ASSORTMENT OF SOLUTIONS FOR VARIABLE TASKS IN MULTI-OBJECTIVE PROBLEMS

Gideon Avigad, Erella Eisenstadt and Uri Ben Hanan
Ort Braude College of Engineering, Karmiel, Israel

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Abstract: In the same manner that species are associated with variants in order to survive, and that human communities, apparently in order to survive, are built up from people with different skills and professions, we suggest in this paper to select a set of diverse solutions in order to optimally solve Multi-Objective Problems (MOPs). As a set, the solutions may cover a wider range of capabilities within the multi-objective space than is possible for an individual member of the set. The diversity within the set is a key issue of this paper and hereinafter designated as an *assortment*. In the paper, we suggest a computational tool that supports the selection of such an assortment. The selection is posed as an auxiliary MOP of cost versus variability. The cost is directly related to the size of the assortment, whereas the variability is related to the ability of the assortment to cover the objective space. A previously treated problem is adopted and utilized in order to explain and demonstrate the approach.

1 INTRODUCTION

The use of Evolutionary Multi-objective Optimization, (EMO) is a popular approach for searching for solutions to MOPs (Multi Objective Problems). Commonly when the objectives of a MOP are contradicting the solution to the MOP is the Pareto set. The development of Pareto-based evolutionary algorithms has been initiated by the procedure suggested by Goldberg, (1989). Surveys and descriptions of EMO algorithms can be found in several references (e.g., Deb, 2001).

Selecting a solution out of a Pareto set is commonly based on the designers' preferences. Choosing a set of solutions to MOPs instead of selecting a single solution is relatively a new area of research.

Recently a new approach to select conceptual solutions has been investigated (e.g., Mattson, and Messac, 2005). It involves Set-Based Concept (SBC) representation in which a concept is associated with the performances of multiple solutions. When dealing with SBCs, each of the solutions (design alternatives), of the SBC is assumed to be associated with a point in the objective space, representing its performances. Therefore the concept performances can be evaluated based on a cluster of points in the objective space, where each of the points of the cluster is associated with the performances of at least one of the design alternatives (solutions) of the SBC. According to that approach, each concept has its related front. The global front, which is the non dominated set over all the objective space, is the s-Pareto (Mattson, and Messac, 2005). An approach for choosing a concept, (a set) which has representatives on the s-Pareto, has been suggested in Mattson and Messac, (2005). There, it has been assumed that, the more representatives a concept has on the s-Pareto, the more flexible it is in corresponding to uncertainties. Avigad and Moshaiov (2009) have highlighted some pitfalls of considering just the s-Pareto and suggested an auxiliary MOP of optimality versus variability to compare between the concepts, based on their entire individual Pareto fronts.

Apart from selecting a set, the evolution of sets has also been considered. For instance, there are studies that use set domination to search for the best approximation of the Pareto front (e.g., the Indicator-based Evolutionary Algorithm, -IBEA of Zitzler and Künzli, (2004). Such a search is based on assigning a value to the degree of domination between sets of competing approximations of the Pareto set. For example, such an assignment is performed using the binary additive indictor, which

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was introduced by Zitzler et al. (2003). The binary additive-indicator of two Pareto set approximations is equal to the minimum distance among the dimensions of the objective space by which one Pareto set approximation needs to move or can be moved such that the compared approximation is weakly dominated by it.

Within the context of this paper, it is important to consider another assessment measure that allows a comparison between two sets. This measure is the hyper-volume measure or *S* metric, which has been proposed by Zitzler and Thiele, (1998), who called it the *'size of the space covered'* or the *'size of dominated space*.' Van Veldhuizen and Lamont (2000), described it as the Lévesque measure of the union of hyper-cubes defined by a non-dominated point and a reference point. When engineering design is considered, the reference point might be related to specified boundaries in the objective space, namely within a *'region-of-interest'*. According to Mattson and Messac (2005), in order to define a region-of-interest, the designer should specify a single point in the objective space.

