FRAMEWORK OF AN ESTIMATION ALGORITHM OF TIME VARYING MULTIJOINT HUMAN ARM VISCOELASTICITY

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Abstract: The paper concerns a framework of an estimation of multijoint human arm viscoelasticity in a small sufficient time period. The uncertainties have to be considered in estimating the viscoelasticity of the multijoint human arm. In general, the uncertainties existing in the structure of the human arm and the motor command from the central nervous system are subject to the non-Gaussian noises. A generalized Gaussian ratio function is brought in to deal with the non-Gaussian noises. The momotonicity of the generalized Gaussian ratio function is studied based on the approximation formula of Gamma functions, then a robust condition is proposed for the computation of even moments using shape parameters. That is, we can guarantee the accuracy of the simulation results and experimental results by the robust condition. The effectiveness of the proposed method is confirmed by the experimental results.

1 INTRODUCTION

One of the key characteristics that human beings surpass the animals is the human arm. According to the central nervous system (CNS), human can do many things by making full use of human arm. Simply, when human beings want to do something by arm, (e.g. taking something in front of him) the arms move forward following the guide of the CNS, adjust the direction and gradually reduce the distance between the hand and the object, finally the object can be taken. Actually, the human arm is derived by the multijoint muscle generated torque, which is assumed to be a function of angular position, velocity and motor command of CNS (Gomi and Kawato, 1996; Gomi and Kawato, 1997). The change of the torque is caused by multijoint arm viscoelasticity which consists of joint stiffness. Joint stiffness is regulated by muscle inherent spring-like properties, neural feedbacks, and viscosity. In the fields of industrial robots and medical service, the study of the human arm viscoelasticity plays an important part. For example, if the knowledge of how the arm moves according to the CNS is known, some artificial limbs can be designed to help the disabled. So in order to get some corresponding knowledge about the moving human arms, the estimation of the human arm viscoelasticity is discussed in this paper.

The estimation of the viscoelasticity of the human arm has been studied by many researchers (Deng, Inoue, Gomi and Hirashima, 2006; Gomi and Kawato, 1996; Gomi and Kawato, 1997; Deng, Saijo, Gomi and Inoue, 2006; Kim, Kang, Kim and Park, 2009). A high-performance manipulandum was developed to measure human arm stiffness based on the equilibrium-point control hypothesis (Gomi and Kawato, 1996). The authors discussed the manipulandum in details in (Gomi and Kawato, 1997): by using the manipulandum and a new estimation algorithm, human multi-joint arm stiffness parameters during multi-joint point-to-point movements on a horizontal plane were measured. Later, online estimation algorithm of the human arm viscoelasticity was proposed in (Deng et al, 2006), (Deng, Inoue and Zhu) and (Iseki, Deng, Inoue and Bu, 2009). An integrated procedure to study on real time estimation of time varying multijoint human arm viscoelasticity was proposed in (Deng et al.) concerning the uncertainty factor consisting of time-varying motor command from central nervous system, measurement noises and modeling error of the rigid body dynamics. However, there exist some problems, for example, in (Gomi and Kawato, 1996) and (Gomi and Kawato, 1997), the stiffness, viscosity, and inertia pa-

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rameters of the human arm are estimated by applying small perturbations which means a apparatus of a very high cost. This may hinder many researchers from the studying of human arm. So in order to reduce the cost of the experiment, we consider to estimate the viscoelasticity of the human arm in a small sufficient time period without perturbation. Moreover, the online estimation is based on the former experimental data, and there may be some difference or some inaccurate information from the actual value. So in this study, the monotonicity of the generalized Gaussian function is proved to be monotonically decreasing and a robust condition of the computation of the even moment is given, that is: when the varying of the shape parameter is bounded, the variance of the even moment is guaranteed to be bounded. Then we can guarantee the accuracy of the simulation results and experimental results by the robust condition.

The outline of the paper is given as follows: the human arm model and the estimation filter of viscoelasticity for human arm model are introduced in Section 2; Section 3 includes the main results, the monotonicity of the even moments and the robust condition; The experimental results are shown in Section 4, and the last part is the conclusion.

