AN OPTIMAL SILVICULTURAL REGIME MODEL USING COMPETITIVE CO-EVOLUTIONARY GENETIC ALGORITHMS

Oliver Chikumbo
Scion, Private Bag 3020, Rotorua, New Zealand

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Abstract: A competitive co-evolutionary genetic algorithm was successfully employed to determine an optimal silvicultural regime for the South African *Pinus patula* Schl. Et Cham. The solution to the silvicultural regime included: initial planting density; frequency, timing and intensity of thinnings; final crop number; and rotation length. The growth dynamics for *P.patula* were estimated using dynamical models, the building blocks of the combined optimal control and parameter selection formulation, with a single objective function that was maximised for value production. The results were compared against a silvicultural regime determined using Pontryagin's Maximum Principle. Both the regimes were then compared against the recommended silvicultural regime determined from years of experimental trials. The genetic algorithms regime was superior to the other two.

1 INTRODUCTION

A plantation forest is an investment for goods (entities with a market value) and services (entities important to our livelihood with no market value yet). The trees that grow on it are harvested for their timber or retained for other values, which may be conflicting and incommensurable, such as carbon sequestration, biodiversity, water quality, water quantity, and so on. For example, since biomass increases with stand age, postponing harvesting to the age of biological maturity may result in the formation of a large carbon sink but delayed income for harvestable trees. Stand level information is vital for understanding the flow and extent of goods and services over a forest estate or region. To simultaneously satisfy conflicting and incommensurable values for a forest stand requires an understanding of forest stand dynamics and how responsive these dynamics are to management controls and different physical environmental characteristics.

This paper looks at a decision framework that determines a management control for optimising the value of a stand, which in this case only looks at maximising value for timber. The thinking behind this development is that if we can identify a decision framework that determines a management control for a single value, it may be less of an effort to extend the framework to include more values. The management control actions are time-dependent and include choosing the appropriate initial planting density, when to thin (timing), how much to thin (intensity), how often to thin (frequency), final crop number prior to clear-felling and rotation length. However, determining optimality for economic value for a forest stand is elusive, partly because of the difficulty in simulating and forecasting growth dynamics of a forest stand, price of harvested timber, and costs of haulage and harvesting. For instance, if a forest stand is harvested too soon then the price of timber fetched may not be enough to cover the costs of harvesting. On the other hand, if harvesting is delayed for too long then the mature trees may not be growing fast enough to justify their occupation of the land (Mesterston-Gibbons, 1995).

1.1 Methods Employed

Mathematical models and techniques are the default for simulation, forecasting and optimisation. Most invariably, the conveniency of models introduces a jumble of consequences i.e., the decision-making process based on the outcomes of simulation, forecasting and optimisation, may actually be

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simplified but with complications that may range from problem formulation to handling large computations. For an economic value of a forest stand there are two dominant dynamic trends that may be modelled, i.e., growth and economic dynamics. Both dynamics may be integrated into an optimisation formulation such that optimal initial stand density, thinning strategy (i.e. frequency, timing and intensity of thinning), final crop number and rotation length may be predicted. Due to the time-dependent and multi-stage nature of the optimisation problem, dynamic programming (Bellman, 1957) has been employed to the task where net present value of the harvested timber was maximised (Yin and Newman, 1995). Chen *et al.*, (1980) gave a generic formulation of this modelling approach and pointed out to its weaknesses that included inappropriate growth models and the pesky "curse of dimensionality", which meant an inability to do an exhaustive search. The same weaknesses were reiterated by Arthaud and Pelkki (1997), Arthaud and Warnell (1994), Filius and Dul (1992), Pelkki (1994), Pelkki and Arthaud (1997).

Another approach employed by many to maximise the economic value of a forest stand, involves determining silviculturally sound stand strategies using simulation models. A financial analysis based on net present value is then applied to the strategies. The strategy with the highest net present value is then chosen as the optimum (Kuboyama and Oka, 2000). Such an approach may mean sub-optimal outcomes because the silvicultural and financial decisions are made independently. It also severely limits the outcomes that may be simulated and there is no way of telling how close or far away the tried outcomes are from the "true" optimal outcome.

