage to approach any analytical

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nonlinear systems and general enough to model many physical systems (Benhadj Braiek and Rotella, 1992) (Benhadj Braiek and Rotella, 1994) (Benhadj Braiek et al., 1995) (Benhadj Braiek and Rotella, 1995) (Benhadj Braiek, 1995) (Bouzaouache and Benhadj Braiek, 2006). In another hand, we propose the use of the LMI approach in terms of minimization problems (Belkhiria Ayadi and Benhadj Braiek, 2005) , to derive a new sufficient LMI stabilization conditions, which resolution yields a stabilizing polynomial control law involving the quadratic stabilization of the polynomial closed-loop systems and the maximization of the nonlinearity bounds. Notice that, in recent years, various methods have been developed in field of system analysis and control amount to compute the controllers which enlarge the domain of attraction of equilibrium points of polynomial systems through LMI approach (Chesi et al., 1999). Mainly based on LMI relaxation for solving polynomial optimizations (Chesi et al., 2003) (Chesi, 2004), these methods proposes a convex optimization solutions with LMI constraints for a chosen Lyapunov functions (Chesi, 2009).

An additional contribution of this paper is to apply the versatile tools of LMI for the design of robust feedback Turbine-Governor control (Anderson and Fouad, 1977) (Elloumi, 2005). Our primary reason for selecting this type of control is the underlying system model, which can be bounded in a way that conforms to quadratic bounds of nonlinearity. The proposed method has an advantage such as the control design of our power system is formulated as a convex optimization problem, which ensures computational simplicity, and guarantees the existence of a solution. The optimal gains matrices are obtained directly, with no need for tuning parameters or trial and error procedures.

This paper is organized as follows: section 2 is devoted to introduce the description of the nonlinear studied systems and problem formulation. In the third section, the LMI sufficient condition for robust quadratic stabilization of polynomial systems is proposed. Applications to power systems are then considered in section 4. Finally some concluding remarks are given in the last section.

2 DESCRIPTION OF THE STUDIED SYSTEMS AND PROBLEM FORMULATION

In the present paper, we focus on analytical nonlinear polynomial control-affine systems under nonlinear perturbations described by the following state space equation:

$$\begin{aligned} \dot{X} &= f(X(t)) + GU(t) + h(X(t),t) \\ &= \sum_{i=1}^{r} F_i X^{[i]} + GU(t) + h(X(t),t), \end{aligned} \tag{1}$$

where:
$\forall t \in \mathbb{R}^+$; $X \in \mathbb{R}^n$ is the state vector, $U(t) \in \mathbb{R}^m$ is the input vector.
For $i = 1, ..., r$, $X^{[i]} \in \mathbb{R}^{n^i}$ is the *i-th* Kronecker power of the vector X and $F_i \in \mathbb{R}^{n \times n^i}$ are constant matrices. G is a constant $(n \times m)$ matrix and the polynomial degree r is considered odd: $r = 2s-1$, with $s \in \mathbb{N}^*$.
$h : \mathbb{R}^{n+1} \to \mathbb{R}^n$ is the nonlinear perturbations. The crucial assumption about nonlinear function $h(t,X(t))$ is that it is uncertain and all we know is that, in the domains of continuity, it satisfies the quadratic inequality:

$$h^T(t,X)h(t,X) \leq \alpha^2 X^T H^T H X, \tag{2}$$

where $\alpha > 0$ is the bounding parameter and H is a constant matrix. For simplicity, we use $h(t,X)$ instead of $h(t,X(t))$.
A large amount of works have been developed in the robust quadratic stabilization area, considering a particular class of nonlinear systems, where the nonlinearities are totally in the perturbation terms (Stipanovic and Siljak, 2001) (Siljak and Zecevic, 2005) (Zuo and Wang, 2005). The basic idea addressed in this paper is the consideration of a nonlinear uncertain systems described by a polynomial part with nonlinear perturbations which present the uncertainties (Mtar et al., 2007). The present work is an attempt towards expanding the robust quadratic stabilization approach of the considered nonlinear systems in the literature to polynomial ones. The aim of the proposed approach is to guarantees on the one hand, the stabilization of the linear part of the polynomial system, and on the other hand to weaken the perturbation which provide the maximization of the domain of uncertainties.

