Robust Navigation for an Autonomous Helicopter with Auxiliary Chattering-free Second Order Sliding Mode Control

S. Vite-Medécigo, Ernesto Olguín-Díaz and Vicente Parra-Vega

Robotics and Advanced Manufacturing
Research Center for Advanced Studies–CINVESTAV, Saltillo, Mexico

Abstract. This paper presents a novel technic for autonomous flight and navigation control of AAVs, particularly useful for helicopters. Three servo-loop controller are introduced to yield stable robust regulation. The inner control loop is based on an LQR regulator designed over the linearized plant at hover to guarantee close-loop stability. The middle loop is a feedback linearization controller based on the close-loop linearized system to cope with the underactuated nature of the helicopter, by guaranteing an asymptotically stable zero dynamics. Finally the outer control loop enforces a tracking second-order sliding-mode for cartesian position and heading navigation outputs. The simplicity of this control proposal allows easier and intuitive guidelines to tune feedback gains while the chattering-free sliding-mode fulfills basic robustness properties, ideal for this complex systems subject to external disturbances like wind gusts.

1 Introduction

Automatic flying vehicles, also known as Autonomous Aerial Vehicle (AAV), represents a huge field of applications in particular for advanced automatic control techniques because human intervention is considered difficult or dangerous. There are wide civil and military interests in helicopters, like traffic surveillance, air pollution monitoring, area mapping, agricultural applications, exploration, scientific data collection, search and rescue.

Among the AAVs, the rotary wing AAVs such as the helicopter has the advantage of having the ability to perform different flight regimes like hover, backward, lateral of pure vertical flight, in contrast to fix wing such as typical airplanes. However, helicopters are underactuated mechanisms whose dynamic model exhibits high nonlinearities with physical parameters hard to measure precisely. The operational versatility of helicopters requires complex controllers to achieve such flight regimes.

We can classify two type of controllers. One uses the full dynamic modeling with simple model-free controllers; the second assumes simple dynamic modeling used in complex controllers design. In the former case, due to the complexity of the full dynamic model of helicopters and unknown aerodynamic/aeroelastic parameters, model-based controllers are hard to implement and then simpler control laws based on linearized plant are preferred. Since this approach is prone to instability due to the un-

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knowns of the dynamic plant, an auxiliary ν controller is added, typical PID-like controller, which introduces limited performance because of the well-known limitations of these PID-like controllers. However this type of studies has been useful to understand better the complexity and structural properties on real applications because they employ the full model with simple controllers providing clear intuitive understanding on the stability properties of the closed-loop system. The latter case uses simpler dynamic models, based on restrictive academic assumptions, such as the helicopter is constrained to move only in a subset of $\Re^6$, exhibiting pseudo-flying conditions with model-based controllers [4]. This approach guarantees very limited performance in real conditions, with limited scope of real applications.

In this paper, we focus our attention in the full dynamical model of the scalar R/C X-cell90 helicopter, and propose a novel auxiliary controller based on a chattering-free sliding modes, which increases the closed-loop performance because it is a tracking-designed controller with inherent robustness capabilities. This allows to guarantee better closed-loop performance in comparison to auxiliary controllers based on PID-like controllers. Simulations under external disturbances like wind gusts, wherein clearly verifies the validity of the proposed approach.

2 Relevant Background

Complex helicopter models, [9, 12], based in the Newtonian model of a free flying rigid objet are restricted to measurements on the center of mass, which indeed can vary in real conditions, neglecting at small velocities the Coriolis effects, thus this model is not useful in aggressive maneuvers or wide range of operational flight conditions. More over, dissipative effects on the fuselage are not taken in account that would be important during the navigation. In [6] this Coriolis effects are taken in account but simplifies the 6-DOF Inertia-Matrix to be completely diagonal. More over, a spring model is included to describe the main rotor forces mapping to the main body rigid object modeling. Nonetheless these models neglect the blade's kinetic energy, which can be up to 20 times the one of the fuselage [1]. Thus, in hover regime this energy must be taken in account to give rise to a dynamic model of more than the 6 degrees of freedom (DOF) of a rigid free flying object, showing the complexity of the main rotor itself. This model is more relevant in practice since it includes this important energy.

