SYNCHRONIZATION OF MODIFIED CHUA'S CIRCUITS IN STAR COUPLED NETWORKS

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Abstract: In this paper, we use Generalized Hamiltonian systems approach to synchronize dynamical networks of modified fourth-order Chua's circuits, which generate hyperchaotic dynamics. Network synchronization is obtained among a single master node and two slave nodes, with the slave nodes being given by observers.

1 INTRODUCTION

The synchronization problem of two chaotic oscillators has received a lot of attention in last decades, see e.g. this example in order to achieve the highest quality possible (Pecora and Carroll, 1990); (Nijmeijer and Mareels, 1997); (López-Mancilla and Cruz-Hernández, 2005); (López-Mancilla and Cruz-Hernández, 2008); (Cruz-Hernández and Nijmeijer, 2000); (Boccaleti and et. al., 2002); (Luo, 2008); (Cruz-Hernández, 2004) and references therein. This interest increases by practical applications in different fields, particularly in secure communications, see e.g. (Cruz-Hernández, 2004); (López-Mancilla and Cruz-Hernández, 2005); (Aguilar-Bustos and Cruz-Hernández, 2008); (Cruz-Hernández and N.Romero-Haros, 2008). Hyperchaotic dynamics characterized by more than one positive Lyapunov exponent are advantageous over simple chaotic dynamics. However, hyperchaos synchronization is a much more difficult problem, see e.g. (Aguilar-Bustos and Cruz-Hernández, 2008) for two coupled oscillators.

In (Posadas-Castillo and et.al.(a), 2007) was developed an experimental study on practical realization to synchronize dynamical networks of Chua's circuits globally coupled. While in recent works (Posadas-Castillo and et. al.(b), 2007); (Posadas-Castillo and et. al., 2008); (H. Serrano Guerrero, 2009) was obtained synchronization in coupled star networks with chaotic nodes given by Nd:YAG lasers and 3*D* CNNs, respectively; by using the approach given in (Wang, 2002). Some literature devoted on synchronization of complex networks (Manrubia and et.al., 2004); (Pogromsky and Nijmeijer, 2001); (Wang, 2002).

Network synchronization of coupled star nodes can be applied to transmit encrypted messages, from a single transmitter to multiple receivers in network communication systems, if the coupled nodes are chaotics. The aim of this paper is to synchronize three modified fourth-order Chua's circuits (which exhibit hyperchaotic behavior) studied in (Thamilmaran and et.al., 2004) in star coupled networks via Generalized Hamiltonian forms and observer design proposed in (Sira-Ramírez and Cruz-Hernández, 2001). This approach presents several advantages over the existing synchronization methods reported in the current literature.

2 PROBLEM SETTING

Consider the following set of N interconnected iden-

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tical dynamical systems

$$x_i = f(x_i) + u_i, \quad i = 1, 2, ..., N, \tag{1}$$

where $x_i = (x_{i1}, x_{i2}, ..., x_{in})^T \in \mathbb{R}^n$ is the state vector and $u_i = u_{i1} \in \mathbb{R}$ is the input signal of the system i, defined by

$$u_{i1} = c \sum_{j=1}^{N} a_{ij} \Gamma \mathbf{x}_j, \quad i = 1, 2, \ldots, N, \tag{2}$$

the constant $c > 0$ represents the *coupling strength*, and $\Gamma \in \mathbb{R}^{n\times n}$ is a constant 0-1 matrix linking coupled states.Whereas, $\mathbf{A} = (a_{ij}) \in \mathbb{R}^{n\times n}$ is the *coupling matrix*, which represents the coupling configuration in (1)-(2). If there is a connection between node i and node j, then $a_{ij} = 1$; otherwise, $a_{ij} = 0$ for $i \neq j$. Note that, if $u_{i1} = 0$, $i = 1, 2, ..., N$, in (1) we have a set of N isolated dynamical systems, operating with their own dynamics. While, if $u_{i1} \neq 0$ the set constitutes a *dynamical network* and each dynamical system i is called *nodo i*; and under appropiates u_{i1} the dynamical networks can be achieve collective behaviors. It is clear that, the input singal u_{i1} determines the kind of coupling among nodes in the networks. The coupling matrix for star coupled networks is given by

$$A = \begin{pmatrix} N-1 & -1 & -1 & \cdots & -1 \\ -1 & 0 & 0 & \cdots & 0 \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ -1 & 0 & 0 & \cdots & 0 \\ -1 & 0 & 0 & \cdots & 0 \end{pmatrix} \tag{3}$$

The star coupled configuration for N nodes is shown in Fig. 1, with the common or central node 1.