2 HUMAN ARM DYNAMIC MODEL AND ESTIMATING FILTER FOR VISCOELASTICITY

Two-link rigid human arm dynamics on the horizontal plane can be described as the following equation:

$$\Psi(\ddot{q},\dot{q},q)=\tau_{in}(\dot{q},q,u) \qquad (1)$$

Here, $\Psi(\cdot)$ denotes a two-link arm dynamics, and q, $\dot{q}$ and $\ddot{q}$ are angular position, velocity and acceleration vector, respectively. τ_{in} can be regarded as a function of angular position, velocity, and motor command, u descending from the supraspinal central nervous system, where

$$\begin{aligned} q &= (\theta_1(t),\theta_2(t))^T \\ \tau_{in} &= (\tau_s,\tau_e)^T \end{aligned} \qquad (2)$$

$\theta_1(t)$ is shoulder angle and $\theta_2(t)$ is elbow angle shown in Fig. 2, where $\tau_s=\tau_1$, $\tau_e=\tau_2$. Taking the derivative of (1):

$$\begin{aligned} &\frac{\partial\Psi}{\partial\ddot{q}}\frac{d\ddot{q}}{dt}+\frac{\partial\Psi}{\partial\dot{q}}\frac{d\dot{q}}{dt}+\frac{\partial\Psi}{\partial q}\frac{dq}{dt} \\ = \; &\frac{\partial\tau_{in}}{\partial\dot{q}}\frac{d\dot{q}}{dt}+\frac{\partial\tau_{in}}{\partial q}\frac{dq}{dt}+\frac{\partial\tau_{in}}{\partial u}\frac{du}{dt} \end{aligned} \qquad (3)$$

If the arm is assumed to be rigid body serial link system, such that:

$$\Psi(\ddot{q},\dot{q},q)=I(q)\ddot{q}+H(\dot{q},q) \qquad (4)$$

where, D and R present muscle viscosity and stiffness matrix, and

$$\begin{aligned} -\frac{\partial\tau_{in}}{\partial\dot{q}} &= D=\begin{pmatrix} D_{ss} & D_{se} \\ D_{es} & D_{ee}\end{pmatrix} \\ -\frac{\partial\tau_{in}}{\partial q} &= R=\begin{pmatrix} R_{ss} & R_{se} \\ R_{es} & R_{ee}\end{pmatrix} \end{aligned} \qquad (5)$$

The subscript ss of D and R represent the shoulder single-joint effect on each coefficient. Similarly, se and es denote cross-joint effects, and ee denotes the elbow single-joint effect. Then according to (3) (4) and (5), the following equation can be established:

$$\begin{aligned} &I(q)\frac{d\ddot{q}}{dt}+\frac{\partial H(\dot{q},q)}{\partial\dot{q}}\ddot{q}+[\frac{\partial I(q)\ddot{q}}{\partial q}+\frac{\partial H(\dot{q},q)}{\partial q}]\dot{q} \\ &\quad =-D\ddot{q}-R\dot{q}+\frac{\partial\tau_{in}}{\partial u}\frac{du}{dt} \end{aligned} \qquad (6)$$

Here the corresponding parameters of I and H are given as follows:

$$I = \begin{bmatrix} I_{11} & I_{12} \\ I_{21} & I_{22}\end{bmatrix} \qquad (7)$$

$$\begin{aligned} I_{11} &= m_1 l_{g1}^2+m_2(l_1^2+l_{g2}^2)+\tilde{I}_1+\tilde{I}_2+2m_2l_1l_{g2}\cos\theta_2 \\ &= Z_1+2Z_2\cos\theta_2 \\ I_{12} &= I_{21}=m_2l_{g2}^2+\tilde{I}_2+m_2l_1l_{g2}\cos\theta_2 \\ &= Z_3+Z_2\cos\theta_2 \\ I_{22} &= m_2l_{g2}^2+\tilde{I}_2=Z_3 \end{aligned} \qquad (8)$$

$$\begin{aligned} H &= \begin{bmatrix} -m_2l_1l_{g2}\sin\theta_2(\dot{\theta_2}^2+2\dot{\theta_1}\dot{\theta_2}) \\ m_2l_1l_{g2}\dot{\theta_1}^2\sin\theta_2 \end{bmatrix} \\ &= \begin{bmatrix} -Z_2\sin\theta_2(\dot{\theta_2}^2+2\dot{\theta_1}\dot{\theta_2}) \\ Z_2\dot{\theta_1}^2\sin\theta_2 \end{bmatrix} \end{aligned} \qquad (9)$$

from the above equations (6)-(9), we can get the following relationship:

$$\tau_{in}=-D\dot{q}-Rq+\int\frac{\partial\tau_{in}}{\partial u}du \qquad (10)$$