It is noted that choosing a set might also be related to the notion of community of robots. In that case, the aim is to find a set of robots which are communicating in order to perform a task or tasks. The idea might be related to swarms (see http://www.swarm-robotics.org) or to a multi-agent design (e.g., Bensaid, and Matheieu, 1998).

In contrast to previous studies, which utilize a set of robots in order to execute an aggregated mission, the current paper suggests choosing a set such that its members do not always participate in the mission, but are rather "called for" based on the mission at hand. To elucidate the problem that will be attended by the current paper, refer to the following illustrative example: Suppose that robotic platforms are operating in a multi-task environment. This means that sometimes a fast action is needed and a robot should quickly move from one place to the other. In another scenario heavy loads should be transferred by a robot from one place to the other. It is clear that if optimality is considered, ideally the two different tasks should be performed by different robots. Choosing the optimal variety of robots (solutions) is the scope of the current paper.

The paper is organized as follows. The next section lays out the background on which we rely in order to introduce the suggested approach. Section 3 describes the motivation for this paper, having its origins in biology, sociology, and engineering design. Section 4 is the methodology, where the problem, its solutions, and the search approach are explained and formulated. In Section 5, an example is utilized in order to demonstrate the applicability of the approach in choosing an optimal assortment.

2 BACKGROUND

Recently in Avigad et al. (2009), we have introduced a new problem within the context of MOPs. The problem treated in Avigad et al. (2009), although defined as a common uncertain MOP, differs inherently from that problem. This difference influences both the search procedure as well as the multi criteria decision making. In the following, a brief outline of the problem and its solution as given in Avigad et al. (2009) are described. Consider the following MOP:

$$\underset{(x)}{\text{Minimize}}\, F(x,d) \tag{1}$$

where $F(x,d)=[f_1(x,d),f_2(x,d),.....f_K(x,d)]^T\,;K\geq 2$

$$x \in \Omega \subseteq R^n,\; x=[x_1,x_2,....x_n]^T$$

$$d \in \Gamma, \subseteq R^m,\; d=[d_1,d_2,....d_m]^T$$

$$x_i^{(L)} \leq x_i \leq x_i^{(U)} \text{ and } d_j^{(L)} \leq d_j \leq d_j^{(U)}$$

where Ω is the design parameters space (parameters that are to be chosen) and Γ is the model's environmental parameters space (which are not chosen but might be uncertain). x is a solution $x=[x_1,..x_i,...x_n]^T$, $x \in \Omega \subset R^n$ where $x_i^{(L)} \leq x_i \leq x_i^{(U)}$ and R^n is the design space (controlled parameter space). The interval given for x is commonly related to an uncertainty, which is related to the realization of an exact value of x. Factors such as machine precision are the basis for the interval, which is in fact a tolerance given for each design parameter. In Avigad et al. (2009), the origin of the interval is fundamentally different. The interval is a span of possible values associated with choosing a specific parameter. For example choosing a specific motor (with no uncertainty) is like choosing a span of output torques (as well as weight, size, etc.). Each of the possible values for all design parameters is performing within an environmental situation $d \in \Gamma \subseteq R^m$ $d=[d_1,...d_j....d_m]^T$ such that $d_j^{(L)} \leq d_j \leq d_j^{(U)}$ and R^m are the environmental space (uncontrolled parameter space). If $S_x \subseteq \Omega\times\Gamma$ then $s^x \in S_x$ is a scenario of x (which is a vector in $R^{n\times m}$). A scenario's vector of performances in a K objective space is $y_{s^x} = F(s^x)$ where $F(s^x) = [F_1(s^x), F_2(s^x), ...F_K(s^x)]^T$