3 ROBUST STABILIZING CONTROL SYNTHESIS USING THE LMI APPROACH

When the linear part of the perturbed polynomial system (1) (defined by F_1) is not stable, we can introduce a nonlinear feedback to stabilize the overall system and, at the same time, maximize its tolerance to uncertain nonlinear perturbations. The considered polynomial control law is described by the following equation:

$$U = k(X) = \sum_{i=1}^{r} K_i X^{[i]}, \tag{3}$$

where $K_{i,i=1,\ldots,r}$ are constant gains matrices, which stabilizes asymptotically and globally the equilibrium $(X = 0)$ of the considered system.
When we apply the feedback (3) to the open-loop system (1), we obtain the closed-loop system:

$$\begin{aligned} \dot{X} &= (f + Gk)(X) + h(t,X) \\ &= a(X) + h(t,X) \\ &= \sum_{i=1}^{r} A_i X^{[i]} + h(t,X) \end{aligned} \tag{4}$$

where:

$$A_i = F_i + GK_i \tag{5}$$

is the closed-loop system matrix.
We define the following set:

$$\mathcal{S}(h,H,\alpha) = \{h : h^T(t,X)h(t,X) \le \alpha^2 X^T H^T HX\}. \tag{6}$$

For any given matrix H, our purpose is to establish robust quadratic stabilization of the system (4-5) and meanwhile make the set $\mathcal{S}(h,H,\alpha)$ as large as possible.

Definition 1. *The system (1) is robustly stabilized by the control law (3) if the closed-loop system (4) is robustly stable with degree α for all $h(t,X)$ satisfying constraint (2).*

Using the quadratic Lyapunov function:

$$V(X) = X^T PX, \tag{7}$$

which is positive definite when P is a symmetric positive definite $(n \times n)$ matrix and computing the derivative $\dot{V}(X)$ along the trajectory of the system (4), lead to the sufficient condition of the global asymptotic stabilization of the perturbed polynomial system. Useful mathematical transformations have allowed the formulation of the obtained condition as an LMI optimization problem according to the polynomial system parameters, given by the following Theorem 1:

Theorem 1. *The system (4) is robustly stabilized by control law (3) if the following optimization problem is feasible:*

$$\textit{minimize} \quad \eta = \frac{1}{\alpha^2}$$

subject to $\mathcal{D}_S(P) > 0,\ \gamma > 0$

$$\begin{bmatrix} \Pi(P) & \star & \star & \star & \star \\ \Lambda\mathcal{D}_S(P)\tau & -I & 0 & 0 & 0 \\ \mathcal{D}_S(P)\tau & 0 & -\frac{1}{\gamma}I & 0 & 0 \\ \mathcal{G}\tilde{\mathcal{M}}(k)\tau & 0 & 0 & -\frac{1}{\gamma}I & 0 \\ H\Lambda\tau & 0 & 0 & 0 & -\eta I \end{bmatrix} < 0 \tag{8}$$

where: $\eta = \frac{1}{\alpha^2}$ *and* $\tilde{\mathcal{M}}(k) = \gamma^{-1}\mathcal{M}(k)$.

$\star$: *denotes the elements below the main diagonal of a symmetric block matrix.*

The relative notations of the Theorem 1 are mentioned in Appendix A.3.
To prove the Theorem 1, we need the two following lemmas:

Lemma 1. *(Yakubovich, 1977)*
Let $\Omega_0(x)$ and $\Omega_1(x)$ be two arbitrary quadratic forms over $\mathbb{R}^n$, then $\Omega_0(x) < 0$ for all $x \in \mathbb{R}^n - \{0\}$ satisfying $\Omega_1(x) \le 0$ if and only if there exist $\sigma \ge 0$ such that:

$$\Omega_0(x) - \sigma\Omega_1(x) < 0, \quad \forall x \in \mathbb{R}^n - \{0\} \tag{9}$$

Lemma 2. *(Zhou and Khargonedkar, 1988)*
For any matrices A and B with appropriate dimensions and for any positive scalar $\gamma > 0$, one has:

$$A^T B + B^T A \le \gamma A^T A + \gamma^{-1} B^T B \tag{10}$$

Proof of Theorem 1:
Let us consider the quadratic Lyapunov function (7) and differentiating along trajectory of the system (4), we have:

$$\begin{aligned} \dot{V}(X) &= \dot{X}^T PX + X^T P\dot{X} \\ &= X^T P(\sum_{i=1}^{r} A_i X^{[i]} + h(t,X)) \\ &+ (\sum_{i=1}^{r} A_i X^{[i]} + h(t,X))^T PX \\ &= \sum_{i=1}^{r} (X^T PA_i X^{[i]} + X^{[i]T} A_i^T PX) \\ &+ h^T PX + X^T Ph \\ &= 2\sum_{i=1}^{r} X^T PA_i X^{[i]} + h^T PX + X^T Ph. \end{aligned} \tag{11}$$