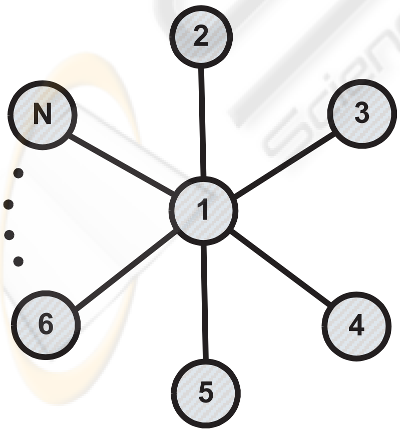

Figure 1: Star coupled configuration with N nodes.

The complex dynamical network (1) is said to achieve (asymptotically) *synchronization*, if (Wang 2002):

$$x_1(t) = x_2(t) = ... = x_N(t) \text{ as } t \to \infty. \tag{4}$$

The synchronization state in (1) can be an equilibrium point, a periodic orbit or, a chaotic attractor. This paper addresses the synchronization problem of dynamical networks (1) with coupled nodes in star topologies. In particular, by choosing a master node with the objective of to impose a particular collective behavior in (1). For illustrative purposes only, we consider three isolated nodes ($N = 3$) to be synchronized (which are described in Section 4), see Fig. 2(a). In Fig. 2(b) is shown this dynamical network with master node $N1$ and two slave nodes $N2$ and $N3$. Our objective is the synchronization of this network, when the coupled nodes are given by modified fourth-order Chua's circuits, to be described in Section 4. This particular coupling topology is important for its application to network communication systems, to transmit messages from a single transmitter to multiple receivers (Chow T.W.W. and Ng, 2001).

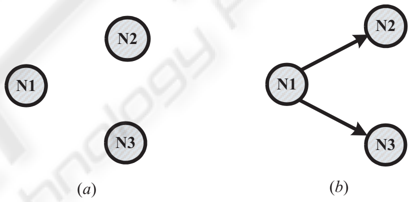

Figure 2: (a) Three isolated nodes. (b) Star coupled network with master node N1.

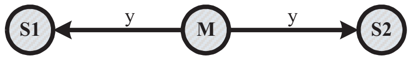

Figure 3: Single master node M and two slave nodes S1 and S2 configuration.

3 SYNCHRONIZATION VIA HAMILTONIAN FORMS

To solve the network synchronization problem stated in previous section, we appeal to synchronization (of two chaotic oscillators) via Hamiltonian forms and observer design reported in (Sira-Ramírez and Cruz-Hernández, 2001). In the sequel, we show that this approach is appropriate to synchronize a coupled star network with three nodes shown in Fig. 2(b). By using the proposed synchronization scheme shown in Fig. 3, where M is given in Hamiltonian form (Eq. (7)) and $S1$ and $S2$ being two observers for M given by Eq. (8).

Consider the following isolated dynamical system

$$\dot{x} = f(x), \tag{5}$$

where $x(t) \in \mathbb{R}^n$ is the state vector, $f : \mathbb{R}^n \rightarrow \mathbb{R}^n$ is a nonlinear function.

In (Sira-Ramírez and Cruz-Hernández, 2001) is reported how the dynamical system (5) can be written in the following Generalized Hamiltonian canonical form,

$$\dot{x} = \mathcal{J}(x)\frac{\partial H}{\partial x} + \mathcal{S}(x)\frac{\partial H}{\partial x} + \mathcal{F}(x), \quad x \in \mathbb{R}^n, \tag{6}$$

$H(x)$ denotes a smooth energy function which is globally positive definite in $\mathbb{R}^n$. The gradient vector of H, denoted by $\partial H/\partial x$, is assumed to exist everywhere. We use quadratic energy function $H(x) = (1/2)x^T Mx$ with M being a, constant, symmetric positive definite matrix. In such case, $\partial H/\partial x = Mx$. The matrices, $\mathcal{J}(x)$ and $S(x)$ satisfy, for all $x \in \mathbb{R}^n$, the properties: $\mathcal{J}(x) + \mathcal{J}^T(x) = 0$ and $S(x) = S^T(x)$. The vector field $\mathcal{J}(x)\partial H/\partial x$ exhibits the conservative part of the system and it is also referred to as the workless part, or work-less forces of the system; and $S(x)$ depicting the working or nonconservative part of the system. For certain systems, $S(x)$ is negative definite or negative semidefinite. Thus, the vector field is considered as the dissipative part of the system. If, on the other hand, $S(x)$ is positive definite, positive semidefinite, or indefinite, it clearly represents, respectively, the global, semi-global, and local destabilizing part of the system. In the last case, we can always (although nonuniquely) descompose such an indefinite symmetric matrix into the sum of a symmetric negative semidefinite. matrix $R(x)$ and a symmetric positive semidefinite matrix $N(x)$. Finally, $F(x)$ represents a locally destabilizing vector field.