The corresponding set of all the scenarios' performances of the solution x is designated as: Y_x, $Y_x \subseteq T \subseteq R^K$. This means that a solution is represented by a cluster of points (each representing a vector) in objective space. The cluster of possible scenarios in the work of Avigad et al. (2009), is built of scenarios that are related to the same solution whereas in the uncertainty MOP case, each scenario is a different realized solution. So if all are possible scenarios, then a comparison between the solutions should be based on the best. In a multi objective space, the best might be a set of best scenarios. The set of best scenarios of a solution x, RS^x and related front RSF^x has been defined in Avigad et al. (2009) as follows:

$$RS^x := \{s^x \in S_x \,|\, \neg\exists\, s^{x'} \in S_x : F(s^{x'}) \preceq F(s^x)\}$$
$$RSF^x := \{y_{s^x} \in \Gamma \,|\, y_{s^x} = F(s^x) : s^x \in RS^x\} \quad (2)$$

The set of optimal solutions P* and their representation in objective space, the Pareto Layer, PL, are defined as follows:

$$P^* := \{x \subseteq \Omega \,|\, \neg\exists\, RS^{x'} : RS^{x'} \preceq RS^x\}$$
$$PL := \{F(s^x) \in RSF^x \,|\, x \in P^*\} \quad (3)$$

The PL is associated with sets of representative sets, each related to a solution. This front is not a clear-cut front but rather a cloud of scenarios' performances and therefore, it has been termed in Avigad et al. (2009) as the Pareto layer (PL). Such a PL possesses solution scenarios' performances that are dominated by the performances of other solutions' scenario's performances. Nevertheless, the representative sets of the optimal solutions do not dominate each other.

In Avigad et al. (2009), an MOEA (Multi Objective Optimization Algorithm), which applies a search for finding the PL, has been suggested and investigated.

The current paper deals with selecting sub-sets of the Pareto layer set, based on the motivation explained in the following section.

3 MOTIVATION

This paper approach is motivated by the apparent diversity within species in nature and by the diversity of professions and expertise within human societies. The genetic diversity carried in natural populations is a key factor in evolution (e.g., Mayr 1982) and is one of the fundamentals of what is termed as *modern evolutionary synthesis*. The importance of population diversity is highlighted in many nature related studies (e.g., Booy, G. et al 2000). According to Booy, G. et al 2000, "Such genetic variation within a population may allow species to change over time and thereby survive changing environmental conditions." According to Boer et al. 1993, "… a population can only achieve its adaptability by distribution of the variation across its individuals". The above citation clearly implies the importance of variability within a species. The above biologically related differences are associated with diversity in the genotype. A question that needs to be investigated involves phenotypic differences. Naturally, the dissimilarity of human faces is one example. However, here we are more interested in the behavior aspects. A clear dissimilarity between individuals within a human community is the existence of different trades, such that different people within the community are experts in different fields of knowledge (e.g., a medic, a coal miner and a fisherman are trades commonly practiced by different people).

When optimality is considered, this is somewhat comparable to the fact that there is no world heavy weight champion winning a 100m run against Usain Bolt (the current world champion). These two extremes (strong and fast) are not the only cases. A decathlon athlete should possess characteristics that will allow him competing both in speed and strength. Neither the decathlon athlete, the runner, nor the heavy lifter athletes, is superior to each other if the bi-objective space of speed versus strength is considered. This issue is the base for choosing the members of an Olympic team. Instead of choosing one solution (a single superstar that performs reasonably in all Olympic professions), a set of solutions (several athletes, each expert at his own field of profession) are selected. Thus a multi-objective problem of optimizing all objectives (i.e., running the fastest, lifting the heaviest, etc.) is solved through using a set of solutions (athletes).

Motivated by the apparent importance of diversity in species and human communities, we suggest searching for a set of diversified engineering solutions such that they may optimally comply with their set related tasks.

Before going on to present the methodology, we would like to note the following two remarks.

1. Choosing a name for the set of solutions within the context of this paper was not an easy task. Community, group, team, unit, and other names were considered. The main drawback of all of these notions is the inherent interaction between their members. The definition for an assortment seems to best fit the idea. According to en.wiktionary.org/wiki/assortment an assortment is "a collection of varying but related items." No interaction is reminded in that definition, which is fine, with the relation possibly being interpreted here as the relation to the same objective space.