Using the rule of the *vec-function* (see Appendix A.1), one obtains:

$$\dot{V}(X) = 2\sum_{i=1}^{r} \Psi_i^T X^{[i+1]} + h^T PX + X^T Ph, \tag{12}$$

where:

$$\Psi_i = vec(PA_i). \tag{13}$$

Then, we have:

$$\begin{aligned} \dot{V}(X) &= 2X^T \mathcal{D}_S(P)\mathcal{M}(a)X + h^T PX + X^T Ph \\ &= X^T[\mathcal{D}_S(P)\mathcal{M}(a) + \mathcal{M}(a)^T \mathcal{D}_S(P)]X \\ &+ h^T PX + X^T Ph, \end{aligned} \tag{14}$$

where $\mathcal{D}_S(P)$ and $\mathcal{M}(a)$ are defined in Appendix, and $\mathcal{X}$ is expressed by the following equation:

$$\mathcal{X} = \begin{bmatrix} X^T & X^{[2]^T} & \dots & X^{[s]^T} \end{bmatrix}^T \quad (15)$$

For more details, the transitions between the inequalities (12) to (14) are detailed in (Benhadj Braiek, 1996) (Belhaouane et al., 2008).
When considering the nun-redundant form, the vector $\mathcal{X}$ can be written as:

$$\mathcal{X} = \tau\tilde{\mathcal{X}}, \quad (16)$$

where τ is mentioned in Appendix A.3.
Consequently, $\dot{V}(X)$ can be expressed as:

$$\begin{array}{rcl} \dot{V}(X) & = & \widetilde{\mathcal{X}}^T\tau^T(\mathcal{D}_S(P)\mathcal{M}(a)+\mathcal{M}(a)^T\mathcal{D}_S(P))\tau\widetilde{\mathcal{X}} \\ & + & h^TPX+X^TPh \end{array} \quad (17)$$

Which can be written as:

$$\begin{array}{rcl} \dot{V}(X) & = & \widetilde{\mathcal{X}}^T\tau^T(\mathcal{D}_S(P)\mathcal{M}(a)+\mathcal{M}(a)^T\mathcal{D}_S(P))\tau\widetilde{\mathcal{X}} \\ & + & h^T\Lambda\mathcal{D}_S(P)\tau\widetilde{\mathcal{X}}+\widetilde{\mathcal{X}}^T\tau^T\mathcal{D}_S(P)\Lambda h, \end{array} \quad (18)$$

with Λ is mentioned in Appendix A.3.
A sufficient condition of the global quadratic stabilization of the equilibrium ($X = 0$) is that (18) is negative definite. Considering the obtained result, we can derive LMI sufficient conditions for global asymptotic stabilization of the studied system by employing some LMI techniques given by the following development:
Using the S-procedure method, presented by the Lemma1, the inequality (18) with the constraint $h^Th-\alpha^2\widetilde{\mathcal{X}}^T(H\Lambda\tau)^T(H\Lambda\tau)\widetilde{\mathcal{X}} \leq 0$ derived from (2), is equivalent to the existence of a $\mathcal{D}_S(P)$ matrix and a scalar $\varepsilon \geq 0$ such that:

$$\begin{array}{l} \widetilde{\mathcal{X}}^T\tau^T(\mathcal{D}_S(P)\mathcal{M}(a)+\mathcal{M}(a)^T\mathcal{D}_S(P))\tau\widetilde{\mathcal{X}}+h^T\Lambda\mathcal{D}_S(P)\tau\widetilde{\mathcal{X}} \\ +\widetilde{\mathcal{X}}^T\tau^T\mathcal{D}_S(P)\Lambda h-\varepsilon[h^Th-\alpha^2\widetilde{\mathcal{X}}^T(H\Lambda\tau)^T(H\Lambda\tau)\widetilde{\mathcal{X}}] < 0 \end{array}$$

which can be written as:

$$\begin{bmatrix} \widetilde{\mathcal{X}}^T \\ h^T \end{bmatrix}^T \begin{bmatrix} Q_{11}+\varepsilon\alpha^2(H\Lambda\tau)^T(H\Lambda\tau) & \star \\ \Lambda\mathcal{D}_S(P)\tau & -\varepsilon I \end{bmatrix} \begin{bmatrix} \widetilde{\mathcal{X}} \\ h \end{bmatrix} < 0 \quad (19)$$

where:
$Q_{11} = \tau^T(\mathcal{D}_S(P)\mathcal{M}(a)+\mathcal{M}(a)^T\mathcal{D}_S(P))\tau$. It should be noted that inequality (19) is a non-strict LMI since $\varepsilon \geq 0$. For the minimization problem, it is well-known in (Boyd et al., 1994), that the minimization result under non-strict LMI constraints is equivalent to that under strict LMI constraints. Thus we can substitute $\varepsilon > 0$ by $\varepsilon \geq 0$. Then, the inequality (19) is further equivalent to the existence of a matrix $\hat{\mathcal{D}}_S(P)$ so that:

$$\hat{\mathcal{D}}_S(P) > 0$$

$$\begin{bmatrix} \hat{Q}_{11}+\alpha^2(H\Lambda\tau)^T(H\Lambda\tau) & \star \\ \Lambda\hat{\mathcal{D}}_S(P)\tau & -I \end{bmatrix} < 0 \quad (20)$$

where $\hat{P} = \varepsilon^{-1}P$ and $\hat{\mathcal{D}}_S(P) = \varepsilon^{-1}D_S(P)$.
In what follows, the matrix $\hat{\mathcal{D}}_S(P)$ is replaced by $\mathcal{D}_S(P)$, to relieve the writing.
According to the following relation (see the lemma given in (Benhadj Braiek et al., 1995)):

$$\mathcal{M}(a) = \mathcal{M}(f+Gk) = \mathcal{M}(f)+\mathcal{G}\mathcal{M}(k), \quad (21)$$

we can write:

$$\begin{bmatrix} S_{11}+R_{11} & \star \\ \Lambda\mathcal{D}_S(P)\tau & -I \end{bmatrix} < 0 \quad (22)$$

where:
$S_{11} = \Pi(P)+\alpha^2(H\Lambda\tau)^T(H\Lambda\tau)$,
$R_{11} = \tau^T(\mathcal{D}_S(P)\mathcal{G}\mathcal{M}(k)+(D_S(P)\mathcal{G}\mathcal{M}(k))^T)\tau$
and $\Pi(P) = \tau^T(\mathcal{D}_S(P)\mathcal{M}(f)+\mathcal{M}(f)^T\mathcal{D}_S(P))\tau$.
$\mathcal{G}$, $\mathcal{D}_S(P)$, $\mathcal{M}(f)$ and $\mathcal{M}(k)$ are mentioned in Appendix A.3.
Using the well known matrix inequality given by Lemma 2, it follows that (22) holds if there exist a constant symmetric matrix $\mathcal{D}_S(P) > 0$ and a positive scalar $\gamma > 0$, such that:

$$\begin{bmatrix} S_{11}+R'_{11} & \star \\ \Lambda\mathcal{D}_S(P)\tau & -I \end{bmatrix} < 0 \quad (23)$$

where:

$$R'_{11} = \gamma\tau^T\mathcal{D}_S(P)^T\mathcal{D}_S(P)\tau+\gamma^{-1}\tau^T\mathcal{M}(k)^T\mathcal{G}^T\mathcal{G}\mathcal{M}(k)\tau_1.$$

Relying on the generalized Schur Complement, equation (23) can be rewritten as:

$$\begin{bmatrix} \Pi(P)+\alpha^2(H\Lambda\tau)^T(H\Lambda\tau) & \star & \star & \star \\ \Lambda\mathcal{D}_S(P)\tau & -I & 0 & 0 \\ \mathcal{D}_S(P)\tau & 0 & -\frac{1}{\gamma}I & 0 \\ \mathcal{G}\mathcal{M}(k)\tau & 0 & 0 & -\gamma I \end{bmatrix} < 0 \quad (24)$$