Since the sampling time $\Delta t\to 0$, then $du\to 0$, so the value of $\int\frac{\partial\tau_{in}}{\partial u}du\to 0$. By using a band-pass filter for (10), the high frequency and low frequency measurement noise can be removed. The filtered torque τ_{in}^f, position $\theta_1^f(t)$ and $\theta_2^f(t)$, vecocities $\dot{\theta}_1^f(t)$ and $\dot{\theta}_2^f(t)$ satisfy the following relationship:

$$\tau_{in}^f=XU+\Delta+\zeta_1 \qquad (11)$$

where X is the regression vector, U is the time-varying parameter vector to be estimated, and

$$X = \begin{pmatrix} \theta_1\ \theta_2\ \dot{\theta}_1\ \dot{\theta}_2\ 0\ \ 0\ \ 0\ \ 0 \\ 0\ \ 0\ \ 0\ \ 0\ \theta_1\ \theta_2\ \dot{\theta}_1\ \dot{\theta}_2 \end{pmatrix}$$
$$U = \begin{pmatrix} R_{ss}\ R_{se}\ D_{ss}\ D_{se}\ R_{es}\ R_{ee}\ D_{es}\ D_{ee} \end{pmatrix}^T \quad (12)$$

where $\Delta = [\Delta_1, \Delta_2]^T$ consists of the structural uncertainties which are assumed to be Gaussian. $\zeta_1 = [\zeta_{11}, \zeta_{22}]^T$ is the non-Gaussian measurement error matrix of filtered measurement noise.

First, the above model needs to be converted into its discrete time state-space form as follows.

$$U(t+1) = U(t) + \zeta_2, \ \ t = 1, 2, \cdots$$
$$\tau_{in}^f(t+1) = X(t+1)U(t+1) + \Delta(t+1) + \zeta_1(t+1)$$

where, ζ_2 is white noise. The shape parameters of the probability density function (pdf) are known to control the shape of the distribution. For the generalized Gaussian uncertainty factor $\Delta_i(t) + \zeta_{ii}(t)(i = 1, 2)$ with zero mean, variance σ_i^2 and shape parameter γ_i is given by:

$$p_i(x_i; \sigma_i, \gamma_i) = \frac{\alpha_i(\gamma_i)\gamma_i}{2\sigma_i\Gamma(1/\gamma_i)} e^{-[\alpha_i(\gamma_i)|x_i/\sigma_i|]^{\gamma_i}}$$
$$x_i \in R, \ \ i = 1, 2 \quad (13)$$
$$\alpha_i(\gamma_i) = \sqrt{\frac{\Gamma(3/\gamma_i)}{\Gamma(1/\gamma_i)}} \quad (14)$$

where $\Gamma(\cdot)$ is the Gamma function. In this paper, the generalized Gaussian ratio function is given as follows (Niehsen, 1999; Niehsen, 2002; Sharifi and Leon-Garcia, 1995).

$$\phi_i^{(2m)}(\gamma_i) = \frac{\Gamma(\frac{2m+1}{\gamma_i})\Gamma^{m-1}(1/\gamma_i)}{\Gamma^m(3/\gamma_i)}, m = 1, 2, \cdots \quad (15)$$

where

$$\sigma_i^2 = \sigma_{\Delta_i}^2 + \sigma_{\zeta_{ii}}^2 \quad (16)$$

$E(\tau_i^{2m})$ is a function of $\sigma_{\Delta_i}^2$, γ_{Δ_i}, $\sigma_{\zeta_{ii}}^2$ and $\gamma_{\zeta_{ii}}$. Variables $\sigma_{\Delta_i}^2$, γ_{Δ_i}, $\sigma_{\zeta_{ii}}^2$ and $\gamma_{\zeta_{ii}}$ are variance of Δ_i, shape parameter of Δ_i, variance of ζ_{ii} and shape parameter of ζ_{ii}, respectively. The odd moments vanish because of the symmetrical pdf.