In the context of observer design, we consider a special class of Generalized Hamiltonian forms (to be considered as the master node M) with linear output map $y(t)$, given by

$$\begin{aligned} \dot{x} &= \mathcal{J}(y)\frac{\partial H}{\partial x} + (\mathcal{I} + \mathcal{S})\frac{\partial H}{\partial x} + \mathcal{F}(y), \ x \in \mathbb{R}^n, \\ y &= \mathcal{C}\frac{\partial H}{\partial x}, \quad y \in \mathbb{R}^m, \end{aligned} \tag{7}$$

where S is a constant symmetric matrix, not necessarily of definite sign. The matrix I is a constant skew symmetric matrix, and $\mathcal{C}$ is a constant matrix.

We denote the estimates of the state $x(t)$ by $\hat{x}_i(t)$, $i = 1,2$ and consider the Hamiltonian energy function $H(\hat{x}_i)$ to be the particularization of H in terms of $\hat{x}_i(t)$. Similarly, we denote by $\eta_i(t)$, $i-1,2$ the estimated outputs, computed in terms of the estimated states $\hat{x}_i(t)$. The gradient vector $\partial H(\hat{x}_i)/\partial\hat{x}_i$ is naturally, of the form $M\hat{x}_i$ with M being a, constant, symmetric positive definite matrix.

Two *nonlinear state observers* for M (7) are given by

$$\begin{aligned} \dot{\hat{x}}_i &= \mathcal{J}(y)\frac{\partial H}{\partial \hat{x}_i} + (\mathcal{I} + \mathcal{S})\frac{\partial H}{\partial \hat{x}_i} + \mathcal{F}(y) + K_i(y - \eta_i), \\ \eta_i &= \mathcal{C}\frac{\partial H}{\partial \hat{x}_i}, \quad \eta_i \in \mathbb{R}^m, \quad i = 1,2, \end{aligned} \tag{8}$$

with $\hat{x}_i \in \mathbb{R}^n$ and K_i is the observer gain.

The state estimation errors, defined as $e_i(t) = x(t) - \hat{x}_i(t)$ and the output estimation error, defined as $e_{iy}(t) = y(t) - \eta_i(t)$, are governed by

$$\begin{aligned} \dot{e}_i &= \mathcal{J}(y)\frac{\partial H}{\partial e_i} + (\mathcal{I} + \mathcal{S} - K\mathcal{C})\frac{\partial H}{\partial e_i}, \ e_i \in \mathbb{R}^n, \\ e_{iy} &= \mathcal{C}\frac{\partial H}{\partial e_i}, \quad e_{iy} \in \mathbb{R}^m, \quad i = 1,2, \end{aligned} \tag{9}$$

where the vectors $\partial H/\partial e_i$ actually stands, with some abuse of notation, for the gradient vector of the modified energy functions, $\partial H(e_i)/\partial e_i = \partial H/\partial x - \partial H/\partial \hat{x}_i = M(x - \hat{x}_i) = Me_i$. We set, when needed, $I + S = \mathcal{W}$.

A necessary and sufficient condition for global asymptotic stability to zero of the estimation errors (9) is given by the following theorem.

Theorem 1 (Sira-Ramírez and Cruz-Hernández, 2001). The state $x(t)$ of the master node M (7) can be globally, exponentially, asymptotically estimated, by the states $\hat{x}_i(t)$, $i = 1,2$ of the observers (8) if and only if, there exist constant matrices K_i such that the symmetric matrices

$$\begin{aligned} [\mathcal{W} - K_i\mathcal{C}] + [\mathcal{W} - K_i\mathcal{C}]^T &= [\mathcal{S} - K_i\mathcal{C}] + [\mathcal{S} - K_i\mathcal{C}]^T \\ &= 2\left[\mathcal{S} - \frac{1}{2}(K_i\mathcal{C} + \mathcal{C}^T K_i^T)\right] \end{aligned}$$

are negative definite.

