OPTIMAL CONTROL OF HAZARDOUS MATERIALS TRAFFIC FLOW
The Case of Transport through a Critical Infrastructure

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Abstract: In this work, a preliminary study as regards the possibility to define optimal control strategies for the hazmat (hazardous material) traffic flowing towards one critical road infrastructure (e.g. as in the case study a tunnel) at the macroscopic level is introduced. Specifically, the simplified model that is studied is related to part of a highway, on which the hazmat traffic can flow from one entrance. The control variables are represented by the number of vehicles that are allowed to enter the highway during a specific time interval, while the state variables are the queue of vehicles before the entrance, the number of vehicles in the various tracts of the highway, and the number of vehicles that enter the tunnel. The objective function to be minimized is characterized by three main terms: the queue, the hazard over the road, and the hazard related to the tunnel.

1 INTRODUCTION

Hazardous materials cover a wide range of products (explosives, gases, flammable liquids and solids, radioactive materials, hazardous wastes, etc. (Verter and Kara, 2008)). Transportation of these materials (that is, in general, multi-modal: road, pipelines, railway, ship) is a relevant problem to be considered because of the significant amount of material that flows among roads, territory and infrastructures (Bersani et al., 2008). Defining strategies for hazardous materials (hazmat) transportation management is a complex task because it is necessary to take into account different objectives (minimize risks, satisfy goods demand transportation), different decision makers (fleet managers, local authorities, infrastructures managers), and different approaches (mainly based on the different spatial-temporal scales to be considered: strategic planning, tactical planning, operational management).

In the literature of hazardous materials transportation on road, there are few, thought important and relevant, works on this subject (for example: Berman et al., 2007; Verter and Kara, 2007; Kara and Verter, 2004; Sadjadi, 2007; Bell, 2009; Bell and Cassir, 2002, Bersani et al., 2008a;

Serafini, 2006; Beroggi and Wallace, 1994). The majority of these works is based on optimization models for planning and design purposes. The preliminary approach presented in this work is instead based on real time operational management (like the work presented by Bersani et al., 2008b) with specific reference to the case of critical infrastructures.

The transportation of hazardous materials (hazmat) on road has important consequences in the overall traffic management (Minciardi et al., 2008). This fact is more evident when a vehicle requires to move towards a critical road infrastructure, such as a tunnel or a bridge. The control of traffic networks has been the subject of a great amount of literature from different viewpoints. The main articles related to the case of a tunnel are reported in (Minciardi et al., 2008). The aim of this preliminary study regards the possibility to define optimal control strategies for the hazmat traffic flowing towards one critical road infrastructure (e.g. as in the case study a tunnel).

A given number of vehicles transporting hazardous material has to use a highway and to reach one critical infrastructure (e.g. a tunnel). They can stop in a park before the highway entrance and start their travel according to the exigencies of a decision maker that can be identified as the tunnel manager. The park may be taken into account as an

inventory in which the state of the system is represented by the vehicles that are present at a specific time instant. The flow dynamics of hazardous material vehicles on the highway has also to be modelled. In particular, the problem is defined at a macroscopic level, in which the state and the control variables correspond to the number of vehicles, for which the integrity condition may be relaxed, in order to obtain a continuous-variable decision problem. The control variables are represented by the number of vehicles that are allowed to enter the highway during a specific time interval, while the state variables are the queue of vehicles before the entrance, the number of vehicles in the various tracts of the highway, and the number of vehicles that enters the tunnel. The objective function to be minimized is characterized by three main terms: the queue, the hazard over the road, and the hazard related to the tunnel.

The resulting optimal control problem is linear quadratic with non-negativity constraints over the state and control variables. A receding horizon control scheme is used to derive the solution and to allow the model to be suitable in real time decision frameworks. An optimization package (Lingo 9.0, www.lindosystems.com) is used to solve the problem at each step.

In fact, the explicit form of the optimal control law of a given linear, discrete-time, time-invariant process subject to a quadratic cost criterion is well known in the unconstrained case, while, even for simple constraints, solution is hard to achieve. In (Castelein and Johnson, 1989), the authors use the controllable block companion transformation and derive sufficient conditions on the weighting matrices of the cost criterion to ensure that the closed-loop response of the original process with the standard, unconstrained optimal feedback law will be nonnegative. Bertsimas and Brown (2007) assess that the celebrated success of dynamic programming for optimizing quadratic cost functions over linear systems is limited by its inability to tractably deal with even simple constraints, and present an alternative approach based on results from robust optimization to solve the stochastic linear-quadratic control (SLQC) problem.