3 MAIN RESULTS

Before the robust condition is given, some mathematical preliminaries are given as follows:

First, the form of the Stirling's formula is given as follows:

$$n! \approx \sqrt{2\pi n}\left(\frac{n}{e}\right)^n \quad (17)$$

The Stirling's formula can be applied to estimate the Gamma function $\Gamma(z)$, if $Re(z) > 0$. The corresponding approximation formula is in the following form:

$$\Gamma(z) = \sqrt{\frac{2\pi}{z}}\left(\frac{z}{e}\right)^z\left(1 + O(\frac{1}{z})\right) \quad (18)$$

where O denotes the Big O notation. In 2007, an estimation form was proposed by Gergo Nemes (Stirling approximation) which has the same computational accuracy with formers', but it is simpler for calculator.

$$\Gamma(z) \approx \sqrt{\frac{2\pi}{z}}\left[\frac{1}{e}\left(z + \frac{1}{12z - \frac{1}{10z}}\right)\right]^z$$
$$= \sqrt{\frac{2\pi}{z}}\left[\frac{1}{e}\frac{120z^3 + 9z}{120z^2 - 1}\right]^z \quad (19)$$

Next, let's prove that the generalized Gaussian ratio function decreases with the increasing of the shape parameter γ.

Based on the former results about generalized Gaussian ratio function, the following equality is established:

$$\phi^{(2m)}(\gamma) = \frac{\Gamma(\frac{2m+1}{\gamma})\Gamma^{m-1}(\frac{1}{\gamma})}{\Gamma^m(\frac{3}{\gamma})} \quad (20)$$

when $m = 2$, we get the equation (21):

$$\phi^{(4)}(\gamma) = \frac{\Gamma(\frac{5}{\gamma})\Gamma(\frac{1}{\gamma})}{\Gamma^2(\frac{3}{\gamma})} \quad (21)$$

From the equation (21), we can find that the following equation is satisfied if $z = \frac{1}{\gamma}$ and named the equivalent of $\phi^{(4)}(\gamma)$ to be $\Phi(z)$:

$$\Phi(z) = \frac{\Gamma(5z)\Gamma(z)}{\Gamma(3z)}$$
$$= \frac{3}{\sqrt{5}}(\frac{5}{9})^{5z}(\frac{1}{9})^z \frac{[\frac{1000z^2+3}{120\times 25z^2-1}]^{5z}[\frac{40z^2+3}{120z^2-1}]^z}{[\frac{120z^2+1}{120\times 9z^2-1}]^{6z}} \quad (22)$$

For simplicity, we consider the natural logarithm of $\Phi(z)$:

$$f(z) = ln(\Phi(z)) = ln3 - \frac{1}{2}ln5 + (5ln\frac{5}{9} + ln\frac{1}{9})z$$
$$+ [5ln(1000z^2 + 3) + ln(40z^2 + 3)$$
$$+ 6ln(120 \times 9z^2 - 1)]z - [5ln(120 \times 25z^2 - 1)$$
$$+ ln(120z^2 - 1) + 6ln(120z^2 + 1)]z \quad (23)$$

and take the derivative of the above function $f(z)$:

$$\begin{aligned}f'(z) = & 5ln\frac{5}{9} + ln\frac{1}{9} + 5ln\frac{1000z^2+3}{120\times 25z^2-1} \\ & + ln\frac{40z^2+3}{120z^2-1} + 6ln\frac{120\times 9z^2-1}{120z^2+1} \\ & - \frac{30}{1000z^2+3} - \frac{6}{40z^2+3} + \frac{12}{120\times 9z^2-1} \\ & - \frac{10}{120\times 25z^2-1} - \frac{2}{120z^2-1} + \frac{12}{120z^2+1}\end{aligned}$$

We can assume that

$$\begin{aligned}F(z) = & 5ln\frac{1000z^2+3}{120\times 25z^2-1} + ln\frac{40z^2+3}{120z^2-1} \\ & + 6ln\frac{120\times 9z^2-1}{120z^2+1} \\ H(z) = & -\frac{30}{1000z^2+3} - \frac{6}{40z^2+3} + \frac{12}{120\times 9z^2-1} \\ & - \frac{10}{120\times 25z^2-1} - \frac{2}{120z^2-1} + \frac{12}{120z^2+1}\end{aligned}$$