More recently, other analysts have tackled this forest stand problem using discrete-time dynamical models for simulating the growth dynamics and Pontryagin's Maximum Principle (PMP), for an exhaustive search, averting the curse of dimensionality (Chikumbo *et al.*, 1997, Chikumbo and Mareels, 2003). Discrete-time dynamical models, commonly used in systems engineering, are based on a common axiom that the current observation at time t is dependent on the previous observation at time t-$1$for a first-order model (Ljung 1987). This means that dynamical models are expressed in terms of orders of magnitude of their previous values. The choice for using discrete-time dynamical models in this type of formulation is that their mathematical structure has linear parameters that control shape and scale of a variable trend, making it easier to reflect change in the dynamics of the trend, where the parameters themselves are a function of control actions (Chikumbo *et al.*, 1999).

A combined optimal control and parameter selection formulation made it possible to estimate the initial stand density (one parameter), final crop number (another parameter), thinning strategy (optimal control) and the rotation length of a forest stand. Therefore, the problem was transformed into a terminal constraint problem by specifying the final crop number in the formulation. As a result the formulation became sensitive to terminal time, making it possible to determine the optimal rotation length. The terminal time was determine by incrementing the rotation length in small yearly steps and solving the problem, until ill-conditioning made it impossible to solve the problem. Therefore, determining the terminal time was never a case of the model being "clever" enough to know when to terminate a rotation, but rather sensitive to the terminal time obtained through a trial and error exercise identify the optimal rotation length.

In general, the task of designing and implementing algorithms for the solution of optimal control problems is a nontrivial one(Anderson and Moore, 1989; Michalewicz, 1999). This is because the optimal control problems are quite difficult to deal with numerically and therefore many dynamic optimisation programs available for general users are typically an offspring of static packages (Brooke *et al.*, 1988) or forward recursive heuristics (Chikumbo, 1996). Only recently are genetic algorithms being applied to optimal control problems in a systematic way (Michalewicz *et al.*, 1992).

1.2 Genetic Algorithms

In this paper the author ups the ante to solve the combined optimal control and parameter selection problem using a competitive co-evolutionary genetic algorithm (GA). Such an approach has many advantages in that, unlike PMP, which is calculus-based and therefore dependent on the restrictive requirements of continuity and derivative existence of functions, a GA uses payoff (objective function) information, making it robust, as in, a wider problem domain application (Goldberg 1989). Because derivatives are not a feature of a GA, the formulation lends itself to greater flexibility in defining the objective function or multiple objectives. The organisation of this paper is outlined here. Data description is brief and is followed by a section with the models that were used for the control design, *viz*., state equations. The optimisation criterion is then defined and the results of the GA optimisation presented and discussed in the context of the PMP results and currently recommended silvicultural strategies for the South African *P. patula*. In the final section conclusions are drawn and further work indicated.

2 DATA

The re-measurement data used for developing the discrete-time dynamical models came from *Pinus patula* Schl. et Cham., correlated curve trend spacing trials, Nelshoogte, Transvaal, South Africa. Detail of the spacing trials and how the data were prepared for model development may be found in Chikumbo and Mareels (1995). A theoretical economic aggregate was used in the formulation because of lack of data. The economic aggregate was based on stumpage, the maximum price that a competitive buyer is willing to pay for harvested timber, less the expected costs in harvesting and haulage. Theory has it that a forest plantation increases in stumpage value with time and follows a sigmoidal trend, its slope increasing up to an inflection point (Chikumbo, 1996). This was easily represented as a second-order dynamical model with an asymptotic limit of one. Using real world data will only change the two time constants and asymptotic limit of the second-order dynamical model, retaining the characteristic signature of the stumpage trend (Chikumbo and Mareels, 2003). The thinking therefore, is that the theoretical stumpage model will influence the control model in a similar way as a realistic one.