On one hand the forces acted in the rigid free flying object (the fuselage) are given mainly by the forces exerted at the main and tail rotors. The forces at the tail rotor is a simple thrust in the direction perpendicular to the tail rotor whose magnitude changes with the tail collective. On the other hand, the main rotor provides 3 Cartesian components of the main rotor thrust given by the main rotor collective and two azimuth angles also known as lateral and longitudinal cyclic. Then, even for the most simplest model, *i.e.* 6-DOF, the full system is underactuated because the control dimension is 4.

The problem of control design for this kind of systems even for complex models including all or some of the full main rotor dynamics as been addressed extensively in the literature, however the control of the underaction remains open, though it has been addressed in [2, 13]. In particular, [14] proposes LQR-BDU techniques a the linearized model, concluding a robust regulator in a small neighborhood of the linearized point.

LQR feedback control scheme plus an additional PID-like regulators loop is a popular choice because the unknown parameters and external disturbances, like gust of wind, deviates the operational point; however the popular integral-loop may increase the sensitivity of the system under commonly time-varying disturbances. In this paper, the additional servoloop is based on a robust chattering-free sliding mode controller to provide wider operational conditions, with better performance.

3 Mathematical Model

In contrast to the Lagrange method, the equations obtained via Newton's laws expressed with velocities and acceleration measured at the body (relative to the body's frame and not to the inertial one) result in a simpler representation. The difference in these representations arise from the fact that the generalized coordinates needed in the Lagrange method, while having a physical meaning in the pose, the generalized velocity does not have a physical meaning and neither the generalized force vector; at least part of them. Equivalences between these two different representations can be obtained via the kinematic equation, *i.e.* using the mapping operator that express the physical meaning of velocity wrench used in Newton formulation out of the generalized velocity vector used in Lagrange one [5, 10].

The kinematic of a rigid single body in space is represented only by the pose (position and attitude) of the body with respect to an inertial (fixed) frame Σ_0, where Σ_v is the frame rigidly attached to the object. See Fig. 1.

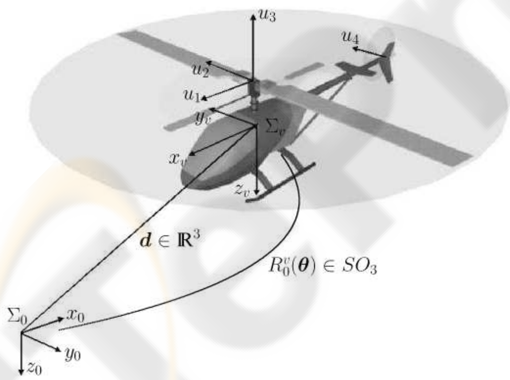

Fig. 1. Inercial frame Σ_0 and object frame Σ_v.

The rotation matrix $R_0^v \in SO_3$ transfers a 3D vector from its representation in frame Σ_v to the inertial frame Σ_0. The generalized position of the object, expressing both position and attitude of the object is then defined as

$$q \triangleq \begin{pmatrix} d \\ \theta_v \end{pmatrix} \in \Re^6 \tag{1}$$

where $d = (x, y, z)^T \in \Re^3$ is the object inertial position with respect to the Σ_0 given by the inertial Cartesian coordinates of the origin of frame Σ_v and $\theta_v =$

$(\phi_x, \theta_y, \psi_z)^T \in [-\pi, \pi] \times [-\pi/2, \pi/2] \times [-\pi, \pi]$ is the set of attitude parameters (in this case the roll-pith-yaw Euler angles) of Σ_v with respect to Σ_0. For this very set of attitude parameter the form of the rotation matrix R has a particular expression that can be found in either [5, 10]. The vector $\nu \in \Re^6$ is the velocity twist which defines the linear and angular velocity of Σ_v expressed in the non-inertial frame Σ_v, i.e. the velocity measured from the object

$$\nu \triangleq \begin{pmatrix} v \\ \omega \end{pmatrix} \in \Re^6 \tag{2}$$

where $v = R_0^{v\,T} \dot{d} \in \Re^3$ is the lineal velocity of the object and $\omega \in \Re^3$ is the angular velocity of frame Σ_v, both vectors expressed in the non-inertial frame Σ_v. In strictly mathematical sense $R_0^{v\,T} \dot{\theta}_v \neq \omega$, however there is a relationship given by $\omega = R_0^{v\,T} J_\theta \dot{\theta}_v$, where $J_\theta \in \Re^{3\times 3}$ is a linear operator given by attitude parameters. Then a relationship between ν and $\dot{q}$ is found as follows