2. The current paper is bound to the ideas presented in Avigad et al. (2000). Here a solution may have a span of possible performance vectors rather than a single performance vector. Consider a 100 meters runner that runs slower than his/her best, or would run slower if s/he carries a load (or if s/he runs uphill). This is comparable to a design of a cart to move as fast as possible and to carry the highest loads. Carrying heavier loads means moving slower and vice versa. We could think of choosing a set of cars that could carry as high as possible loads and move as fast as possible or on a set of telescopic arms that should carry high as possible loads to most distant horizontal locations. In these cases there is a fundamental contradiction in the objective space. However, more importantly, each solution has a span of possible performances. This is why the methodology and the example are built upon the Pareto Layer notion.

4 METHODOLOGY

4.1 An Assortment of Solutions

An assortment A_s is a sub-set of all possible solutions $A_s \subseteq \Omega$, $A_s = \bigcup_{i=1}^{n_s} x^i$ where $n_s = |A_s|$.

Notes:

1. *An assortment might possess a single member.*
2. *A solution might be a member of more than one assortment.*
3. *An assortment might possess identical members.*

The performance of an assortment, Y_s is represented in the problem objective space by the union of the representative sets of the assortment's members, $Y_s = \bigcup_{i=1}^{n_s} RSF^i$ where RSF^i is the representative-set related front (see Section 2) of the i-th member of the assortment.

In the current paper we shall follow some assumptions, which are given and explained here.

1: Following the motivation for optimality, we only consider solutions that belong to the Pareto set as candidates for members in an assortment.

2: The boundaries of the performances within the objective (task) space are known beforehand. In other words, the task WOI is given *a priori* to the design process. For example, it is assumed that the maximal carried load is known.

3: The maximal cost involved with the assortment is known. This might be based on costs of manufacturing and transportation, among others.

Based on the above assumptions, the assortment set and related performances are:

$$A_s \subseteq P^* \text{ and } Y_s \subseteq PL$$

In the current paper, we assume that the Pareto set and related Pareto Layer are given.

4.2 The Competency of an Assortment

It is suggested here that comparing and selecting an assortment out of all possible assortments is carried out by considering their performances. The performances of an assortment is termed here as the competency of the assortment. There might be several measures used to assess this competency. In the current paper, we consider just two: The first is a straight forward one, the cost of the assortment. As the number of members within an assortment increase, so does its cost. The cost of an assortment is the sum of the individual members' costs:

$$Cost(A_s) = \sum_{i=1}^{n_s} Cost(x_s^i) \tag{4}$$

The second measure is the variability of the assortment, V_{ss}. It is a measure of the capability of the assortment to cover the objective space. The issue of variability has been extensively treated in Avigad and Moshaiov, (2009). Here it is the hyper-volume rendered by the community related scenarios as formulized for an assortment A_s as follows: Let F_A be the union of all representative sets of an assortment such that: $F_A = \bigcup_{i=1}^{n_s} RS^i$. Thus the variability measure may be defined as:

$$V_{ss} = \frac{\bigcup_i^{|F_A|} (HV_i^{s^s} - \bigcap_{j=1}^{i-1} (HV_i^{s^x})(HV_j^{s^x}))}{\prod_{k=1}^{K} y_{WOI}^k} \tag{5}$$

where $HV_i^{s^x}$ is the hyper-volume measure of the i-th scenario belonging to the set F_A. To elucidate the measure, refer to Figure 1. The figure depicts the representative sets of three solutions within a WOI (designated by dashed lines). The size of the grey area in the figure is the variability measure. As it grows, the assortment may comply with more tasks within the WOI.