Pre-multiplying and post-multiplying:

$$\Phi = diag(I,I,I,\gamma^{-1}I)$$

for both sides of (24), we have:

$$\begin{bmatrix} \Pi(P)+\alpha^2(H\Lambda\tau)^T(H\Lambda\tau) & \star & \star & \star \\ \Lambda\mathcal{D}_S(P)\tau & -I & 0 & 0 \\ \mathcal{D}_S(P)\tau & 0 & -\frac{1}{\gamma}I & 0 \\ \mathcal{G}\tilde{\mathcal{M}}(k)\tau & 0 & 0 & -\frac{1}{\gamma}I \end{bmatrix} < 0 \quad (25)$$

where $\tilde{\mathcal{M}}(k) = \gamma^{-1}\mathcal{M}(k)$ and $\tilde{K}_i = \gamma^{-1}K_i$.
Finally, by using the Schur complement, we get:

$$\begin{bmatrix} \Pi(P) & \star & \star & \star & \star \\ \Lambda\mathcal{D}_S(P)\tau & -I & 0 & 0 & 0 \\ \mathcal{D}_S(P)\tau & 0 & -\frac{1}{\gamma}I & 0 & 0 \\ \mathcal{G}\tilde{\mathcal{M}}(k)\tau & 0 & 0 & -\frac{1}{\gamma}I & 0 \\ H\Lambda\tau & 0 & 0 & 0 & -\eta I \end{bmatrix} < 0 \quad (26)$$

where $\eta = \frac{1}{\alpha^2}$.
To establish robust quadratic stabilization in the sense

of Definition 1 of the system (4) under the constraint (2) with maximal α, it comes the following optimization problem:

$$\left\{ \begin{array}{l} minimize \quad \eta = \frac{1}{\alpha^2} \\ s.t. \quad (26) \\ \gamma > 0, \mathcal{D}_S(P) > 0, P > 0 \end{array} \right. \quad (27)$$

translated by the Theorem1, which ends the proof.

4 APPLICATION TO POWER SYSTEM CONTROL

To illustrate how the proposed LMI approach can be applied for the robust feedback control of power systems, we will consider a electrical mono-machine system with steam valve control (see Figure 1). The parameters of the considered system are given by the following list of symbols:
δ : rotor angle for machine, in radian;
ω : relative speed for machine, in radian/s;
P_m : mechanical power for machine, in pu;
P_c : power control input of machine, in pu;
X_e : steam valve opening for machine, in pu;
H : inertia constant for machine, in second;
D : damping coefficient for machine, in pu;
T_m : time constant of machine's turbine of machine, in second;
T_e : time constant of machine's speed governor, in second;
K_m : gain of machine turbine;
R : regulation constant of machine, in pu;
E : internal transient voltage for machine, in pu;
B : nodal susceptance for machine, in pu;
ω_0 : the synchronous machine speed, in radian/s;
δ_0, P_{m_0} and X_{e_0} are the initial values of $\delta(t)$, $P_m(t)$ and $X_e(t)$ respectively.

The generator dynamics are described as (Sauer and Pai, 1998) (Siljak and Zecevic, 2002):

$$\begin{array}{rcl} \dot{\delta} & = & \omega \\ \dot{\omega} & = & -\frac{D}{2H}\omega + \frac{\omega_0}{2H}(P_m - EVBsin\delta). \end{array} \quad (28)$$

The equation linking the mechanical power P_m to the steam valve opening of turbine X_e for synchronous machine is:

$$\dot{P}_m = -\frac{1}{T_m}P_m + \frac{K_m}{T_m}X_e. \quad (29)$$

The mechanical-hydraulic speed governor can be represented as first order system:

$$\dot{X}_e = -\frac{K_e}{T_e R \omega_0}\omega - \frac{1}{T_e}X_e + \frac{1}{T_e}P_c, \quad (30)$$

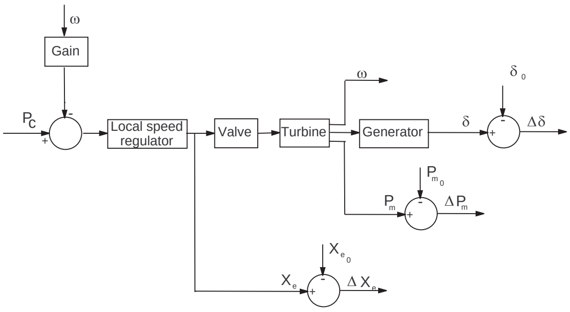

Figure 1: Diagram Bloc representation of Electrical Mono-machine System.