4 HYPERCHAOTIC CHUA'S CIRCUIT LIKE NODE

Consider the modified fourth-order Chua's circuit described by (Thamilmaran and et.al., 2004):

$$\begin{aligned} \dot{x}_1 &= \alpha_1(x_3 - f(x_1)), \\ \dot{x}_2 &= -\alpha_2 x_2 - x_3 - x_4, \\ \dot{x}_3 &= \beta_1(x_2 - x_1 - x_3), \\ \dot{x}_4 &= \beta_2 x_2, \end{aligned} \tag{10}$$

with nonlinear function given by

$$f(x_1) = bx_1 + \frac{1}{2}(a-b)(|x_1+1| - |x_1-1|). \tag{11}$$

With the paramerter values: $\alpha_1 = 2.1429$, $\alpha_2 = -12.83$, $\beta_1 = 0.0393$, $\beta_2 = 0.0015$, $a = -0.0299$, and

$b = 1.995$ the modified Chua's circuit (10)-(11) exhibits hyperchaotic behavior, with two positive Lyapunov exponents. By using the initial conditions $x(0) = (1.1, 0.1, -0.5, 0.01)$, Figs. 1, 2, 3, and 4 show the hyperchaotic attractors $x_1$vs x_2, $x_2$vs x_3, $x_3$vs x_4, and $x_1$vs x_4, respectively.

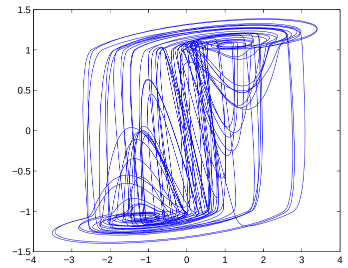

Figure 4: Hyperchaotic attractor projected onto the (x_1, x_2)-plane.

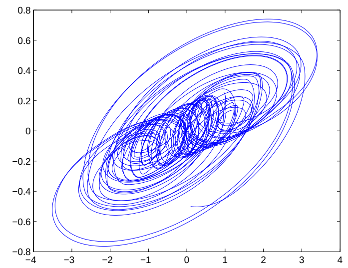

Figure 5: Hyperchaotic attractor projected onto the (x_2, x_3)-plane.

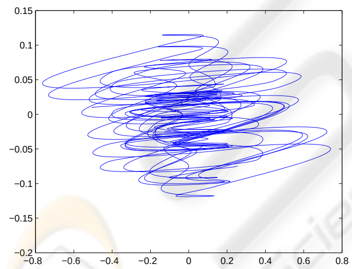

Figure 6: Hyperchaotic attractor projected onto the (x_3, x_4)-plane.

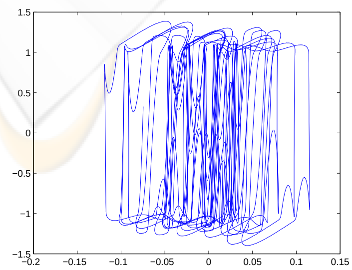

Figure 7: Hyperchaotic attractor projected onto the (x_1, x_4)-plane.

Next, we show the arrangement for star dynamical network by using as coupled node to hyperchaotic Chua's circuit defined by (10)-(11).

5 SYNCHRONIZATION OF HYPERCHAOTIC CHUA'S CIRCUITS IN A STAR NETWORK

In this section, we show the synchronization of three hyperchaotic Chua's circuits in a star coupled network, via Generalized Hamiltonian forms and observer design proposed in (Sira-Ramírez & Cruz-Hernández 2001). Firstly, we rewrite the modified fourth-order Chua's circuit (10)-(11) for the master node as follows.

Taking as Hamiltonian energy function to

$$H(x) = \frac{1}{2}\left(\frac{1}{\alpha_1}x_1^2 + x_2^2 + \frac{1}{\beta_1}x_3^2 + \frac{1}{\beta_2}x_4^2\right). \qquad (12)$$

Modified fourth-order Chua's circuit (10)-(11) in Generalized Hamiltonian form (**master node**, M) according to Eq. (7) is given by

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{pmatrix} = \begin{pmatrix} 0 & 0 & \alpha_1\beta_1 & 0 \\ 0 & 0 & -\beta_1 & -\beta_2 \\ -\alpha_1\beta_1 & \beta_1 & 0 & 0 \\ 0 & \beta_2 & 0 & 0 \end{pmatrix} \frac{\partial H}{\partial x} + \qquad (13)$$

$$\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & -\alpha_2 & 0 & 0 \\ 0 & 0 & -\beta_1^2 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \frac{\partial H}{\partial x} + \begin{pmatrix} -\alpha_1 f(x_1) \\ 0 \\ 0 \\ 0 \end{pmatrix}.$$