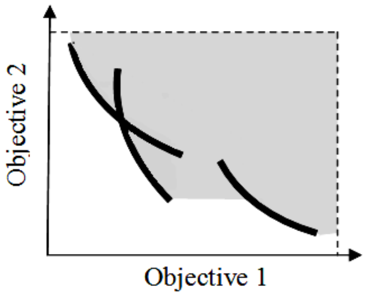

Figure 1: Assortment related variability.

The two measures explained above are mapping an assortment, which is represented by its related representative sets' fronts in the problem objective space, into an auxiliary space, which represents the competency of the assortment. The competency of an assortment is a point in the auxiliary space representing the cost associated with the assortment and its variability. Therefore, the problem of comparing and selecting between communities is transformed from the original objective space where each assortment is represented by a sub-Pareto layer into the auxiliary objective space where each assortment is represented by a single competency point of variability and cost.

4.3 Problem Definition

The problem of finding a sub-set of a Pareto set (an assortment) is defined as follows:

Find $A_s \subseteq \Omega$, In order to

$$\max_{A_s}(\Psi(A_s)) \quad , \; \Psi(A_s) = (V_{ss}, \frac{1}{Cost_{ss}}) \tag{6}$$

Following assumption 1 in Section 4.1, the problem of Equation 6 may be restated as a search for a sub-set of the Pareto set:

$$\text{Find } A_s \subseteq P^*, \tag{7}$$

In order to $\max_{A_s}(V_{ss}, \frac{1}{|Cost_{ss}|})$ s.t:

$\forall x \in A_s \exists s^x \in RS_x \mid y_{WOI} \preceq F(s^x)$

Observing Figure 1, it may be understood that as the size of the assortment grows, its variability also grows. This means that the MOP that is defined in Equation 7 involves contradicting objectives. Therefore the solution, (which is defined in Section 4.4) may involve a Pareto front within the auxiliary MOP of variability versus cost.

4.4 Problem Solution

The solution to the problem, which has been defined by Equation 7, is an assortment A_s* and related competency Pareto front in the auxiliary MOP, FC*:

$$\begin{aligned} &A_s^* := \{A_s \subseteq P^* \mid \neg\exists\, A'_s : \Psi(A'_s) \preceq \Psi(A_s)\} \\ &FC^* := \{Z^* \in Z \mid Z^* = \Psi(A_s) : A_s \equiv A_s^*\} \end{aligned} \tag{8}$$

To elucidate the notions of Equation 8, refer to Figure 2. In order to simplify the example, suppose that the cost is the number of members in an assortment.

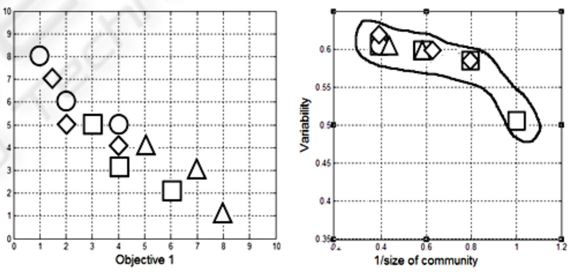

Figure 2: The PL and the optimal assortments.

4.5 Auxiliary MOP's Boundary Solutions

The boundary solutions of the problem of Equation 8 are; a single solution with the maximal hyper volume on one side and a set of all the solutions, which have the scenario/s' performances on global Pareto front on the other side. This might be seen by inspecting Figure 3, in which a PL in a bi-objective space is depicted.

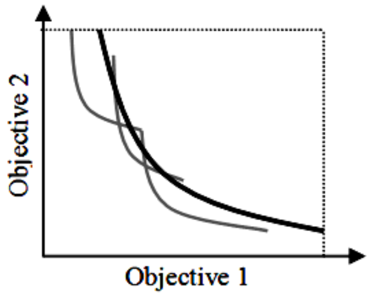

Figure 3: Four PL's solutions RSFs.