For this reason, interesting developments of this work will be devoted to the definition of methodologies to find efficient solutions for the optimal control strategies.

In the next subsections, the system model is described in detail. Then, the decision problem is formalized. Finally, results and conclusion are drawn.

2 THE SYSTEM MODEL

Figure 1 shows the schematic representation of the decision framework: the highway directed towards one critical infrastructure is modelled as a line divided in highway tracts. As a simplification, two highway tracts have been considered.
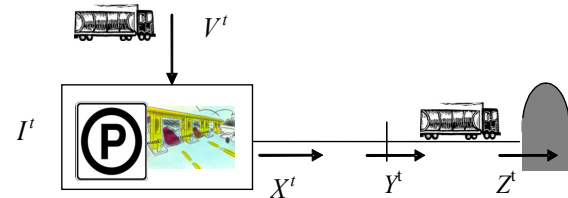

Figure 1: The considered system.

The physical inputs of the whole system are the quantities V^t, i.e., the (known) number of vehicles entering the park near the highway entrance in time interval $(t, t+1)$, $t = 0,\ldots,T$-1. The control variables correspond to the number of vehicles that enter the highway X^t in a specific time interval $(t, t+1)$, while the state variables correspond to the number of vehicles in the inventory/queue, I^t, the number of vehicles per tract of the highway (N_1^t, N_2^t), and the number of vehicles going out from tract 1 and entering the tunnel (Y^t, Z^t).

Two different kinds of state equations have to be introduced, regarding, respectively, the queue in the park at the highway entrance, and the highway tracts. Moreover, the hazard has been formalized as a function of the state and control variables.

2.1 The Queue State Equation

The state equation is:

$$I^{t+1} = (I^t + V^t - X^t) \qquad t=0,\ldots, T\text{-}1 \qquad (1)$$

where:

- I^t is the number of vehicles stored, at time instant t, in the park near the entrance, i.e., the inventory of the entrance park area, in time interval $(t, t+1)$;
- X^t is the number of vehicles that enter the highway in time interval $(t, t+1)$, from the entrance park area;
- V^t is the (known) number of vehicles that enters the entrance park in time interval $(t, t+1)$.

2.2 The Highway Tract State Equations

These state equations describe the evolution over time of a state variable that represents the number of hazmat vehicles (per unit length) present in a specific tract of the highway. The speed of these vehicles is related to the overall vehicle density over the considered tract. It is assumed that the vehicle flow can be represented through an average speed, which is common to hazmat and non-hazmat vehicles. In agreement with the literature dealing with traffic models, it is assumed that the (average) vehicle speed is never so high to allow the complete covering of a highway tract within a single time interval (of course, this may be also seen as a constraint over the space discretization of the highway). The equations are given by

$$N_1^{t+1} = N_1^t + \frac{X^t}{L_1} - \frac{Y^t}{L_1} \qquad t=0,\ldots,T\text{-}1 \qquad (2)$$

$$N_2^{t+1} = N_2^t - \frac{Z^t}{L_2} + \frac{Y^t}{L_2} \qquad t=0,\ldots,T\text{-}1 \qquad (3)$$

with

$$Y^t = N_1^t vel_1^t \Delta t \qquad t=0,\ldots,T\text{-}1 \qquad (4)$$

$$Z^t = N_2^t vel_2^t \Delta t \qquad t=0,\ldots,T\text{-}1 \qquad (5)$$

where:

- N_1^t, N_2^t are the number of (hazmat) vehicles per unit length that is present in the highway road in tracts 1 and 2, in time instant t;
- L_1, L_2 are the tracts lengths;
- Δt is the time interval length;
- vel_1^t, vel_2^t are the (average) velocities in the tracts in time interval $(t, t+1)$, which is assumed to be imposed by the ordinary traffic (i.e., non hazmat), assuming that the hazmat vehicle flow is only a negligible part of the overall traffic flow;
- Y^t is the number of vehicles that passes from tract 1 to tract 2 in time interval $(t, t+1)$;
- Z^t is the number of vehicles that reaches the tunnel in time interval $(t, t+1)$.