where the term P_c represents the control input. Defining new states:

$$\left[\begin{array}{cccc} \Delta\delta(t) & \omega(t) & \Delta P_m(t) & \Delta X_e(t) \end{array} \right]^T \quad (31)$$

as deviations from the equilibrium,
where:

$$\Delta\delta(t) = \delta(t) - \delta_0 \quad ; \quad \Delta P_m(t) = P_m(t) - P_{m_0}$$
$$\Delta X_e(t) = X_e(t) - X_{e_0}.$$

Then, we obtain the modified system given by the following state space equation:

$$\dot{X}(t) = AX(t) + B_0 U + h(t,X), \quad (32)$$

where:

$$A = \left[\begin{array}{cccc} 0 & 1 & 0 & 0 \\ 0 & \frac{-D}{2H} & \frac{\omega_0}{2H} & 0 \\ 0 & 0 & -\frac{1}{T_m} & \frac{K_m}{T_m} \\ 0 & -\frac{K_e}{T_e R \omega_0} & 0 & -\frac{1}{T_e} \end{array} \right]$$

$B_0 = (0 \quad 0 \quad 0 \quad \frac{1}{T_e})^T$ and

$$h(t,X) = \left[\begin{array}{c} 0 \\ -\omega_0 EVB/2H \\ 0 \\ 0 \end{array} \right] g(X(t)),$$

with $g(X(t)) = sin\delta(t) - sin\delta_0$, represents the nonlinearity of system (32).
The nonlinear system (32) can be developed into a polynomial form by a Taylor series expansions, then we obtain the new state space representation given by the following form:

$$\dot{X} = F_1 X + F_2 X^{[2]} + F_3 X^{[3]} + GU(t) + \overline{h}(t,X), \quad (33)$$

where:

$$F_1 = \left[\begin{array}{cccc} 0 & 1 & 0 & 0 \\ -\omega_0 EVB/2H & \frac{-D}{2H} & \frac{\omega_0}{2H} & 0 \\ 0 & 0 & -\frac{1}{T_m} & \frac{K_m}{T_m} \\ 0 & -\frac{K_e}{T_e R \omega_0} & 0 & -\frac{1}{T_e} \end{array} \right]$$

$$F_2 = \mathbf{0}_{4\times16},\ F_3(2,1) = \frac{\omega_0 EVB}{12H}.$$

For the others values:

$$F_3(i,j) = 0\ \forall i,j : (i = 1,...4; j = 1,...64)$$

$G = B_0$ and

$$\bar{h}(t,X) = \begin{bmatrix} 0 \\ -\omega_0 EVB/2H \\ 0 \\ 0 \end{bmatrix} \bar{g}(X(t)), \qquad (34)$$

where:

$$\bar{g}(X(t)) = sin\delta(t) - \delta(t) + \frac{\delta(t)^3}{6}. \qquad (35)$$

According the values of the machine parameters (Sauer and Pai, 1998) indicated in Table 1, the evolution of the state variables of system (33) is shown in the Figure 2. Since the nonlinearity (34) of system (33) satisfy the quadratic inequality (2) with $H = I$, the LMI robust stabilizing control given by Theorem 1 can be applied to considered power system in order to maximize the domain of nonlinearity while ensuring the stability of the overall system.

Table 1: Machine system parameters.

Symbols of parameters	Values
$H(s)$	5.1
$D(pu)$	3
$T_m(s)$	0.35
$T_e(s)$	0.1
R	0.05
K_m	1
K_e	1
$\omega_0(rad/s)$	314.159

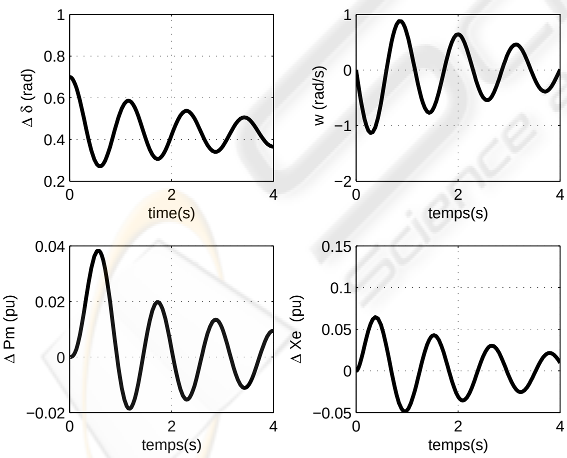

Figure 2: Evolution of state variables towards an perturbation on the variable δ.