The PL is associated with four RSF's of the solutions. It can be seen that the black related RSF possesses the highest variability (biggest hyper volume) when just one member for the assortment is sorted. Nevertheless, the highest possible variability would be if all three grey RSFs are combined, unfortunately at the expanse of cost (three members). The question is how to find the solutions which are not the boundary solutions. Here we suggest using EMO for that search.

4.6 The Evolutionary Search

In this paper, it is assumed that the Pareto set and the related PL, are given. Therefore, in the current paper, the focus is on the search for sub-sets of the optimal solutions' set in order to comply with Equation 8. The evolutionary search involves a single chromosome integer value code for each individual. The length of the individual is predefined and is usually constrained by transportation volume or maximal cost boundary. Decoding an individual results in both the size of the assortment as well as which of the solutions (found by using the procedure of (Avigad et al., 2009)) are to be used for the assortment. For example, depict the coded individual of Figure 4.

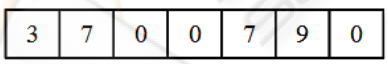

Figure 4: An Individual.

Decoding the individual of Figure 4, results in a four member assortment with solutions 3, 7 (twice) and 9 as its members. Any MOEA may be used for the evolutionary search. Here we have used the NSGA-II (Deb et al. 2002), which is given in the following with some added details that relate the algorithm to the current methodology:

Store the RSFs of all P solutions (see equations… and (*Avigad et al. 2000) *for details how to evolve them).*

1. *Initialize a population* P_t *with n individuals. create* $Q_t = P_t$
2. *Create a combined population* $R_t = P_t \cup Q_t$.
3. *Decode* R_t *and compute the competency of all assortments using equations 4 and 5.*
4. *Perform a non-dominated sorting for* Z^* *and find fronts,* Fr_i, $i=1,\ldots,n_r$ *where* n_r *is the number of fronts in a generation.*
5. *Initialize a new parent population* $P_{t+1} = \emptyset$. *Set a non-dominance level counter i=1. While* $|P_{t+1}| + |Fr_i| \leq n$, *include the i-th front in the new parent population:* $P_{t+1} = P_{t+1} + Fr_i$ *and set i=i+1.*
6. *Perform the Crowding Sort procedure (see (Deb et al., 2002)), and complete the filling of* P_{t+1} *with the most widely spread* $n - |P_{t+1}|$ *solutions using the Crowding Distance measure of (Deb et al., 2002).*
7. *Create offspring population* Q_{t+1}^{*} *from* P_{t+1} *by Tournament Selection.*
8. *Perform crossover to obtain* Q_{t+1}^{**} *from* Q_{t+1}^{*}.
9. *Perform mutation to obtain* Q_{t+1} *from* Q_{t+1}^{**}.
10. *If last generation Go to 12*
11. *Go to 2*
12. *Introduce the FC* (Equation 8) to decision makers.*

5 EXAMPLE

The example described in this section proceeds from Avigad et al. (2000). A cart of mass m(m') driven by a motor and gear with a mass of m' is to be designed. m(m'), meaning that as the chosen motor gets heavier, the carrying cart should be bigger and heavier in order to support the motor. The cart is carrying a load M as depicted in Figure 5.

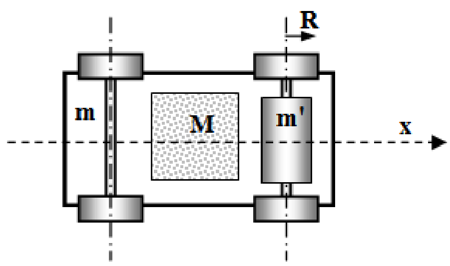

Figure 5: The cart.