According to the monotonicity of the functions $F(z)$ and $H(z)$, we can find that when $z \in [\frac{1}{8}, \frac{2}{3}]$, $H(z) \in [-0.84, -0.1558]$ and $F(z) \in [ln792, ln1280]$, then the estimation of the $f'(z)$ can be obtained:

$$\begin{aligned}f'(z) & \geq 5ln5 - 12ln3 + ln792 - 0.84 \\ & \geq ln\frac{5^5\times 792}{3^{12}} - 0.84 \\ & \geq ln4.657 - 0.84 \\ & \geq 0.16 \geq 0\end{aligned}$$

Therefore, the function $f(z)$ is monotonically increasing in $z \in [\frac{1}{8}, \frac{2}{3}]$, so the even moment $\phi^{(4)}(\gamma)$ is monotonically decreasing with the shape parameter γ in $\gamma \in [\frac{3}{2}, 8]$, so the similar results can be obtained for the case of $m = 3$.

Then, the robust condition can be obtained as follows:

Using the result in (Deng et al, 2006), the following relationship is established:

$$\begin{aligned}E(\tau_i^6)(\gamma) = & \sigma_{\Delta_i}^6 \phi_i^{(6)}(\gamma_{\Delta_i}) + 15\sigma_{\Delta_i}^4 \phi_i^{(4)}(\gamma_{\Delta_i})\sigma_{\zeta_{ii}}^2 \\ & + 15\sigma_{\Delta_i}^2 \phi_i^{(4)}(\gamma_{\zeta_{ii}})\sigma_{\zeta_{ii}}^4 + \sigma_{\zeta_{ii}}^6 \phi_i^{(6)}(\gamma_{\zeta_{ii}})\end{aligned}$$

where $\phi^{(4)}(\gamma_{\Delta_i}) = 3$ and $\phi^{(6)}(\gamma_{\Delta_i}) = 15$. So the relationship can be simplified to be:

$$\begin{aligned}E(\tau_i^6)(\gamma_i) = & 15\sigma_{\Delta_i}^6 + 45\sigma_{\Delta_i}^4\sigma_{\zeta_{ii}}^2 + 15\sigma_{\Delta_i}^2\phi_i^{(4)}(\gamma_{\zeta_{ii}})\sigma_{\zeta_{ii}}^4 \\ & + \sigma_{\zeta_{ii}}^6\phi_i^{(6)}(\gamma_{\zeta_{ii}}) \end{aligned} \quad (24)$$

Therefore, the similar results can be obtained for the case of $\gamma_{\zeta_{ii}} = 1.5$, where $\phi^{(4)}(1.5) = 3.76$ and $\phi^{(6)}(1.5) = 26.7$.

$$\begin{aligned}E(\tau_i^6)(1.5) = & 15\sigma_{\Delta_i}^6 + 45\sigma_{\Delta_i}^4\sigma_{\zeta_{ii}}^2 + 56.4\sigma_{\Delta_i}^2\sigma_{\zeta_{ii}}^4(1.5) \\ & + 26.7\sigma_{\zeta_{ii}}^6(1.5) \end{aligned} \quad (25)$$

Then from the equations (24) and (25), the difference between $E(\tau_i^6)(1.5+\Delta\gamma)$ and $E(\tau_i^6)(1.5)$ is as follows, where $\gamma = 1.5 + \Delta\gamma$:

$$\begin{aligned}\Delta E = & E(\tau_i^6)(1.5+\Delta\gamma) - E(\tau_i^6)(1.5) \\ = & 45\sigma_{\Delta_i}^4[\sigma_{\zeta_{ii}}^2(1.5+\Delta\gamma) - \sigma_{\zeta_{ii}}^2(1.5)] + 15\sigma_{\Delta_i}^2 \\ & [\phi_i^{(4)}(1.5+\Delta\gamma)\sigma_{\zeta_{ii}}^4(1.5+\Delta\gamma) - 3.76\sigma_{\zeta_{ii}}^4(1.5)] \\ & + [\sigma_{\zeta_{ii}}^6(1.5+\Delta\gamma)\phi_i^{(6)}(1.5+\Delta\gamma) - 26.7\sigma_{\zeta_{ii}}^6(1.5)] \end{aligned} \quad (26)$$

