MIMO INSTANTANEOUS BLIND IDENTIFICATION BASED ON THIRD-ORDER CUMULANT

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Keywords: Instantaneous blind identification, Third-order cumulant, Steepest descent method, Nonlinear, Undetermined problem.

Abstract: This paper presents a new MIMO instantaneous blind identification algorithm based on third-order temporal property. Third-order temporal structure is reformulated in a particular way such that each column of the unknown mixing matrix satisfies a system of nonlinear multivariate homogeneous polynomial equations. The nonlinear system is solved by improved steepest descent method. We construct a general goal of the nonlinear system and convert the nonlinear problem into an optimal problem. The optimal solutions are obtained one by one by adding a penalty item to the general goal, which is Gaussian function characterized with valley-filled feature. Our algorithm allows estimating the mixing matrix for scenarios with 3 sources and 2 sensors, etc. Finally, simulations and comparisons show its effectiveness.

1 INTRODUCTION

Multiple-input multiple-output (MIMO) instantaneous blind identification (MIBI) is one of the attractive blind signal processing (BSP) problems, where a number of source signals are mixed by an unknown MIMO instantaneous mixing system and only the mixed signals are available, i.e., both the mixing system and the original source signals are unknown. The goal of MIBI is to recover the instantaneous MIMO mixing system from the observed mixtures of the source signals (Cichocki, A., Amari S I. 2002) (van de Laar J, Moonen M, Sommen P C W., 2008) (Shen Xizhong, Hu Dachao, and Meng Guang., 2009). In this paper, we focus on developing a new algorithm to solve the MIBI problem by using third-order statistics.

Many researchers have investigated the use of third-order cumulant temporal structure (TOCTS) for MIBI (Cichocki, A., Amari S I. 2002). The greater majority of the available algorithms are based on the generalized eigenvalue decomposition or joint approximate diagonalization of three- or fourth- order cumulant-based matrix for different lags and/or times arranged in the conventional manner. Most of them can only identify overdetermined problem. An MIBI based on second order temporal structure (SOTS) (van de Laar J, Moonen M, Sommen P C W., 2008) (Shen Xizhong, Hu Dachao, and Meng Guang., 2009) has been proposed to be applied to the estimation of the more columns than sensors. Our work is a continuation of their work presented in (van de Laar J, Moonen M, Sommen P C W., 2008) and we apply third-order cumulant to construct our algorithm.

In this paper, we exploit TOCTS by considering third-order cumulant. Then we project the MIBI problem on the system of homogeneous polynomial equations of degree three. At last steepest descent method is improved to estimate the columns of the mixing matrix, which is different from the algorithm in (van de Laar J, Moonen M, Sommen P C W., 2008) which applied SOTS. The MIBI method presented in this paper allows estimating the mixing matrix for several underdetermined mixing scenarios with 3 sources and 2 sensors. Simulations show its effectiveness.

Shen X., Hu D. and Meng G. (2010).
MIMO INSTANTANEOUS BLIND IDENTIFICATION BASED ON THIRD-ORDER CUMULANT.
In *Proceedings of the Third International Conference on Bio-inspired Systems and Signal Processing*, pages 127-132
DOI: 10.5220/0002581401270132

2 MIBI MODEL AND ITS ASSUMPTIONS

Let us use the usual model (Cichocki, A., Amari S I., 2002) (van de Laar J, Moonen M, Sommen P C W., 2008) in MIBI problem as follows

$$\mathbf{x}(t)=\mathbf{A}\mathbf{s}(t)+\mathbf{v}(t) \tag{1}$$

where $\mathbf{A}=[\mathbf{a}_1,\cdots,\mathbf{a}_m]\in\mathbb{R}^{n\times m}$ is an unknown mixing matrix with m n-dimentional array response vectors $\mathbf{a}_j=\begin{pmatrix}a_{1j} & \cdots & a_{nj}\end{pmatrix}^{\mathrm{T}}, j=1,2,\cdots,m$, $\mathbf{s}(t)=[s_1(t),s_2(t),\cdots,s_m(t)]^{\mathrm{T}}$ is the vector of source signals, $\mathbf{v}(t)=[v_1(t),\cdots,v_n(t)]^{\mathrm{T}}$ is the vector of noises, and $\mathbf{x}(t)=[x_1(t),x_2(t),\cdots,x_n(t)]^{\mathrm{T}}$ is the vector of observations. Without knowing the source signals and the mixing matrix, the MIBI problem is to identify the mixing matrix from the observations by estimating $\mathbf{A}$ as $\hat{\mathbf{A}}$.