The destabilizing vector field calls for $x_1(t)$ to be used as the output $y(t)$, of the master node M (13). The matrices C, S, and I are given by

$$C^T = \begin{pmatrix} \alpha_1 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \quad S = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & -\alpha_2 & 0 & 0 \\ 0 & 0 & -\beta_1^2 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix},$$

$$I = \begin{pmatrix} 0 & 0 & \alpha_1\beta_1 & 0 \\ 0 & 0 & -\beta_1 & -\beta_2 \\ -\alpha_1\beta_1 & \beta_1 & 0 & 0 \\ 0 & \beta_2 & 0 & 0 \end{pmatrix}.$$

Next, we design two state observers (slave nodes $S1$ and $S2$, see Fig. 3) for master node (13). The first nonlinear state observer for the Generalized Hamiltonian system (13) (according to Eq. (8) as **slave node**

S1 is given by

$$\begin{pmatrix}\dot{\hat{x}}_{11}\\ \dot{\hat{x}}_{12}\\ \dot{\hat{x}}_{13}\\ \dot{\hat{x}}_{14}\end{pmatrix} = \begin{pmatrix} 0 & 0 & \alpha_1\beta_1 & 0\\ 0 & 0 & -\beta_1 & -\beta_2\\ -\alpha_1\beta_1 & \beta_1 & 0 & 0\\ 0 & \beta_2 & 0 & 0\end{pmatrix}\frac{\partial H}{\partial \hat{x}} + \tag{14}$$

$$\begin{pmatrix} 0 & 0 & 0 & 0\\ 0 & -\alpha_2 & 0 & 0\\ 0 & 0 & -\beta_1^2 & 0\\ 0 & 0 & 0 & 0\end{pmatrix}\frac{\partial H}{\partial \hat{x}} +$$

$$\begin{pmatrix} -\alpha_1 f(x_1)\\ 0\\ 0\\ 0\end{pmatrix} + \begin{pmatrix} k_{11}\\ k_{12}\\ k_{13}\\ k_{14}\end{pmatrix} e_{1y},$$

$$\eta_1 = \hat{x}_{11},$$

the second state observer (**slave S2**) is described by

$$\begin{pmatrix}\dot{\hat{x}}_{21}\\ \dot{\hat{x}}_{22}\\ \dot{\hat{x}}_{23}\\ \dot{\hat{x}}_{24}\end{pmatrix} = \begin{pmatrix} 0 & 0 & \alpha_1\beta_1 & 0\\ 0 & 0 & -\beta_1 & -\beta_2\\ -\alpha_1\beta_1 & \beta_1 & 0 & 0\\ 0 & \beta_2 & 0 & 0\end{pmatrix}\frac{\partial H}{\partial \hat{x}} + \tag{15}$$

$$\begin{pmatrix} 0 & 0 & 0 & 0\\ 0 & -\alpha_2 & 0 & 0\\ 0 & 0 & -\beta_1^2 & 0\\ 0 & 0 & 0 & 0\end{pmatrix}\frac{\partial H}{\partial \hat{x}} +$$

$$\begin{pmatrix} -\alpha_1 f(x_1)\\ 0\\ 0\\ 0\end{pmatrix} + \begin{pmatrix} k_{21}\\ k_{22}\\ k_{23}\\ k_{24}\end{pmatrix} e_{2y},$$

$$\eta_2 = \hat{x}_{21},$$

where $e_{1y} = x_1 - \hat{x}_{11}$ ($e_{11} = y - \eta_1$) and $e_{2y} = x_1 - \hat{x}_{21}$ ($e_{21} = y - \eta_2$). From master node (13) and slave nodes (14) and (15), we have that the synchronization error dynamics among the master node and two slave nodes (observers) is governed by

$$\begin{pmatrix}\dot{e}_{i1}\\ \dot{e}_{i2}\\ \dot{e}_{i3}\\ \dot{e}_{i4}\end{pmatrix} = \begin{pmatrix} 0 & \frac{k_{i2}\alpha_1}{2} & \gamma_i & \frac{k_{i4}\alpha_1}{2}\\ -\frac{k_{i2}\alpha_1}{2} & 0 & -\beta_1 & -\beta_2\\ -\gamma_i & \beta_1 & 0 & 0\\ -\frac{k_{i4}\alpha_1}{2} & \beta_2 & 0 & 0\end{pmatrix}\frac{\partial H}{\partial e_i} +$$