$$\nu = J_v(q)\dot{q} \tag{3}$$

with $J_v(q) \in \Re^{6\times 6}$ being the linear operator of the kinematic equation. The Kirchhoff formulation for the equation of motion of a rigid object is nothing but the moment conservation equations expressed in the non-inertial frame in terms of the kinetic energy as

$$\frac{d}{dt}\frac{\partial K}{\partial v} + \omega \times \frac{\partial K}{\partial v} = f \tag{4}$$

$$\frac{d}{dt}\frac{\partial K}{\partial \omega} + \omega \times \frac{\partial K}{\partial \omega} + v \times \frac{\partial K}{\partial v} = n \tag{5}$$

where f and n are the forces and torques respectively that acts over the object, including gravity, dissipative forces and any external input force acting on the object, and K is the kinetic energy as $K = \frac{1}{2}\nu^T M \nu$, where matrix $M \in \Re^{6\times 6}$ is the *Inertia Matrix* with respect to the origin of frame Σ_v, defined as follows:

$$M \triangleq \begin{bmatrix} mI_3 & -m[r_c \times] \\ m[r_c \times] & I_g \end{bmatrix} \tag{6}$$

which is by construction constant, positive definite and symmetric $M = M^T > 0$. The terms of this Inertia Matrix are the total mass m of the object, the distance from the origin of frame Σ_v to the center of mass of the body r_c, expressed in the body's frame, the inertia moment matrix I_g computed from the origin of Σ_v, and the skew symmetric matrix representation of the cross product $[a\times]b = a \times b$.

Equations (4)-(5), after proper algebraic manipulation and using the kinetic energy expression above, can also be expressed in a single vectorial equation as

$$M\dot{\nu} + c(\nu) = F,$$

where matrix $M \in \Re^{6\times 6}$ is the *Inertia Matrix* with respect to the origin of frame Σ_v, the vector $c(\nu)$ regroups all the nonlinear terms and is known as the Coriolis vector, and

$F \triangleq \left(f^T, n^T\right)^T = F_G + F_D + F_T$ is the force wrench consisting in gravity, dissipation and thrust wrenches respectively.

Because of the quadratic nature in terms of velocity Coriolis vector it can also be expressed as product of a matrix and the velocity wrench: $c(\nu) = C(\nu)\nu$. The matrix $C(\nu)$, referred as the Coriolis matrix may have many different representations, but at last one of them fulfills the skew-symmetry property $C(\nu) + C(\nu)^T = 0$.

F_G, being the gravity force wrench in the objects frame, can be computed rotating the gravity influence to the objects frame $f_g = mgR_0^{v\,T}\boldsymbol{k}$. The gravity vector is defined then as $g(q) \triangleq \left(f_g^T; 0\right)^T$. Then $F_G = -g(q)$, where the negative sign comes from the fact that the positiveness of the vertical axis z_0 is pointing downward, to the center of the earth, due to convention in vessel engineering.

F_D are the dissipation aerodynamic forces and these are by nature quadratic and homogeneous to the velocity wrench. Then a possible approach to model these forces can be given as $F_D = -D(\|\nu\|)\nu$, where the damping matrix should be definite positive $D > 0$ to fulfill passivity [10].

Finally, F_T are thrust aerodynamical wrench and are given by the influences of the forces exserted by both rotors. There are 3 forces at the center of the main rotor given by longitudinal cyclic (u_1), the lateral cyclic (u_2) and the collective (u_3). There is also a fourth force at the center of the tail rotor (u_4) (See Figure 1). This mapping is given by a constant operator $B_e \in \Re^{6\times 4}$ that can be computed from the geometry of the rotors with respect to vehicle's frame Σ_v as $F_T = B_e u$, with $u = (u_1,\, u_2,\, u_3,\, u_4) \in \Re^4$ and B_e a column full rank matrix.