The mixing matrix is identifiable in the sense of two indeterminacies, which are unknown permutation of indices of each column of the matrix and its unknown magnitude. The usual convention is to assume that each column $\mathbf{a}_j$ satisfy the normalization conditions, i.e.

$$\sum_{i=1}^{n} a_{ij}^{\ 2}=1, j=1,2,\cdots,m; \tag{2}$$

and leave the permutation undetermined.

To solve the MIBI problem, we define the following concepts Def 1~2 for the derivation of the algorithm, and make the following assumptions AS 1~4 (van de Laar J, Moonen M, Sommen P C W.,2008) on noise-free region of support (ROS) Ω.

Def 1 Third-order cumulant $c_{s,ijk}(t,\tau_1,\tau_2)$ of $s_i(t), i=1,2,\cdots,m$ with zero mean at time instant t and lag τ_1,τ_2 is defined as, $\forall t,\tau_1,\tau_2\in\mathbb{Z}$ and $\forall i,j,k=1,2,\cdots,m$

$$\begin{aligned} c_{s,ijk}(t,\tau_1,\tau_2) &\triangleq \mathrm{cum}\left(s_i(t),s_j(t-\tau_1),s_k(t-\tau_2)\right) \\ &= \mathrm{E}\left[s_i(t)s_j(t-\tau_1)s_j(t-\tau_2)\right]. \end{aligned} \tag{3}$$

When $i=j=k$, that is, auto-cumulant, it is defined as

$$c_{s,i}(t,\tau_1,\tau_2)\triangleq \mathrm{E}\left[s_i(t)s_i(t-\tau_1)s_i(t-\tau_2)\right]. \tag{4}$$

Def 2 Included angle between the j-th column $\mathbf{a}_j$ of $\mathbf{A}$ and its estimate $\hat{\mathbf{a}}_j$ is defined as

$$\theta_j=\frac{\langle \mathbf{a}_j,\hat{\mathbf{a}}_j\rangle}{\|\mathbf{a}_j\|\cdot\|\hat{\mathbf{a}}_j\|}, \forall j=1,2,\cdots,m, \tag{5}$$

where $\langle\cdot,\cdot\rangle$ is dot product and $\|\cdot\|$ is norm-2 of a vector.

AS 1 Source signals have zero cross-cumulant, that is, for $\forall i\neq j, j\neq k,$ or $k\neq i$,

$$c_{s,ijk}(t,\tau_1,\tau_2)=0. \tag{6}$$

AS 2 Auto-cumulants of source signals are linearly independent

$$\begin{aligned}\sum_{j=1}^{m}\xi_j c_{s,jjj}(t,\tau_1,\tau_2)=0 \Rightarrow \xi_j=0, \\ \forall j=1,2,\cdots,m\end{aligned} \tag{7}$$

AS 3 The noise signals have zero auto- and cross-cumulants,

$$c_{v,ijk}(t,\tau_1,\tau_2)=0, \forall 1\le i,j,k\le m. \tag{8}$$

AS 4 The cross-cumulant between the source and noise signals are zero:

$$\begin{aligned} c_{ssv,ijk}(t,\tau_1,\tau_2) &= c_{svv,ijk}(t,\tau_1,\tau_2) \\ &= 0, \forall i,j,k \end{aligned} \tag{9}$$

The procedure of our proposed algorithm includes two steps, that is, step 1 is that the problem of MIBI is formulated as the problem of solving a system of homogeneous polynomial equations; and step 2 is that steepest descent method is improved to solve the system of polynomial equations on the unit vector. We detail these steps respectively in section 3 and 4.