3 STATE EQUATIONS

Discrete-time dynamical models were used to represent the state of the forest stand (which is the system) and included, the stand basal area model (Chikumbo *et al*., 1999), average height function (Chikumbo, 1996) and the economic aggregate model (Chikumbo and Mareels, 2003). Application of dynamical models to forest growth is not new and has been demonstrated in the last 15 years (Chikumbo *et al*., 1992).

The optimisation was designed as a state space representation (Ljung, 1987) where the input was the management control action (to be estimated) that influenced the dynamics of the forest stand (represented by the state equations or dynamical models) with an output that was represented by a volume function (Chikumbo, 1996) as shown in Figure 1.

The state equations were as follows:

$$sph(t) = sph(t-1) - u(t) \tag{1}$$

$$sba(t) = a_1(sph(t-1)) \; sba(t-1) + \ldots b_1(sph(t-1)) \tag{2}$$

$$ht(t) = a_2(sph(t-1))\, ht(t-1) + \ldots \; b_2(sph(t-1))\, d_q(t-1) \tag{3}$$

$$stpge(t) = a_3 stpge(t-1) + a_4 stpge(t-2) \ldots + b_3(1 - a_3 - a_4) \tag{4}$$

$$a_1(y) = 0.93 + 0.01\, y - 0.047\, y^2 + 0.01\, y^3 \tag{5}$$

$$\begin{aligned} a_2(sph(t)) &= 0.782, \\ &\text{for } sph(t) \geq 1000 \ldots \text{ stems ha}^{-1} \\ &= 0.85, \\ &\text{for } 1000 > sph(t) \geq \ldots 400 \text{ stems ha}^{-1} \\ &= 0.913, \\ &\text{for } 400 > sph(t) \geq \ldots 124 \text{ stems ha}^{-1} \end{aligned} \tag{6}$$

$$a_3 = 1.566 \tag{7}$$

$$a_4 = -(a_3)^2/4 \tag{8}$$

$$b_1(y) = 2.32 + 4.24y - 0.0035y^2 \tag{9}$$

$$\begin{aligned} b_2(sph(t)) &= 0.19 + 0.03y, \\ &\text{for } sph(t) \geq \ldots 1000 \text{ stems ha}^{-1} \\ &= 0.095 + 0.05y, \\ &\text{for } 1000 > \ldots sph(t) \geq 400 \text{ stems ha}^{-1} \\ &= 0.035 + 0.1y, \text{ for } 400 > \ldots \\ &sph(t) \geq 124 \text{ stems ha}^{-1} \end{aligned} \tag{10}$$

$$b_3 = 1 \tag{11}$$

$$y = sph/1000 \text{ (for scaling purposes)} \tag{12}$$

where,

d_q = quadratic mean diameter in centimetres.

The parameters of equations (2) and (3), i.e., a_1, a_2, b_1 and b_2 were depended on the initial/residual stand density, making them responsive to changes in growth dynamics before and after thinning (Chikumbo *et al*., 1999).

Figure 1: State space representation of the forest stand model, where, t = time in years; sba = stand basal area in m^2ha^{-1}; ht = stand mean height in metres; $stpge$ = dimensionless stumpage; u = number of trees harvested in stems ha^{-1}; and sph = initial or residual stand density in stems ha^{-1}.

The stand volume function was conveniently developed to fit into a quadratic objective functional formulation and was as follows:

$$V(t) = 0.4047\, sba(t)\, ht(t) \tag{13}$$

where,

V = stand volume in m^3ha^{-1}.