The dynamic modeling of the helicopter without considering the rotors dynamic is then given by [10]:

$$M\dot{\nu} + C(\nu)\nu + D(\|\nu\|)\nu + g(q) = B_e u \tag{7}$$

$$\nu = J_v(q)\dot{q} \tag{8}$$

which can be expressed is state space form using the state definition $x \triangleq (q^T, \nu^T)^T$.

4 Controller Design

A robust control law is necessary due to the environmental nature of AAV, then LQR approach is preferred because it is an optimal criteria for set-point control while minimizing energy consumption [8]. However this technic is based on a linear model or a linearized one, which means it works as supposed only in the operational point x_o, where the linearization was computed with $\tilde{x} = x - x_o$:

$$\dot{\tilde{x}} = A\tilde{x} + Bu \tag{9}$$

$$y = C\tilde{x} \tag{10}$$

In the case of the system (7)-(8) the state realization yields to

$$A(x) = \begin{bmatrix} \frac{\partial}{\partial q}\left(J_v^{-1}(q)\nu\right) & J_v^{-1}(q) \\ -M^{-1}\left(\frac{\partial}{\partial q}g(q)\right) & -M^{-1}\left[C(\nu) + D(\|\nu\|)\right] \end{bmatrix} \in \Re^{12\times 12} \tag{11}$$

$$B = \begin{bmatrix} 0 \\ M^{-1}B_e \end{bmatrix} \in \Re^{12\times 4}, \qquad C = \begin{bmatrix} I & 0 \end{bmatrix} \in \Re^{6\times 12} \tag{12}$$

Remark 1. Clearly, (12) indicates that $CB = [0] \in \Re^{6\times 4}$.

For the particular case where the operation point is *hover*, i.e. $x_o = \left(q_d^T;\ 0\right)$ and $q_d = (x_d, y_d, z_d, 0, 0, 0)^T$ the state matrix becomes constant:

$$A = \begin{bmatrix} 0 & I \\ -M^{-1}\left[\frac{\partial}{\partial q} g(q)\right] & 0 \end{bmatrix} \in \Re^{12x12} \tag{13}$$

for the same pair (B, C). From (13) it can be seen that the linearized model at hover operational point has all the eigenvalues at the origin. This is due to the double integrator nature of the system and the fact that the aerodynamic dissipation forces are quadratic to the velocity which becomes null at the steady state. This explains the high degree of instability of such systems.

Remark 2. Notice that the pair (A, B) is controllable, then a linear state feedback ($u = -Kx$) would enforce a desired closed-loop system stability and performance at the operation state x_o [3].

Remark 3. The product $CAB = M^{-1}B_e \in \Re^{6\times 4}$ is column full rank constant matrix, and column full rank matrix elsewhere: $CA(x)B = J_v^{-1}(x_1)M^{-1}B_e \in \Re^{6\times 4}$.

4.1 Feedback Linearization

Stability of the equilibrium point x_o is only local and valid only in its very narrow neighborhood. When the dynamic model deviates or the system is subject to bounded unmodeled dynamics or bounded disturbances. To cope with that an auxiliary feedback control is commonly proposed [2],

$$u = -Kx + v \tag{14}$$

where K is computed via LQR feedback scheme and v is an additional auxiliary control input. Then, the linearized close-loop system can be written as

$$\dot{x} = [A - BK]\, x + Bv \tag{15}$$

$$\bar{y} = \bar{C}x \tag{16}$$

where $\bar{y}$ is only a part of the originally output ($y = q$), defined, as the Cartesian position and heading only, excluding the roll and pitch attitude angles: $\bar{y} \triangleq (x, y, z, \psi_z)^T$. The output matrix $\bar{C} = [C_1\ 0] \in \Re^{4\times 12}$ with $C_1 \in \Re^{4\times 6}$ has raw full rank. Notice that $\bar{C}B = [0] \in \Re^{4\times 4}$ still holds, consequently the first and second time derivatives of the new output become

$$\dot{\bar{y}} = \bar{C}Ax \tag{17}$$

$$\ddot{\bar{y}} = \bar{C}A\,[A - BK]\, x + \bar{C}ABv \tag{18}$$

Remark 4. Matrix $\bar{C}AB = C_1 M^{-1} B_e \in \Re^{4\times 4}$ is full-rank invertible matrix, thus stable zero dynamics arise, that is the roll and pitch attitude angles are stable, [7].