3 PROJECTION ON POLYNOMIAL EQUATIONS

3.1 Third-order Cumulant Related Matrix Definition and Structure

Consider the following third-order cumulant of sensor signals,

$$c_{x,ijk}(t,\tau_1,\tau_2)=\mathrm{E}\left[x_i(t)x_j(t-\tau_1)x_j(t-\tau_2)\right]. \tag{10}$$

Using AS1~4, it follows on Ω,

$$c_{x,ijk}(t,\tau_1,\tau_2)=\sum_{p=1}^{m}a_{ip}a_{jp}a_{kp}c_{s,p}(t,\tau_1,\tau_2). \tag{11}$$

We now stack all those third-order cumulant values in the n^3-dimensional vector that is defined as,

$$\mathbf{c}_{\mathbf{x}}(t,\tau_1,\tau_2) \triangleq \mathrm{E}\left[\mathbf{x}(t)\otimes\mathbf{x}(t-\tau_1)\otimes\mathbf{x}(t-\tau_2)\right] \quad (12)$$

where $\otimes$ denotes the Kronecker product. When the time-lag three-way (t,τ_1,τ_2) are chosen in a specified ROS Ω, we could obtain the following $n^3 \times N_{\mathrm{ROS}}$ cumulant matrix where N_{ROS} is the length of the ROS,

$$\mathbf{C}_{\mathbf{x}} \triangleq \left[\mathbf{c}_{\mathbf{x}}(\omega_1) \quad \cdots \quad \mathbf{c}_{\mathbf{x}}(\omega_{N_{\mathrm{ROS}}})\right], \quad (13)$$

where $\omega_i = (t,\tau_1,\tau_2)$ is the i th three-way element in Ω.

Likewise, the source cumulant matrix $\mathbf{C}_{\mathbf{s}}$ is defined as follows,

$$\mathbf{C}_{\mathbf{s}} \triangleq \left[\mathbf{c}_{\mathbf{s}}(\omega_1) \quad \cdots \quad \mathbf{c}_{\mathbf{s}}(\omega_{N_{\mathrm{ROS}}})\right], \quad (14)$$

where $\mathbf{c}_{\mathbf{s}}(\omega_i) = \left[c_{s,1}(\omega_i) \quad \cdots \quad c_{s,m}(\omega_i)\right]^{\mathrm{T}}$. The linear space spanned by the rows of the source cumulant matrix in (14) is called the source three-way subspace matrix, which is different from the definition of source subspace formed by source autocorrelation matrix 0. The dimension m of $\mathbf{c}_{\mathbf{s}}(\omega_i)$ equals the rank of $\mathbf{C}_{\mathbf{s}}$ provided that $N_{\mathrm{ROS}} \geq m$ and AS 2, that is,

$$m = \operatorname{rank}(\mathbf{C}_{\mathbf{s}}). \quad (15)$$

From eq.(14) and (13), it follows immediately that

$$\mathbf{C}_{\mathbf{x}} = \mathbf{A}_{3\otimes}\mathbf{C}_{\mathbf{s}}. \quad (16)$$

Here, $\mathbf{A}_{3\otimes} = \left[\mathbf{a}_1 \otimes \mathbf{a}_1 \otimes \mathbf{a}_1 \quad \cdots \quad \mathbf{a}_m \otimes \mathbf{a}_m \otimes \mathbf{a}_m\right]$ is third-order Khatri-Rao product of $\mathbf{A}$ named after second-order Khatri-Rao product (van de Laar J, Moonen M, Sommen P C W., 2008).

Because the kronecker product in $\mathbf{A}_{3\otimes}$ is a vector of length n^3 containing only N unique rows of $\mathbf{A}_{3\otimes}$, where

$$N = \frac{1}{6}n(n+1)(n+2). \quad (17)$$

For simplicity, we use the same symbol $\mathbf{A}_{3\otimes}$ as the matrix combined by unique rows of $\mathbf{A}_{3\otimes}$ without confusion. In general, if the mixing matrix is row full rank and if $m \leq N$, then $\operatorname{rank}(\mathbf{A}_{3\otimes}) = m$. Using matrix analysis (Horn R A, Johnson C R., 1985), it follows from eq.(15) and (16) that

$$\operatorname{rank}(\mathbf{C}_{\mathbf{x}}) = m = \operatorname{rank}(\mathbf{A}_{3\otimes}). \quad (18)$$

The rank of the Khatri-Rao product matrix has been studied in several works, eg,. (Sidiropoulos and R. Bro., 2000).