4 PARAMETER SELECTION AND OPTIMAL CONTROL FORMULATION

Let $J_n(u)$ be defined as the maximum achievable total volume over n periods. Thus

$$J_n(u) = \max_{u(t)} \sum_{t=T-(n-1)}^{T} \frac{u(t)}{sph(t)}[0.4047 sba(t) ht(t)] \tag{14}$$

subject to the constraints, $t \geq T\text{-}(n\text{-}1)$ with upper and lower bounds on the control, $0 \leq u(t) \leq 200, \forall t \in [t_0, T]$. To bias this cost functional to maximize harvested volume and revenue $J_n(u)$ becomes:

$$J_n(u) = \max_{u(t)} \sum_{t=T-(n-1)}^{T} \frac{u(t)}{sph(t)}[0.4047 sba(t) ht(t)] \ldots \frac{sba(t)}{sph(t)} stpge(t) \tag{16}$$

For $n = 1$,

$$J_1(u) = \max_{u(T)} \frac{u(T)}{sph(T)}[0.4047 sba(T) \ldots ht(T)] \frac{sba(T)}{sph(T)} stpge(T) \tag{17}$$

that is, we have a single constrained static optimisation problem that can be solved, if $u(T) = sph(T)$, hence

$$J_1(u) = [0.4047 sba(T) ht(T)] \frac{sba(T)}{sph(T)} stpge(T).$$

Clearly at the final period, a total harvest has to be done. The importance of the harvest is still undetermined as this is a function of all previous control actions.

Consider $n = 2$ and

$$J_2(u) = \max_{u(T-1)} \frac{u(T-1)}{sph(T-1)}[0.407 sba(T-1) \ldots ht(T-1)] \frac{sba(T-1)}{sph(T-1)} stpge(T-1) + \ldots \frac{u(T)}{sph(T)}[0.4047 sba(T) ht(T)] \frac{sba(T)}{sph(T)} stpge(T) \tag{18}$$

and so on, until $n = T+1$ and the original problem has been solved. This is the method of dynamic programming, sometimes called *backwards induction*. Adding the parameter selection constraint for the initial planting density and final crop number,

$$900 \leq z(1) \leq 1900 \tag{19}$$

$$200 \leq z(2) \leq 300 \tag{20}$$

where,

$z(1)$ = initial planting density (in stems ha^{-1}), and

$z(2)$ = final crop number (also in stems ha^{-1}),

makes the computation cumbersome. However, the above problem was solved using PMP such that the first-order necessary condition for optimality would be satisfied because of the inclusion of the Hamiltonian in PMP (Dixon, 1972; Chikumbo and Mareels, 2003). Note that PMP and dynamic programming are essentially the same (Fan and Wang, 1964) although PMP achieves an exhaustive search by breaking the problem into a sequence of sub-problems that are approximated by a constrained nonlinear programming problem, solved by standard mathematical programming algorithms (Teo *et al*., 1989). In this case NLPQL (Schittkowski, 1985), a sequential quadratic programming algorithm for solving constrained nonlinear programming problems, was used to solve the combined optimal control and parameter selection problem. A set of explicit functions and their derivatives with respect to the state, control and parameters need to be supplied to solve the problem.

Note that there is no known method of determining the global maximum (or minimum) for the general nonlinear programming problem. Only when the objective function and the constraints satisfy certain properties, is the global optimum sometimes found (Michalewicz, 1999).

5 THE GENETIC ALGORITHM MODEL

Genetic algorithms (GA) are engineering techniques that mimic chromosomal metaphors and population dynamics for solving optimisation problems, in particular combinatorial ones. For the combined optimal control and parameter selection problem the chromosomal data structure (as shown in Figure 2) was designed in such a way that the first gene was a fixed age *0* for when the planting occurs. The second, third and fourth genes were age ranges in years, *6-8*, *12-15*, and *18-20*, respectively for determining the timing of thinnings. The fifth gene was the terminal time for the rotation length with a range of *25-44* years. The sixth gene was the initial planting density representing *z(1)* as in equation (19). The seventh, eighth and ninth genes were the thinning intensities with ranges *0-300*, *0-200*, and *0-200* stems ha^{-1}, respectively. They corresponded to the age ranges in the second, third and fourth genes. The fitness function was the same as in equation (16). For the genetic operators to produce legitimate offspring at all times (for efficiency), specialized recombination and mutation operators were chosen that maintained the order of the genes. Therefore, the problem became a permutation one.