Considering a movement of the cart along the x axis, and that the overall mass is m*=m(m')+m'+M, the following relaxed equation has been shortly developed in (Avigad et al., 2009)

$$\frac{T(m')}{R} - f_r(\dot{x})m^* g = 0 \quad , \qquad (9)$$

where T, is the driving moment beyond the transmission gear, which depends on the motor size/mass (i.e., the bigger the stronger) and may change such that: $0 \le T \le T_{max}$. R=0.05m is the wheel diameter, and f_r is the rolling resistance force that may be computed for inflation wheel-pressure of 30 psi, $f_r = 0.01 + 0.002(\dot{x})^{2.5}$. Let m(m')=mo+2*m' where mo=4kg. The following bi-objective problem, which maximizes the speed and carried load of the cart, i.e., $\max(\dot{x}, M)$ has been considered. T and m' are motor dependent and are taken from Pittman™ motor data. The resulting PL is depicted in Figure 6, which is borrowed from Avigad et al. (2000).

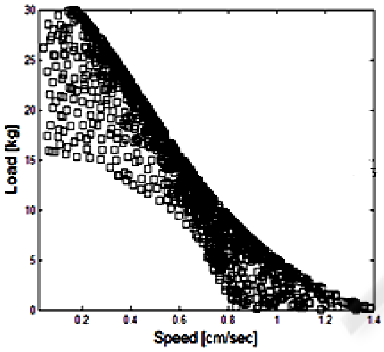

Figure 6: The Pareto Layer of the problem.

The algorithm, which was given in Section 4.6, has been utilized in order to search for the optimal assortments. A population of 100 individuals with 50%, 3% crossover and mutation rates respectively were used. The Pareto front of the auxiliary MOP is found and is depicted in Figure 7.

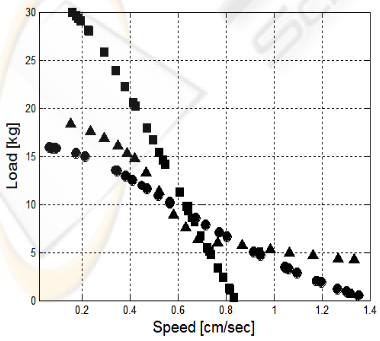

Figure 7: The most variable yet expensive assortment.

It is observed that there are three possible different assortments consisting of one, two or three members. The solution to equation 8 is depicted in Figure 8. In the figure, each square represents an optimal assortment. The leftmost square represents the three member assortment (all solutions of Figure 7). Although its cost is high compared to other assortments, its variability is the largest. This means that if all there carts are available more performance demands may be complied with. The middle square represents the two cart assortment with its medium competency. The rightmost square represents the single cart assortment, which has the least variability but with the least cost. Choosing one of the assortments is up to the DMs, who should consider their available resources, versus the gain of variability.

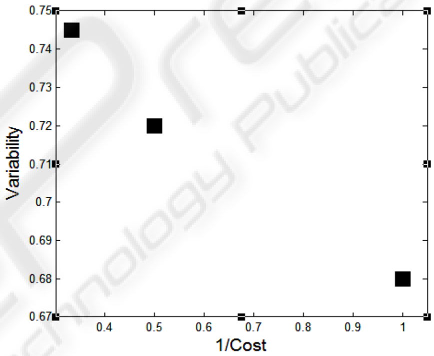

Figure 8: The auxiliary problem Pareto front.

The algorithm was run 50 times for the current problem. The statistical data is depicted in Figure 9.

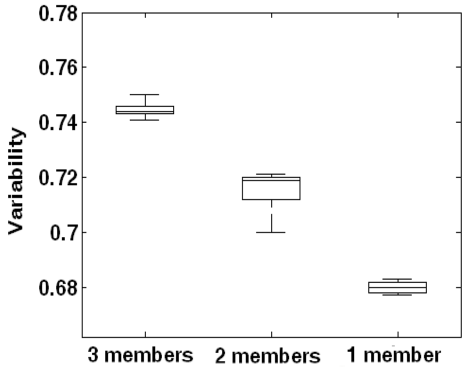

Figure 9: Statistical results for the cart problem.

The Figure depicts the spread of the resulting variability for the three assortments as related to their variability.