3.2 Deriving the System of Homogeneous Polynomial Equations

If the number of rows of the sensor cumulant matrix is larger than the dimension of the subspace spanned by its rows, $\mathbf{C}_{\mathbf{x}}$ has a nonzero left null space $\mathrm{N}(\mathbf{C}_{\mathbf{x}})$. Let $\mathbf{\Phi}$ be a matrix such that its rows form a basis for $\mathrm{N}(\mathbf{C}_{\mathbf{x}})$, that is,

$$\mathbf{\Phi}\mathbf{C}_{\mathbf{x}} = \mathbf{0}. \quad (19)$$

The matrix $\mathbf{\Phi}$ can be determined directly from the singular value decomposition (SVD) of $\mathbf{C}_{\mathbf{x}}$. The maximum number of linearly independent rows of the $\mathbf{\Phi}$ equals $\dim(\mathrm{N}(\mathbf{C}_{\mathbf{x}})) = N - m$. Substituting eq.(16) into eq.(19), and using the fact of eq.(15), it follows immediately that

$$\mathbf{\Phi}\mathbf{A}_{3\otimes} = \mathbf{0}. \quad (20)$$

This system in (20) describes the relation between the unknown coefficients of the mixing matrix $\mathbf{A}$ and the known coefficients of the matrix $\mathbf{\Phi}$. Let $\boldsymbol{\varphi}_q$ be the q th row of $\mathbf{\Phi}$, and for all columns $\mathbf{a}_p$ of the mixing matrix, define the functions

$$f_q(\mathbf{a}_p) \triangleq \sum_{i=1}^{n}\sum_{j=i}^{n}\sum_{k=j}^{n}\varphi_{q,ijk}a_{ip}a_{jp}a_{kp} = 0, \quad (21)$$

with $1 \leq q \leq Q$, and $Q = N - m$.

The MIBI problem has been projected onto the problem of solving the system of equations in (21) for the columns of the mixing matrix.

There are Q unique equations in system (21) by generic consideration in (van de Laar J, Moonen M, Sommen P C W., 2008), and we find the constraint $Q \geq n-1$ by eq.(21). Therefore, the maximum number of columns that can be identified with n sensors equals

$$\begin{aligned} m_{\max} &= N - (n-1) \\ &= \frac{1}{6}n(n+1)(n+2) - (n-1) \end{aligned}. \quad (22)$$

4 APPLICATION AND ITS IMPROVEMENT OF STEEPEST DESCENT METHOD

By steepest descent method (Richard L. Burden; J. Douglas Faires., 2001), a solution at $\mathbf{a}^*$ of the system in (21) satisfies the function g defined by

$$g\left(\mathbf{a}^*\right)=\sum_{i=1}^{n} f_q^{\,2}\left(\mathbf{a}^*\right). \tag{23}$$

To satisfy the constraint (2), we normalize $\mathbf{a}_j$ in each iterative step to unit vector, rather than take it as a penalty item which performs worse in our experiments in the sense that the optimal point is far away the ideal one.

There are many algorithms for the solution of the sequential unconstrained minimization problem (Byrne, C., 2008) in (23) to obtain the sequential optimal solutions one after one. Consider that we have had solutions $\mathbf{a}_i, i=1,2,\cdots,j-1$, and try to find next solution $\mathbf{a}_j \neq \mathbf{a}_i, \forall i$. To avoid converging to the same existing solution, we here improve the objective function in (23) by adding a penalty item for each known solution $\mathbf{a}_i$ to the objective function (23), that is,

$$f_p\left(\mathbf{a}_j\right)=\sum_{i=1}^{j-1}\left(f_g\left(\mathbf{a}_j-\mathbf{a}_i\right)+f_g\left(\mathbf{a}_j+\mathbf{a}_i\right)\right), \quad \forall j=1,\cdots,m. \tag{24}$$

Here, $f_g\left(\mathbf{a}\right)=\frac{1}{\sqrt{2\pi}\sigma}\exp\left(-\frac{\left\|\mathbf{a}\right\|_2^2}{\sigma^2}\right)$ is a Gaussian function, $\|.\|$ is norm-2 of one vector, exp is exponential base and σ is variance coefficient of estimate vector. Then, we get a novel objective function,

$$f_{\text{all}}\left(\mathbf{a}_j\right)=g\left(\mathbf{a}_j\right)+\gamma f_p\left(\mathbf{a}_j\right), \quad \forall j=1,\cdots,m, \tag{25}$$

where γ is penalty factor.