The Feedback Linearization controller (FL), issued from eq. (18) would have the form

$$v = \left[\bar{C}AB\right]^{-1}\left(\bar{v} - \bar{C}A\left[A - \mathbf{BK}\right]x\right), \tag{19}$$

yielding to a closed-loop system $\ddot{\bar{y}} = \bar{v}$, as reported in [2]. However this is rather awkward since the LQR state feedback ($-Kx$) is canceled in (14) by this second loop. Since it is preferable to maintain an optimal stabilizable regulator such as the LQR in the control loop a Partial Feedback Linearization (PFL) is proposed as:

$$v = \left[\bar{C}AB\right]^{-1}\left(\bar{v} - \bar{C}A^2 x\right) \tag{20}$$

which delivers a second order coupled linearized close-loop system

$$\ddot{\bar{y}} = \bar{v} - \bar{C}ABKx \tag{21}$$

Notice that dynamics $-\bar{C}ABKx$ represents the a residual coupled dynamics introduced by the *optimal* LQR regulator and because of the underactuated nature of this system.

4.2 Sliding-Mode Control

Let $\Delta\bar{y} = \bar{y}_d - \bar{y}$ be the output tracking error, where y_d is the desired output signal, and choosing the new second order sliding-mode control law $\bar{v}$ given by

$$\bar{v} \triangleq \ddot{\bar{y}}_d - \alpha\Delta\dot{\bar{y}} + \beta s_0 e^{-\beta t} - K_i tanh(\sigma s_q) - K_d s_r \tag{22}$$

for large enough gains K_d, K_i and small error on initial conditions, with $s_r = s_q + K_i \int \mathrm{sgn}(s_q)$, $s_q = s - s_d$, $s = \Delta\dot{\bar{y}} + \alpha\Delta\bar{y}$ and $s_d = s_0 e^{-\beta t}$, $s_0 = s(t_0)$. The function $tanh(*)$ stands for a the sigmoid hyperbolic tangent function with $\sigma > 0$, not necessarily large. Then, the complete control law is given by

$$u = \left[\bar{C}AB\right]^{-1}\left[\ddot{\bar{y}}_d - \alpha\Delta\dot{\bar{y}} + \beta s_0 e^{-\beta t} - K_i tanh(\sigma s_q) - K_d s_r - \bar{C}A^2 x\right] - Kx \tag{23}$$

Substituting (23) into (9) yields

$$\dot{s}_r = -K_d s_r - \bar{C}ABKx - K_i Z \tag{24}$$

for bounded $Z = tanh(\sigma s_q) - sgn(s_q)$. Finally, we can state the main result.

Theorem 1. *Consider (23) into (9), then the closed loop (24) gives rise to robust exponentially stable dynamics of tracking errors, under a chattering-free second order sliding modes for all time, with stable zero dynamics.*

Proof. It follows closely [11], **QED**.

Remark 5. The state feedback stabilize locally the operation point, decouples the close-loop dynamics of the lateral, longitudinal, vertical, and heading navigation and preserves stability of the zero dynamics. Additionally, the auxiliary control input enables a wider operational region by adding robustness to the overall closed loop control.

5 Results

Consider the nonlinear model of an X-cell90 R/C helicopter. The linear model is computed, for simulation simplification, at the operating point $x_o = (0,0,0,0,0,0)^T$. In Table 1 initial conditions and gain tuning for the output feedback sliding mode are shown. For comparison purposed, simulation using Matlab are also performed commuting the auxiliary control (14) for a properly tuned PD control.

Table 1. Initial conditions and tuning gains for the sliding-mode control.

Initial conditions & SMC-Gains				
	x	y	z	ψ
q_0	-2.1	1.05	0.11	0
$\dot{q}_0$	0	0	0	0
α	3.15	3.15	4.5	15
β	1	1	1	1
k_d	15.6	15.6	56.16	234
k_i	3.51	3.51	27.8	52.65