2.3 Hazard Assessment

The hazard of accidents depends on different structural and environmental parameters that may vary for each time interval and for each highway tract, and on the number of vehicles (Fabiano et al., 2002; Fabiano et al., 2005). In this work, the hazard HAZ^t is simply represented as a time-varying a-dimensional parameter $\eta_{HAZ}{}^t$ multiplied by the number of vehicles in the specific tract. That is,

$$HAZ^t = \eta_{HAZ\,1}{}^t N_1^t L_1 + \eta_{HAZ\,2}{}^t N_2^t L_1 + \eta_{HAZ\,3}{}^t Z^t$$
$$t=0,\ldots,\,T\text{-}1 \qquad\qquad (6)$$

3 THE DECISION PROBLEM

The objective function has to take into account the number of vehicles in the park entrance, the number of vehicles per unit length in each tract of the highway, and the number of vehicles that enter the tunnel. In particular the following terms have to be minimized:

- the number of vehicles waiting in the park entrance;
- the number of vehicles per unit length for tract 1, N_1^t;
- the number of vehicles per unit length for tract 2, N_2^t;
- the difference between the number of vehicles per unit length in tract 1 and tract 2, $N_1^t - N_2^t$;
- the number of vehicles that enter the tunnel.

Thus, the objective function can be expressed as

$$\min \quad \sum_{t=0}^{T-1} \left(I^t\right)^2 + \alpha\left(N_1^t\right)^2 + \beta\left(N_2^t\right)^2 + \\ + \gamma\left(N_1^t - N_2^t\right)^2 + \delta\left(Z^t\right)^2 \qquad (7)$$

where:

- N_1^t, N_2^t are the number of (hazmat) vehicles per unit length that is present in the highway road in tracts 1 and 2, in time instant t;
- I^t is the number of vehicles stored, at time instant t, in the park near the entrance, i.e., the inventory of the entrance park area, in time interval $(t, t+1)$;
- Z^t is the number of vehicles that reaches the tunnel in time interval $(t, t+1)$;
- α, β, γ, δ are specific weighting factors.

4 THE STATEMENT OF THE OPTIMAL CONTROL PROBLEM

The optimal control problem reported in equations (1)-(7) can be expressed in the following form

$$\min_{u_t} \quad \sum_{t=0}^{T-1} \underline{x}_t{}^T Q_t \underline{x}_t \tag{8}$$

where $\underline{x}_t$ is the space vector and Q_t a matrix of time dependent parameters. Specifically,

$$\underline{x}_t = \begin{bmatrix} I^t \\ N_1^t \\ N_2^t \end{bmatrix} \qquad t=0,\dots,T\text{-}1 \tag{9}$$

$$Q_t = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \alpha+\gamma & -\gamma \\ 0 & -\gamma & \beta+\gamma+\delta vel_t^2 \Delta t^2 \end{bmatrix} \quad t=0,\dots,T\text{-}1 \tag{10}$$

s.t.

$$\underline{x}_{t+1} = A_t \underline{x}_t + \underline{b}u_t + \underline{d}_t \qquad t=0,\dots,T\text{-}1 \tag{11}$$

$$u_t \ge 0 \qquad t=0,\dots,T\text{-}1 \tag{12}$$

$$\underline{x}_t \ge 0 \qquad t=0,\dots,T\text{-}1 \tag{13}$$

where $u_t = X_t$ are the control variables, A_t is a matrix of time dependent parameters, $\underline{b}$ a vector of parameters, and $\underline{d}_t$ a vector of time dependent parameters.

$$A_t = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1-\dfrac{vel_1^t \Delta t}{L_1} & 0 \\ 0 & \dfrac{vel_1^t \Delta t}{L_2} & 1-\dfrac{vel_2^t \Delta t}{L_2} \end{bmatrix} \quad t=0,\dots,T\text{-}1 \tag{14}$$

$$\underline{b}_t = \begin{bmatrix} -1 \\ \dfrac{1}{L_1} \\ 0 \end{bmatrix} \qquad t=0,\dots,T\text{-}1 \tag{15}$$

$$\underline{d}_t = \begin{bmatrix} V^t \\ 0 \\ 0 \end{bmatrix} \qquad t=0,\dots,T\text{-}1 \tag{16}$$

The optimal control problem expressed by equations (8)-(16) is a linear-quadratic one, with non negativity constraints over the state and control variables.