The solution of problem (27) yields to the uncertainty bound $\alpha_{max} = 0.4834$ and

$$P = \begin{bmatrix} 10.0734 & 5.3037 & -3.2516 & 1.8013 \\ 5.3037 & 6.0628 & -4.6859 & 1.2175 \\ -3.2516 & -4.6859 & 9.7071 & -2.1715 \\ 1.8013 & 1.2175 & -2.1715 & 12.3111 \end{bmatrix}$$

The control law gain matrices, extracted from $\mathcal{M}(k)$, are given by:

$$K_1 = \begin{bmatrix} -86.592 & -87.171 & -90.911 & -89.173 \end{bmatrix}^T$$

- For $i = 1,...,16$:

$$K_2(1) = -216.909, K_2(2) = K_2(5) = -40.431$$
$$K_2(3) = K_2(4) = K_2(7) = K_2(8) = K_2(9) = -49.443$$
$$K_2(10) = K_2(12) = K_2(13) = K_2(14) = K_2(15) = -49.443$$
$$K_2(11) = K_2(16) = -96.292.$$

- For $i = 1,...,64$:

$$K_3(1) = K_3(17) = K_3(33) = K_3(49) = -216.909$$
$$K_3(2) = K_3(5) = K_3(18) = K_3(21) = -40.431$$
$$K_3(34) = K_3(37) = K_3(50) = K_3(53) = -40.431$$
$$K_3(3) = K_3(4) = K_3(7) = K_3(8) = -49.443$$
$$K_3(9) = K_3(10) = K_3(12) = K_3(13) = K_3(14) = -49.443$$
$$K_3(15) = K_3(19) = K_3(20) = K_3(23) = K_3(24) = -49.443$$
$$K_3(25) = K_3(26) = K_3(28) = K_3(29) = K_3(30) = -49.443$$
$$K_3(31) = K_3(35) = K_3(36) = K_3(39) = K_3(40) = -49.443$$
$$K_3(41) = -49.443 = K_3(42) = K_3(44) = K_3(45) = -49.443$$
$$K_3(46) = K_3(47) = K_3(51) = K_3(52) = K_3(55) = -49.443$$
$$K_3(56) = K_3(57) = K_3(58) = K_3(60) = K_3(61) = -49.443$$
$$K_3(62) = K_3(63) = -49.443$$
$$K_3(6) = K_3(11) = K_3(16) = K_3(22) = K_3(27) = -96.292$$
$$K_3(32) = K_3(38) = K_3(43) = K_3(54) = K_3(59) = -96.292$$
$$K_3(64) = -96.292$$

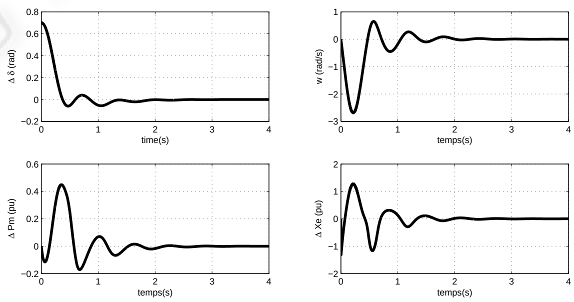

Figure 3: Closed-loop responses of the power system with polynomial control.

From the simulation results shown in Figure 3, it is obvious that the results confirm the validity of the proposed method and the uncertainty bound found by the LMI procedure (27), dominates the maximum of the perturbation function (35) of the system (33). The polynomial robust control can rapidly damp the oscillations of the studied system and greatly enhance transient stability of the mono-machine power system. Besides, the polynomial control is more reassuring in the case of a more aggressive perturbation.

5 CONCLUSIONS

A sufficient LMI condition for robust quadratic stabilization of polynomial systems under nonlinear perturbations has been proposed in this work. This new feedback stabilizing approach is based on the direct Lyapunov method and elaborated algebraic developments using the Kronecker product properties. These developments have been turned into an LMI minimization problem, which can be easily solved by means of numerically efficient convex programming algorithms. A mono-machine power system is considered as an application example of the technique developed in this paper. The numerical simulation results have confirmed the efficiency of the proposed polynomial controller which can rapidly damp the system oscillations and greatly enhance the transient stability of the considered mono-machine power system despite the nonlinear uncertainty affecting the studied system.