We now derive the solution of the optimal problem. The direction of greatest decrease in the value of $g\left(\mathbf{a}_j\right)$ at $\mathbf{a}_j^{(k)}$ with k-th iteration is the direction given by its minus gradient $-\nabla f_{\text{all}}\left(\mathbf{a}_j^{(k)}\right)$ of $f_{\text{all}}\left(\mathbf{a}_j\right)$ (Byrne, C., 2008). The gradient is expressed as

$$\nabla f_{\text{all}}\left(\mathbf{a}_j\right)=2\mathbf{J}\left(\mathbf{a}_j\right)^{\mathrm{T}}\mathbf{F}\left(\mathbf{x}\right)+\gamma\nabla f_p\left(\mathbf{a}_j\right). \tag{26}$$

Here,

$$\nabla f_p\left(\mathbf{a}_j\right)=-\frac{2}{\sigma^2}\sum_{i=1}^{j-1}\Big[f_g\left(\mathbf{a}_j-\mathbf{a}_i\right)\left(\mathbf{a}_j-\mathbf{a}_i\right)^{\mathrm{T}} +f_g\left(\mathbf{a}_j-\mathbf{a}_i\right)\left(\mathbf{a}_j-\mathbf{a}_i\right)^{\mathrm{T}}\Big],$$

$\mathbf{F}\left(\mathbf{x}\right)=\left(f_1\left(\mathbf{x}\right),\cdots,f_Q\left(\mathbf{x}\right)\right)^{\mathrm{T}}$, and $\mathbf{J}\left(\mathbf{a}_j\right)$ is its Jacobian matrix. The objective is to reduce $g\left(\mathbf{a}_j\right)$ to its minimal value of zero, and an appropriate choice for updating $\mathbf{a}_j$ is

$$\mathbf{a}_j^{(k+1)}=\mathbf{a}_j^{(k)}-\alpha_0\nabla f_{\text{all}}\left(\mathbf{a}_j^{(k)}\right), \tag{27}$$

where $\alpha_0=\arg\min_{\alpha} g\left(\mathbf{a}_j^{(k)}-\alpha\nabla f_{\text{all}}\left(\mathbf{a}_j^{(k)}\right)\right)$ is the critical point. We can apply any single-variable function optimal method to find the minimum value of $g\left(\mathbf{a}_j^{(k+1)}\right)$ by an appropriate choice for the value α. In our algorithm, we use Newton's forward divided-difference interpolating polynomial, detailed in (Richard L. Burden; J. Douglas Faires., 2001).

Two things must be noted. One is the tolerance problem. The minimal value of (25) just make the objective reach to the minimal value, but it mustn't make $g\left(\mathbf{a}_j\right)=0, \forall j$, which is the reason why we select steepest descent method because $f_q\left(\mathbf{a}_p\right)\neq 0, \forall q,p$ due to the theorectical errors and estimate errors in (20). The other is the initial problem. We employ the initial solutions as equal distributed vectors in the super space of $\mathbf{a}_j$, for example, in our simulation of mixing matrix with 2×3 sizes,

$$\mathbf{A}=\begin{bmatrix} 1 & 0 & \frac{1}{\sqrt{2}} \\ 0 & 1 & \frac{1}{\sqrt{2}} \end{bmatrix}.$$

5 EXAMPLES WITH SPEECH AND THREE SENSORS

To demonstrate our proposed algorithm, we adopt a system with 2×3 matrix, that is, the system has two mixtures of three speech signals. A large simulations are carried on. Without any loss of generality, we assume that the columns of the mixing matrix have unit Euclidian norms. The speech signals are sampled as 8kHz, consist of

10,000 samples with 1,250ms length, and are normalized to unit variance $\sigma_s = 1$. The signal sequences are partitioned into five disjoint blocks consisting of 2000 samples, and for each block, the third-order cumulants are computed for lags zeros. Hence, in total for each sensor cumulants 5 values are estimated and employed, i.e., the employed ROS in the domain of block-lag pairs is given by

$$\Omega = \{(1,0,0), \cdots, (5,0,0)\},$$

where the first index in each pair represents the block index and the second and the third the lag indices. The sensor signals are obtained from (1) with 2×3 mixing matrix,

$$\mathbf{A} = \begin{bmatrix} 0.7580 & 0.4472 & 0.9094 \\ 0.6523 & -0.8944 & -0.4160 \end{bmatrix}. \quad (28)$$

We set the maximum iterative number is 30, and stop the iteration step if the correction of the estimated is smaller than a certain tolerance 10^{-3}.