The crossover was a discrete integer-valued recombination that performed an exchange of the integer values between the chromosomes. For each position the parent chromosome that contributed its integer value to the offspring was chosen randomly with equal probability. The mutation process that randomly altered populations, involved taking the current population and producing the same number of randomly initialized integer valued chromosomes.

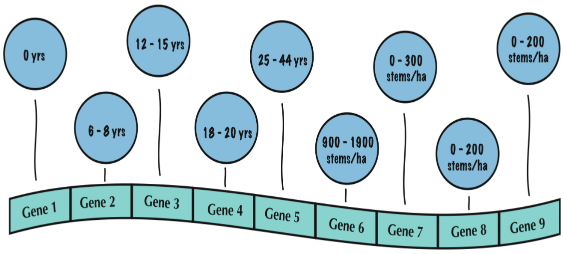

Figure 2: The chromosome structure used in the competitive co-evolutionary GA formulation.

To guard against premature convergence, 5 subpopulations of 100 individuals each were evolved independently in an unrestricted migration topology (where individuals/chromosomes may migrate from any subpopulation to another) that saw exchange (or migration) of genetic material at every10 generation-interval over a 1000 generation-period. This multi-population formulation is sometimes referred to as an island model (Levine, 1994) and happens to be a convenient model for parallel implementation, important for very large problems where computation time may be critical.

The selection process was all that differentiated the 5 subpopulations and all else kept the same. Three selection processes were used that is, stochastic universal sampling, tournament selection, and roulette wheel selection (Pohlheim, 2006). The roulette wheel selection did not seem to perform well and ultimately only two selection processes were utilised amongst the 5 subpopulations. Competition between the subpopulations was enabled such that the size of a subpopulation was made dependent on the current success of that subpopulation, hence the name, competitive co-evolutionary genetic algorithms. Successful subpopulations received more resources and less successful ones transferred resources to where they were most likely to generate the greatest benefit. The cycle of events in each generation is shown in Figure 3 although migration and competition selection were set to occur 10 generation-intervals.

Computation time is always an issue for some, but for this combined optimal control and parameter selection problem, it was solved under 3 minutes on an iMac 2.16 GHz Core 2 Duo with 2 GB SDRAM.

6 RESULTS AND DISCUSSION

The PMP algorithm provided a solution for a *44*-year rotation period with all the upper and lower

Figure 3: The structure of a competitive co-evolutionary genetic algorithm. The processes marked with an asterisk take place after every 10 generations.

control points set as follows: $0 \leq u(t) \leq 200, \forall t \in [0, 44]$. The turnaround time of the execution was under 3 minutes similar to the GA one. The upper and lower control bounds for the PMP formulation could not be replicated for the GA formulation without problems as it meant a numerous amount of "illegal chromosomes" being generated. To make such a formulation for the GA to work, where the chromosome is the control vector for the whole rotation, with the first gene being the initial planting density, would have meant including a "repair algorithm" that would have been costly in computation time. Also such a formulation would have made it hard to determine the optimal rotation length given that the chromosomes had a fixed length. Messy GAs (Goldberg 1993) can handle chromosomes of different lengths but are beyond the scope of this paper. They are able to evaluate the best building blocks from modestly sized populations of longer chromosomes, thereafter cutting down the string length by throwing off the genes of lesser importance. Design calculations are performed for population sizing, selection-deletion timing, and genetic thresholding. One could say this is where the strength of PMP really shows because of the availability of the derivatives of the dynamic system, the PMP will still behave, regardless of the upper and lower bounds on the control vector (within reason of course).

The GA being a heuristic needed a rethink in the design of the chromosome and from a 44-gene structure to a mere 9-gene chromosome produced remarkable results. It is evident here that although the PMP formulation is efficient (providing the functions of the dynamics of the system are smooth and differentiable to the second derivative), a cleverly designed chromosome can be equally efficient. Tables 1-3 show the silvicultural results from the PMP formulation, the recommended silvicultural regime from the Department of Forestry, South Africa (Kassier, 1991), and from the competitive co-evolutionary GA, respectively.