Figure 2 shows the 3D trajectory and the tracking error of both position and attitude for the helicopter when the control law is the two servo-loop, similar to the one presented in [2] (FL-PD), consisting in a Feedback Linearization (which also cancels de LQR inner loop) and a PD controller. As it can be seen this PD controller cannot reject constant disturbances as gravity. Figure 3 shows the same trajectory tracking with the proposed Sliding-Mode robust controller in the place of the PD above (FL-SM). This controller consist in a Feedback Linearization and a second order Sliding-Mode output feedback. It can be seen a good performance on the desired position tracking, including the heading (yaw angle), even in the presence of random disturbance forces (for gust of winds). The roll and pitch angles, which define the zero dynamics, are stable, which is in accordance with the feedback linearization design. Figure 4 shows also the trajectory tracking as in the previous Figures. The difference here is that in this case the middle loop does not cancel the LQR inner loop, and the residual dynamics are coped by the outer second order Sliding-Mode loop. This controller, given by (23), is called in this work as LQR-PFL-SM. Evident differences in the performance of the FL-PD and the FL-SM can be seen mainly because the PD cannot overcome constant disturbances as the gravity effect. Small differences between the FL-SM scheme and LQR-PFL-SM one can be seen at the magnitude level of the Cartesian position tracking error where are smaller in the second, because the Sliding mode acts since the initial conditions, tracking almost perfectly the desired trajectory. In attitude there are no significative differences founded.

6 Conclusions

Control of autonomous helicopters in the presence of environmental and system uncertainties is a challenging task. These uncertainties not only modify the dynamics be-

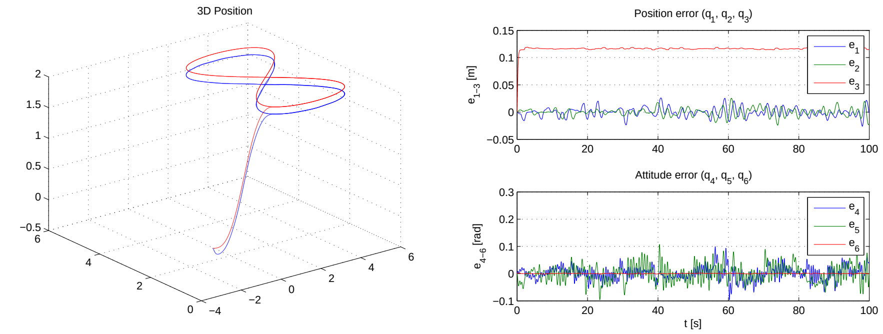

Fig. 2. Space position trajectory tracking in 3D and pose tracking errors for a FL-PD control law.

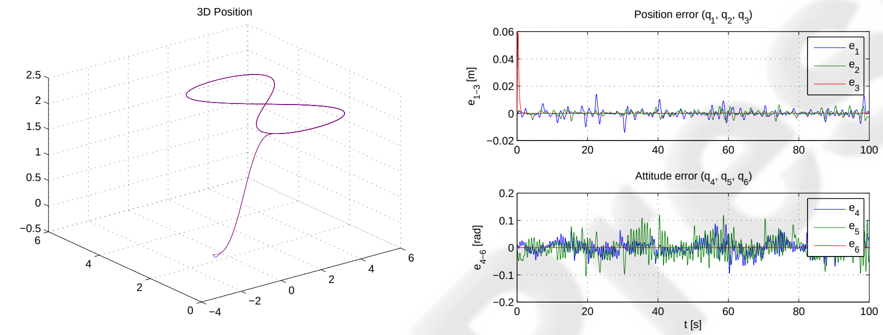

Fig. 3. Space position trajectory tracking in 3D and pose tracking errors for the FL-SM control law.

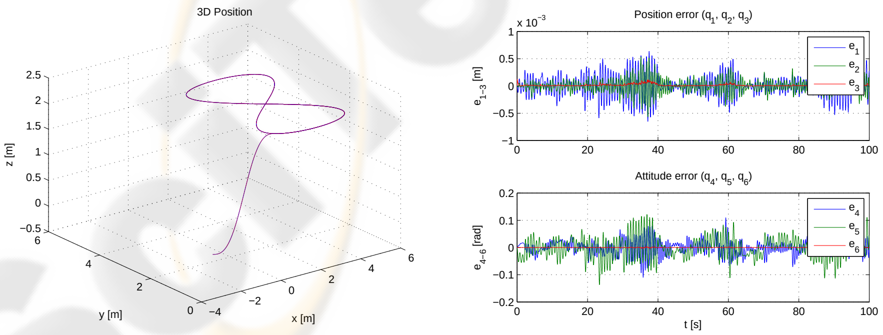

Fig. 4. Space position trajectory tracking in 3D and pose tracking errors for LQR-PFL-SM control law.