5.1 Discussion of Coefficients About Steepest Descent Methods

The penalty factor γ and variance σ are two important coefficients in (24) and (25), depicted in Figure 1. Figure 1 shows that the included angles are affected by different values of γ and σ. θ_1 is unchanged as to eq. (24), θ_2 and θ_3 change with the choice of γ and σ. We set $\gamma = 15, \sigma = 1$ in our algorithm according to the figure.

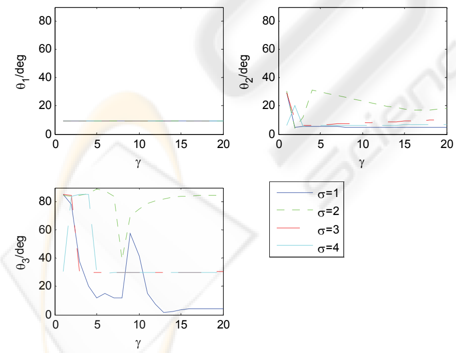

Figure 1: The Included Angles change with γ and σ.

The penalty item $f_p(\mathbf{a}_j)$ in (25) is aimed to avoid converging to one of the estimated optimal points. As the first and second columns of the mixing matrix are relatively simpler than the third one, we discuss the third columns majorly. Table 1 shows the series of estimation of the third column of mixing matrix. Although the initial points have the same initial value, we still get the ideal optimal value under the function of penalty item in (25), where the trajectory of the optimal procedure carefully searches the ultimate point avoiding converging to the previous optimal points. The function in (24) is assigned to valley-filled feature, which make the previous minimum value filled.

Table 1: Estimation of the third column of mixing matrix.

No.	$\mathbf{a}_3$		No.	$\mathbf{a}_3$	
1	0.79	-0.61	16	0.86	-0.51
2	0.79	-0.61	17	0.87	-0.5
3	0.8	-0.6	18	0.87	-0.48
4	0.8	-0.6	19	0.88	-0.47
5	0.81	-0.59	20	0.89	-0.46
6	0.81	-0.59	21	0.89	-0.45
7	0.82	-0.58	22	0.9	-0.44
8	0.82	-0.57	23	0.9	-0.43
9	0.83	-0.56	24	0.91	-0.42
10	0.83	-0.56	25	0.91	-0.41
11	0.84	-0.55	26	0.92	-0.39
12	0.84	-0.54	27	0.92	-0.38
13	0.85	-0.53	28	0.93	-0.37
14	0.85	-0.52	29	0.93	-0.36
15	0.86	-0.51			

5.2 MIBI Problem

Figure 2 depicts column vectors of mixing matrix and its estimation. The column vectors of estimated mixing matrix $\hat{\mathbf{A}}$ are indicated by solid lines and dots, and the column vectors of $\mathbf{A}$ by dashed lines and stars.

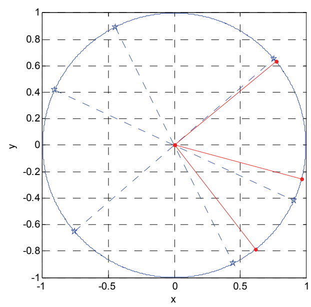

Figure 2: Column vectors of mixing matrix and its estimation. The star represents the column vectors of mixing matrix, and the dot represents their estimations.

The estimated mixing matrix is

$$\hat{\mathbf{A}} = \begin{bmatrix} 0.7741 & 0.6141 & 0.9678 \\ 0.6331 & -0.7893 & -0.2515 \end{bmatrix},$$

and the included angles $\theta_j, j = 1, 2, 3, 4$ are 1.4380, 11.3183, and 10.0130. We see that the estimated columns approximately equal the ideal ones by comparison with the matrix in(28).

6 CONCLUSIONS

A new MIBI algorithm in (20) and (27) is proposed based on third-order temporal property, which is able to estimate underdetermined mixing scenarios with 3 sources and two sensors. The third-order cumulants with different time and lags are considered on a relatively simpler ROS, especially for noise-free region. We then project the MIBI problem in (1) on the system of homogeneous polynomial equations in (21) of degree three. Steepest descent method is improved for estimating the columns of the mixing matrix by adding a penalty item in objective function. Simulations show its effectiveness with more accurate solutions.

ACKNOWLEDGEMENTS

This work has been supported by NSFC with No 10732060, Shanghai Leading Academic Discipline Project with No J51501, and Shanghai Education with No 050Z06.

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