Table 1: The silvicultural regime derived from the PMP formulation.

AGE (years)	Standing trees (stems ha^{-1})	Control (stems ha^{-1} -$u(t)$)
0	1000	
21	600	(400)
33	200	(400)
44	0	(200)

Table 2: The recommended silvicultural regime, Dept., of Forestry, South Africa.

AGE (years)	Standing trees (stems ha^{-1})	Control (stems ha^{-1} -$u(t)$)
0	1372	
8	650	(722)
13	400	(250)
18	250	(150)
25+	0	(250)

Table 3: The silvicultural regime derived from the competitive co-evolutionary GA.

AGE (years)	Standing trees (stems ha^{-1})	Control (stems ha^{-1} -$u(t)$)
0	906	
6	624	(282)
15	454	(170)
20	252	(198)
31	0	(252)

Although the PMP silvicultural regime in Table 1 might be optimal, it somewhat raises questions as to whether it can be implemented successfully. Planting *1000* stems ha^{-1} and trying to retain them until the age of *21* years for a first thinning, creates competition, for light, water and soil nutrients that will ultimately lead to suppressed growth and mortality. In production forestry thinning is designed as a control action in advance of competition in order to reduce mortality. Also a final crop number of *200* stems ha^{-1} at a late age of *33* years maintained until age *44*, may mean less growth gains because the rate of crown expansion and growth vigour at those ages would be approaching or would have reached asymptotic limits (Oliver and Larson, 1990).

The recommended silvicultural strategy in Table 2 looks like it is well thought without the pitfalls of the PMP-derived one. The negatives of the strategy is that starting off with an initial planting of *1372* stems ha^{-1} would be more expensive and the first thinning at age *8* years of *722* stems ha^{-1} seems excessive, which may mean a higher risk of stem damage for the residual trees. The final crop of *250* stems ha^{-1} may be harvested any time from age 25 years.

The GA strategy in Table 3 seems to be the most intuitive one. This is because the initial planting density of *906* stems ha^{-1} would cost comparatively less than the other two strategies. The thinnings at all ages are therefore, not as excessive as in the other two cases, especially in the first thinning where it is only *282* stems ha^{-1}. The final crop number is the same as the one for the recommended strategy. Given that the final thinning is at age *20* years, there is certainly still some growth vigour at that age for the residual trees, to take advantage of growing space, nutrients and water. A clearfell at age *31* years would in no doubt guarantee larger logs that will command a high premium.

Therefore, the strategies from Tables 2 and 3 look similar, although the GA manages to improve recommendations that are based on analysis of years of field experimental trials of *P. patula* in many parts of South Africa. As for the PMP-derived strategy, the state equations may need more state equations such as a mortality function, in order hanging on to too many trees for too long which would induce competition and ultimately mortality. The use of a mortality function in a PMP formulation for finding an optimal silvicultural strategy was found to be critical in highly productive sites for the Australian *Eucalyptus nitens*. It was demonstrated that the PMP formulation predicted a lower initial planting density so as to limit high mortality rates (Chikumbo and Mareels, 2002).

7 CONCLUSIONS

Given the growth dynamics of a forest stand and an economic aggregate, a combined optimal control and parameter selection model may be formulated that will predict an optimal harvesting strategy, initial stand density, final crop number and the rotation length. Though a dimensionless economic aggregate (stumpage) was used in the formulation, it still represented the expected sigmodal or second-order dynamics reminiscent of real world trends for stumpage. The GA generated results that were comparable to the one recommended by the South African Department of Forestry. On close analysis the GA results seemed to show a more practical harvesting strategy than the PMP-derived one. The PMP formulation may need another state equation such as a mortality function in order to predict harvesting strategies that are more practical. Future work will involve multi-objective optimisation were more values (other than just economics) will be taken into consideration for finding a number of non-dominated solutions, the Pareto-optimal set (Horn 1997). The GA has become established as the method at hand for exploring the Pareto-optimal front in multi-objective optimisation problems (Zitler *et al*., 2000).

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