Since the even moments $\phi_i^{(4)}(1.5+\Delta\gamma)$ and $\phi_i^{(6)}(1.5+\Delta\gamma)$ decrease with the increasing of $\Delta\gamma$, and the variance can be estimated using the method in (Iseki et al, 2009), so there exists a bounded M to make the following relationship satisfied:

$$\Delta E \leq M \quad (27)$$

Therefore, for $\Delta\gamma \leq 6.5$, the difference of the even moment $\Delta E \leq M$, so the even moment $E(\tau_i^6)(\gamma)$ is robust for boundedness of the varying of the shape parameter γ.

4 EXPERIMENTAL ISSUES

The experimental system is shown in Fig. 1. In the experiment, the parameters are given as follows, where the external force between the handle and horizontal plane is omitted. The arm parameters of the objective are $l_1 = 0.26m$, $l_2 = 0.30m$. The cut-off frequencies of the third-order band-pass filter to generate τ_{in}^f, $\theta_i^f(t)$ and $\dot{\theta}_i^f(t)$ are 0.5Hz and 9.5Hz. For designing the filter, we use the case of $m = 3$ in (15), then:

$$\begin{aligned}E(\tau_i^6)(\gamma) = & \sigma_{\Delta_i}^6 \phi_i^{(6)}(\gamma_{\Delta_i}) + 15\sigma_{\Delta_i}^4 \phi_i^{(4)}(\gamma_{\Delta_i})\sigma_{\zeta_{ii}}^2 \\ & + 15\sigma_{\Delta_i}^2 \phi_i^{(4)}(\gamma_{\zeta_{ii}})\sigma_{\zeta_{ii}}^4 + \sigma_{\zeta_{ii}}^6 \phi_i^{(6)}(\gamma_{\zeta_{ii}}) \end{aligned} \quad (28)$$

where, $\phi^{(4)}(\gamma_{\Delta_i}) = 3$ and $\phi^{(6)}(\gamma_{\Delta_i}) = 15$. The estimated stiffness and viscosity are shown in Figs. 3 and 4, where $l_{11} = 9.472$, $l_{12} = 7.750$, $l_{21} = l_{22} = 1.034e^9$ are selected. From Figs. 3 and 4, we can find that the proposed method is available to estimate the viscoelasticity of the human arm.

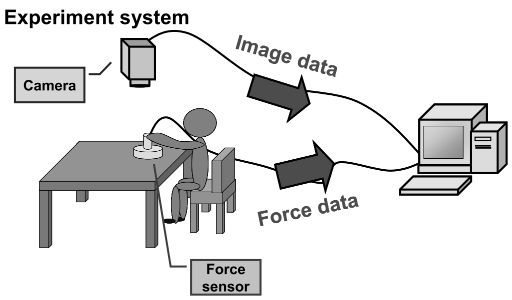

Figure 1: The experimental system.

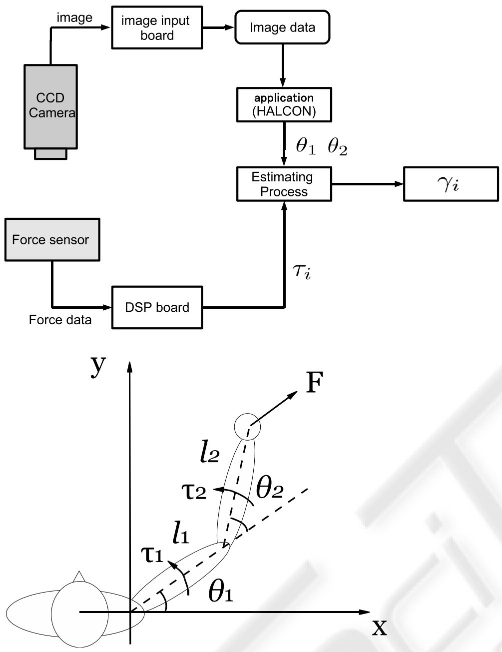

Figure 2: Schema of the experimental system and objective.