5 RESULTS

The space-time discretization of equations (2)-(3) has been chosen in order to avoid instability of the traffic flow (i.e., in the time interval, the vehicles are not allowed to pass the tract length), and in order to have a meaningful time interval for traffic flow simulation (Kotsialos and Papageorgiou, 2004). That is,

$$T = 15$$
$$\Delta t = 10 \quad [s]$$
$$L_1 = 800 \quad [m]$$
$$L_2 = 800 \quad [m]$$
$$vel_1^t = 16.6 \quad [m/s]$$
$$vel_2^t = 16.6 \quad [m/s]$$

Firstly, the optimization problem (1)-(7) has been solved, with the following inputs: $\underline{V} = [10,3,2,0,0,0,0,0,0,2,3,0,0,0,0]$, and the following weights in the objective function: $\alpha = 2 \cdot 10^4, \beta = 2 \cdot 10^4, \gamma = 2 \cdot 10^4, \delta = 2 \cdot 10^5$.

A receding-horizon control scheme has been applied and, in Table 1 and Table 2, the optimization results are reported.

Table 1: Results of the optimization problem: X^t, Z^t, I^t.

Time	X^t	Z^t	I^t
0	8.38	0	0
1	0.56	0	1.62
2	0.6	$0.26 \cdot 10^{-4}$	4.06
3	0.66	$0.26 \cdot 10^{-4}$	5.46
4	0.74	$0.25 \cdot 10^{-4}$	4.8
5	0.86	$0.24 \cdot 10^{-4}$	4.06
6	1	$0.23 \cdot 10^{-4}$	3.2
7	1.19	$0.21 \cdot 10^{-4}$	2.2
8	1	$0.19 \cdot 10^{-4}$	1
9	2	$0.17 \cdot 10^{-4}$	0
10	2.23	$0.14 \cdot 10^{-4}$	0
11	0.77	$0.12 \cdot 10^{-4}$	0.76
12	0	$0.83 \cdot 10^{-5}$	0
13	0	$0.44 \cdot 10^{-5}$	0
14	0	0	0

Table 2: Results of the optimization problem: N_1^t, N_2^t, Y^t.

Time	N_1^t	N_2^t	Y^t
0	0	0	0
1	$0.1 \cdot 10^{-1}$	0	1.74
2	$0.9 \cdot 10^{-2}$	$0.22 \cdot 10^{-2}$	1.49
3	$0.79 \cdot 10^{-2}$	$0.4 \cdot 10^{-2}$	1.3
4	$0.71 \cdot 10^{-2}$	$0.57 \cdot 10^{-2}$	1.17
5	$0.65 \cdot 10^{-2}$	$0.71 \cdot 10^{-2}$	1.08
6	$0.63 \cdot 10^{-2}$	$0.85 \cdot 10^{-2}$	0.04
7	$0.62 \cdot 10^{-2}$	$0.98 \cdot 10^{-2}$	1.03
8	$0.64 \cdot 10^{-2}$	$0.11 \cdot 10^{-1}$	1.06
9	$0.63 \cdot 10^{-2}$	$0.12 \cdot 10^{-1}$	1.05
10	$0.75 \cdot 10^{-2}$	$0.14 \cdot 10^{-1}$	1.25
11	$0.87 \cdot 10^{-2}$	$0.15 \cdot 10^{-1}$	1.45
12	$0.79 \cdot 10^{-2}$	$0.17 \cdot 10^{-1}$	1.31
13	$0.63 \cdot 10^{-2}$	$0.19 \cdot 10^{-1}$	1.04
14	$0.49 \cdot 10^{-2}$	$0.2 \cdot 10^{-1}$	0.82

The overall hazard is (summation over time of equation (6)) equal to 1978, with $\eta_{HAZ\,1}^t = \eta_{HAZ\,2}^t = \eta_{HAZ\,3}^t = 10$.

Then, the non-negativity constraints have been removed. The optimal values are the same like in the constrained case.

Similar results, in the unconstrained case, can be found through the use of the Riccati equation. Instead, for the constrained case an efficient method of solution has to be found. A possible approach can be the one reported in (Bertsimas and Brown, 2007). Otherwise, one can try to use dynamic programming and reduce the explosion of computation that arises.

6 CONCLUSIONS

A preliminary approach for the optimal control of hazardous materials traffic flow has been presented. The novelties of the presented approach in the literature of hazmat transportation have been highlighted, as well as the methodological approaches that might characterize the solution of the optimal control problem.

Future research related to the present work will regard the development of methods to derive the optimal control law to the considered problem in a closed form. After that, the decision problem could be extended to the optimal control of two fleets of hazardous material that have to flow through a tunnel in both competitive and collaborative cases. Moreover, a hierarchical control can be formalized in which a decision maker related to the tunnel has to decide the price to assign to the two fleets on the basis of the costs, the goods demand, and the risk to be minimized in the overall system, while the fleets aim at minimizing their own benefits and hazards.

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