6 CONCLUSIONS

In the paper, we introduce the notion of an assortment, suggest an auxiliary MOP whose solution may aid decision makers in choosing an optimal assortment. Furthermore, an EMO to solve the auxiliary MOP is suggested. The paper contributions are: a. A new kind of a set (the assortment) is represented and motivated, b. A need to choose a set based on a set of sets has been encountered here for the first time by using EC, c. New motivation to correlate nature and sociology to engineering design has been suggested d. A new motivation for variability within engineering design has been highlighted, e. Yet another use of MOEA's has been explored.

Future work should consider searching for assortments based on the auxiliary MOP directly from the beginning without relying on an *a priori* search of the PL. Furthermore, some more examples and test cases should be explored. Among the investigated cases, problems with more objectives, both in the original and the auxiliary MOPs should be interesting. Finally, robustness, while choosing an assortment, should be an important issue. Gaining more robustness may call for a need for overlapping of RSFs, which may reduce variability.

REFERENCES

Avigad G., Eisenstadt, E., and Goldvard A., Report no Br.11709, at http://mech.braude.ac.il/gideonavigad/

Avigad, G., Moshaiov, A., Set-based Concept Selection in Multi-objective Problems: Optimality versus Variability Approach, Journal of Engineering Design, Vol. 20(3) pp: 217 – 242, 2009

Bensaid, N. and Matheieu, P. An Autonomous Agent System to Simulatea Set of Robots Exploring a Labyrinth, In the proceedings of 11th International FLAIRS Conference, pp: 384-388, Sanibel, Florida May, 17-20, 1998.

Boer, P., Szyszko, K., and Vermeulen, R. Spreading the risk of extinction by genetic diversity in populations of the carabid beetle, Netherlands Journal of Zoology, 43, 242-259, 1993.

Booy, G. et al., Genetic Diversity and the Survival of Populations. J. Planet Biology 2, 379-395, 2000.

Deb, K., Pratap, A., Agarwal, S., and Meyarivan, T. A Fast and elitist multiobjective genetic algorithm: NSGA–II, IEEE Transactions on Evolutionary Computation, 6(2):182–197, April 2002.

Deb K., Multi-objective optimization using evolutionary algorithms. J. Wiley & Sons, Ltd, 2001.

Goldberg, D.E., Genetic algorithms in search, optimization and machine learning, Addison-Wesley, 1989.

Marler, R.T., and Arora, J.S. Survey of multi-objective optimization methods for engineering, Structural Multidisciplinary Optimization, 26: 369–395, 2004.

Mattson, C. A., and Messac, A. Pareto frontier Based concept selection under uncertainty, with visualization, Optimization and Engineering, 6: 85–115, 2005.

Mayr E. 1982. The growth of biological thought: diversity, evolution and inheritance. Harvard, Cambs. p567 et seq.

Pareto, V. 1906: Manuale di Economica Politica, Societa Editrice Libraria. Milan; translated into English by A.S. Schwier as Manual of Political Economy, edited by A.S. Schwier and A.N. Page, 1971. New York: A.M. Kelley

Van Veldhuizen, D.A., and Lamont, G.B. Multiobjective evolutionary algorithms: analyzing the State-of-the-Art, Evolutionary Computation, 8(2): 125-147, 2000.

Zitzler, E., Thiele, L.: Multiobjective Optimization Using Evolutionary Algorithms—A Comparative Study. In Eiben, A.E., ed.: Parallel Problem Solving from Nature V, Amsterdam, Springer-Verlag 292–301, 1998.

Zitzler, E., L. Thiele, M. Laumanns, C. M. Fonseca, and V. G. da Fonseca (2003). Performance assessment of multiobjective optimizers: An analysis and review. IEEE Transactions on Evolutionary Computation 7(2), 117–132, 2003.

Zitzler, E., and K¨unzli, S. Indicator-Based Selection in Multiobjective Search, Proceedings of the 8th International Conference on Parallel Problem Solving from Nature (PPSN VIII) September 2004, Birmingham, UK