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APPENDIX

The dimensions of the matrices used in this section are the following: $A(p\times q), C(q\times f), E(n\times p)$

A.1- *vec(.)* function:

An important vector valued function of matrix denoted $vec(.)$ was defined in (Brewer, 1978) as follows:

$$A = \left[\begin{array}{cccc} A_1 & A_2 & \dots & A_q \end{array}\right] \in \mathbb{R}^{p\times q},$$

where

$$\forall i \in \{1,...,q\}, \; A_i \in \mathbb{R}^p$$

$$vec(A) = [A_1 \;\; A_2 \;\; \dots \;\; A_q]^T \in \mathbb{R}^{pq}.$$

We recall the following useful rule (Brewer, 1978) of *vec-function*:

$$vec(EAC) = (C^T \otimes E)vec(A)$$

A.2- *mat*(.) function:

A special function $mat_{(n,m)}(.)$ can be defined as follows:

If V is a vector of dimension $p = n.m$ then $M = mat_{(n,m)}(V)$ is the $(n\times m)$ matrix verifying $V = vec(M)$.

A.3- Notations related to Theorem 1:

(i).

$$\tau = \begin{bmatrix} T_1 & & & & \\ & T_2 & & 0 & \\ & & T_3 & & \\ & 0 & & \ddots & \\ & & & & T_s \end{bmatrix}$$

where:

$$\forall i \in \mathbb{N};\; \exists!\; T_i \in \mathbb{R}^{n^i \times n_i},\; such\ as\;\; X^{[i]} = T_i \tilde{X}^{[i]},$$

with:

$\tilde{X}^{[i]}$ represents the nun-redundant i-power of X (Benhadj Braiek et al., 1995).

(ii).

$$\Lambda = \left[\begin{array}{cccc} I_n & 0_{n\times n^2} & \cdots & 0_{n\times n^s} \end{array}\right],$$

verifying $X = \tilde{X} = \tilde{\Lambda}\tilde{\mathcal{X}},\;\; \Lambda^T P = \mathcal{D}_s(P)\Lambda^T$ where:

$$\tilde{\Lambda} = \Lambda\tau.$$

(iii).

$$\mathcal{D}_s(P) = \begin{bmatrix} P & & & 0 \\ & P\otimes I_n & & \\ & & \ddots & \\ 0 & & & P\otimes I_{n^{s-1}} \end{bmatrix}$$

(iv).

$$\mathcal{G} = \begin{bmatrix} G & & & 0 \\ & G\otimes I_n & & \\ & & \ddots & \\ 0 & & & G\otimes I_{n^{s-1}} \end{bmatrix}$$

(v).

$$\Pi(P) = \mathcal{D}_S(P)\mathcal{M}(f) + \mathcal{M}(f)^T\mathcal{D}_S(P),$$

where for a polynomial vectorial function:

$$z(X) = \sum_{i=1}^{r} Z_i X^{[i]},$$

with $X \in \mathbb{R}^n$ and Z_i are $(n\times n^i)$ constant matrices. We define the $(\upsilon\times\upsilon)$ matrix $\mathcal{M}(z)$ by:

$$\mathcal{M}(z) = \begin{bmatrix} M_{11}(Z_1) & M_{12}(Z_2) & 0 & \dots & 0 \\ 0 & M_{22}(Z_3) & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & 0 \\ \vdots & & \ddots & M_{s-1,s-1}(Z_{2s-3}) & M_{s-1,s}(Z_{2s-2}) \\ 0 & \dots & \dots & 0 & M_{s,s}(Z_{2s-1}) \end{bmatrix},$$

where $\upsilon = n + n^2 + ... + n^s$ and:

- For $j = 1,...,s$

$$M_{j,j}(Z_{2j-1}) = \begin{bmatrix} mat_{(n^{j-1},n^j)}\left(Z_{2j-1}^{1T}\right) \\ mat_{(n^{j-1},n^j)}\left(Z_{2j-1}^{2T}\right) \\ \vdots \\ mat_{(n^{j-1},n^j)}\left(Z_{2j-1}^{nT}\right) \end{bmatrix}$$

- For $j = 1,...,s-1$

$$M_{j,j+1}(Z_{2j}) = \begin{bmatrix} mat_{(n^{j-1},n^j)}\left(Z_{2j}^{1T}\right) \\ mat_{(n^{j-1},n^j)}\left(Z_{2j}^{2T}\right) \\ \vdots \\ mat_{(n^{j-1},n^j)}\left(Z_{2j}^{nT}\right) \end{bmatrix}$$

where Z_k^i is the i^{th} row of the matrix Z_k.