$$\begin{pmatrix} -k_{i1}\alpha_1 & \frac{k_{i2}\alpha_1}{2} & -\frac{k_{i3}\alpha_1}{2} & \frac{k_{i4}\alpha_1}{2}\\ -\frac{k_{i2}\alpha_1}{2} & -\alpha_2 & 0 & 0\\ -\frac{k_{i3}\alpha_1}{2} & 0 & -\beta_1^2 & 0\\ -\frac{k_{i4}\alpha_1}{2} & 0 & 0 & 0\end{pmatrix}\frac{\partial H}{\partial e_i} \tag{16}$$

where $\gamma_i = \alpha_1\beta_1 + \frac{k_{i3}\alpha_1}{2}$. Where the synchronization errors are defined by e_1 and e_2 among master M and slaves 1 and 2, respectively. One may now choose the observer gains $K_i = (k_{i1}, k_{i2}, k_{i3}, k_{4i})^T$, $i = 1,2$ in order to guarantee asymptotic exponential stability to zero of the synchronization errors $e_i(t) = (e_{i1}(t), e_{i2}(t), e_{i3}(t), e_{i4}(t))$, $i = 1,2$ as will be shown in the next section.

6 SYNCHRONIZATION CONDITIONS

Now, we examine the stability of the synchronization errors (16) for the network constructed with master (13) and two slaves (14) and (15), with modified Chua's circuits as coupled nodes. Thus, we invoke Theorem 1, which guarantees global asymptotic stability to zero of $e_i(t)$, $i = 1,2$. In particular, for modified Chua's circuit, the matrices $2\left[\mathcal{S} - \frac{1}{2}(K_i\mathcal{C} + \mathcal{C}^T K_i^T)\right]$, $i = 1,2$ shown in Theorem 1, are give by

$$\begin{pmatrix} -2k_{i1}\alpha_1 & -k_{i2}\alpha_1 & -k_{i3}\alpha_1 & -k_{i4}\alpha_1\\ -k_{i2}\alpha_1 & -2\alpha_2 & 0 & 0\\ -k_{i3}\alpha_1 & 0 & -2\beta_1^2 & 0\\ -k_{i4}\alpha_1 & 0 & 0 & 0\end{pmatrix}, \; i = 1,2 \tag{17}$$

by applying the Sylvester's Criterion -which provides a test for negative definite of a matrix- thus, we have that the mentioned matrices will be negative definite matrices, if we choose $K_i = (k_{i1}, k_{i2}, k_{i3}, k_{i4})^T$, $i = 1,2$ such that the following conditions are satisfied:

$$\begin{aligned} k_{i1} &\leq 1, \quad (18)\\ 4k_{i1}\alpha_1\alpha_2 - k_{i2}^2\alpha_1^2 &\geq 0,\\ 2\left[\alpha_1\beta_1^2\left(\alpha_1 k_{i2}^2 - 4k_{i1}\alpha_2\right) + k_{i3}^2\alpha_1^2\alpha_2\right] &\geq 0,\\ k_{i4} &= 0. \end{aligned}$$

We have used $K_1 = (3.3, 1.5, 0.39, 0)^T$ and $K_2 = (2.3, 1, 0.3, 0)^T$ with initial conditions: for M, $x(0) = (1.1, 0.1, -0.5, 0.01)$ and for $S1$, $\hat{x}_1(0) = (0.5, 0.3, -0.4, 0)$ and for $S2$, $\hat{x}_2(0) = (1, 0, -0.2, 0.04)$. Fig. 8 shows the synchronization among master node (13) and two slave nodes (14) and (15).

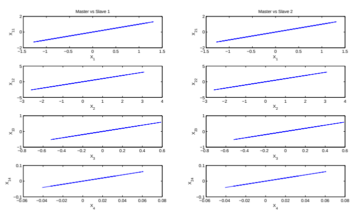

Figure 8: Complete synchronization among states of hyperchaotic master node M and slave nodes S1 and S2.

7 CONCLUSIONS

In this paper, we have presented multiple synchronization of coupled modified fourth-order Chua's circuit, in particular by using star coupled networks.

We have achieve synchronization of three hyperchaotic Chua's circuit (used as fundamental node) in star complex networks, via Generalized Hamiltonian forms and observer design considering a single master node and two slave nodes. This result is particularly interesting given its application in communication network systems, where is required that a single sender transmits simultaneously information to many receivers via a public channel.

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