5 CONCLUSIONS

This paper considered the estimation of the viscoelasticity of human arm and studied the monotonicity of the generalized Gaussian ratio function, then a robust condition of the generalized ratio function is proposed for the varying of the shape parameter. So the accuracy of the experimental data is guaranteed by the robust condition. The effectiveness of the proposed method is confirmed by the experimental results.

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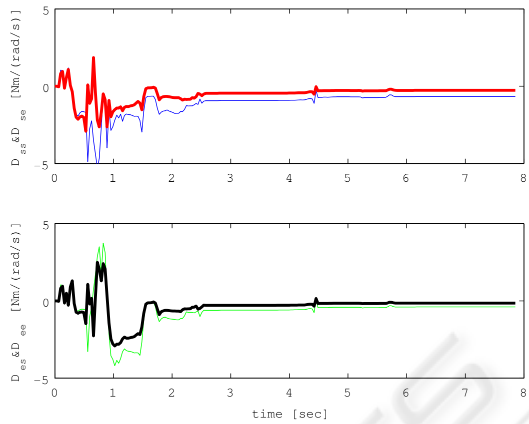

Figure 3: Estimated viscosity by experiment.

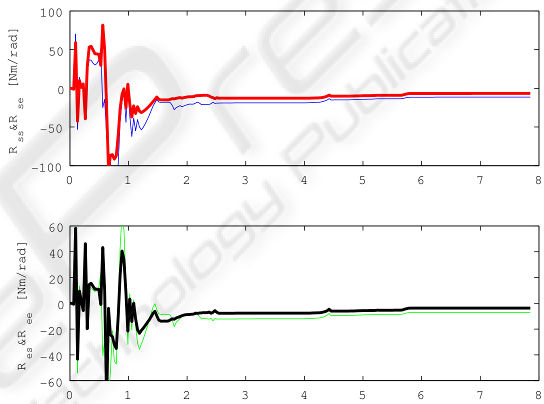

Figure 4: Estimated stiffness by experiment.

REFERENCES

Deng, M., Inoue, A., Gomi, H., & Hirashima, Y. (2006). Recursive Filter Design for Estimating Time Varying Multijoint Human Arm Viscoelasticity. International Journal of. Computers, Systems and Signals, 7(1), 2-18.

Deng, M., Inoue, A., & Zhu, Q. An Integrated Study Procedure on Real Time Estimation of Time Varying Multi-joint Human Arm Viscoelasticity. Transactions of the Institute of Measurement and Control (to appear).

Deng, M., Saijo, N., Gomi, H., & Inoue, A. (2006) A robust real time method for estimating human multijoint arm viscoelasticity. *International Journal of Innovative Computing, Information and Control*, 2(4), 705-721.

Gomi, H., & Kawato, M. (1996). Equilibrium-point Control Hypothesis Examined by Measured Arm-stiffness during Multi-joint Movement. *Science*, 272, 117-120.

Gomi, H., & Kawato, M. (1997). Human Arm Stiffness and Equilibrium-point Trajectory during Multi-joint Movement. *Biological Cybernetics*, 76, 163-171.

Iseki, K., Deng, M., Inoue, A., & Bu, N. (2009). Online Estimation of Multijoint Human Arm Viscoelasticity for the Case of Unknown Variance. *Proceeding of the 7th IEEE International Conference on Control* & *Automation*.

Kim, H., Kang, B., Kim, B., & Park, S. (2009). Estimation of Multijoint Stiffness Using Electromyogram and Artificial Neural Network. *IEEE Transactions on Systems, Man, and Cybernetics, Part A: Systems and Humans*, 39(5), 972-980.

Niehsen, W. (1999). Generalized Gaussian Modeling of Correlated Signal Sources. *IEEE Transactions on Signal Processing*, 47(1), 217-219.

Niehsen, W. (2002). Robust Kalman Filtering with Generalized Gaussian Measurement Noise. *IEEE Transactions on Aerospace and Electronic Systems*, 38(4), 1409-1412.

Sharifi, K., & Leon-Garcia, A. (1995). Estimation of Shape Parameter for Generalized Gaussian Distributions in Subband Decompositions of Video. *IEEE Transactions on Circuits and Systems for Video Technology*, 5(1), 52-56.

Stirling's approximation (n.d.). Retrieved from http://en.wikipedia.org/wiki/Stirling_approximation