havior of the system, but also the trim inputs themselves. What is therefore needed is a viable controller capable of simultaneously accommodating all coupling features, parametric uncertainties, and trim errors. State representation is necessary to perform both tangent linearization for the design of an ideal Optimal stable State Feedback and Partial Feedback Linearization for output decoupling and underaction restrictions. The underactuated nature and the use of some part of the Feedback Linearization control induce undesirable residual dynamics. A second order model-free Sliding-Mode is used to guarantee robust regulation, while preserving zero dynamic stability. Representative simulations provide appreciation of the validity of the proposed approach.

References

1. Avila Vilchis, J.C.: *Modélisation et Commande d'Hélicoptère*, PhD dissertation, Laboratoire d'Automatique de Grenoble (LAG), Septembre 2001, Institut National Polytechnique de Grenoble (INPG), France
2. Bergerman, Marcel; Amidi, Omead; Miller, James Ryan; Vallidis, Nicholas & Dudek, Todd: *Cascaded Position and Heading Control of a Robotic Helicopter*, Proceedings of the 2007 IEEE/RSJ International Conference on Intelligent Robots and Systems, San Diego, CA, USA, Oct 29 - Nov 2, 2007.
3. Chen, Chi-Tsong: *Linear System Theory and Design*, Third Edition, 1999, ISBN:0-19-511777-8.
4. Dzul, A.; Lozano, R. & Castillo: *Adaptative Altitude Control for a Small Helicopter in Vertical Flying Stand*, Proc. of IEEE Conference on Decision and Control, Maui, Dec. 2003.
5. Fossen, Thor Inge: *Guidance and Control of Ocean Vehicles*, Chichester, John Wiley and Sons Ltd, University of Trondheim, Norway, 1994.
6. Gavrilets, V.; Mettler, B. & Feron, E.: *Nonlinear model for a small-size acrobatic helicopter*, in Proceedings of AIAA Guidance Navigation and Control Conference, Montreal, Quebec, Canada, August 2001.
7. Isidori, Alberto: *Nonlinear Control Systems*, Springer-Verlag, London 2003, 3nd ed. ISBN: 3540199160
8. Lin, Feng: *Optimal Control Design: An Optimal Control Approach*, John Wiley & Sons, Ltd, 2007, ISBN:978-0-470-03191-9.
9. Mahony, R.; Hamel, T. & Dzul, A.: *Hover Control via Lyapunov Control for an Autonomous Model Helicopter*, Proc. of IEEE Conference on Decision and Control, Phoenix, Dec. 1999.
10. Olguín-Díaz, Ernesto; Rubio, Luis; Mendoza, Alejandro; Gómez, Nestor & Benes, Bedrich: *Animacin Virtual en 3D de un helicptero de R/C a escala*, 3ra Conferencia Iberoamericana en Sistemas, Ciberntica e Informtica (CISCI 2004). Orlando, FL, July 2004.
11. V. Parra-Vega, S. Arimoto, Y.H. Liu, G. Hirzinger, and P. Akella: *Dynamic sliding PID control for tracking of robot manipulators: Theory and experiments*, IEEE Transaction on Robotics and Automation, Vol. 19, No. 6, pages 976 985. December 2003.
12. Shim, D.H.; Koo, T.J.; Hoffmann, F. & Sastry, S.: *Comprehensive Study of Control Design for an Autonomous Helicopter*, Proc. of IEEE Conference on Decision and Control, Tampa. 1998.
13. Spong, M. W.: *Underactuated Mechanical Systems*, in Control Problems in Robotics and Automation, B. Siciliano and K. P. Valavanis Eds., LNCIS, vol. 230, pp. 135–150, Springer Verlag, London, 1998. http://citeseer.ist.psu.edu/spong98underactuated.html
14. Vite-Medécigo, S. & Olguín-Díaz, Ernesto: *Robust Tracking and Disturbances Rejection with Robust LQR via BDU for a R/C Helicopter*, CIIIEE 2008 - ITA, November 2008, ISBN:978-607-95060-1-8