

XFEM FRAMEWORK FOR CUTTING SOFT TISSUE

Including Topological Changes in a Surgery Simulation

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Abstract: Currently, there are many approaches in computer graphics (CG) that deal with topological changes; some of these are non-interactive animations, unstable or not precise enough to medical applications. It has been found that the Extended Finite Element Method (XFEM) is stable, accurate, with excellent performance and suitable for virtual surgery in real time; nevertheless, to maintain the provided advantages, the selection and creation of a set of CG methods is required that fulfill the requirements of the XFEM. We propose an embedded mapping method that enables the relation of the XFEM elements, with the visual and collision meshes, making the user interaction more dynamic. Furthermore, based on this new mapping method an interactive cutting algorithm is suggested considering a geometric analysis. The XFEM, as a core of our framework, efficiently simulates the topological changes consequently, making the interaction in real time possible, which will allow the creation of more complex simulations of higher impact in the medical area.

1 INTRODUCTION

From the perspective of computer graphics in medicine, the computer can allow surgeons or medical students to analyze patients before performing the surgery by simulating the physical behavior of the body, giving the possibility to train the procedures, avoiding the risks caused to patients by the lack of experience. In order to simulate the incisions in a realistic manner, the physical based methods are required, these give the adequate *accuracy*; Nevertheless, an *interactive* and *real time* simulation is desirable; therefore, it is necessary to look for efficiency in the methods and the *stability* in the simulation. Despite many approaches deal with topological changes, these do not fulfill the necessary characteristics previously described.

We identify the XFEM as a method that ensures a robust simulation; however, there is no framework to exploit its benefits to make it applicable to complex simulations as surgery simulation. Hence, it is necessary to design a framework completely focused on the XFEM in order to obtain efficient responses.

This paper explains how to introduce the XFEM in a surgery simulation including a geometric analysis of our cutting method considering how this affects the collision algorithm, mapping method and the remeshing process. Moreover, many other aspects have been

considered in order to achieve a real time simulation such as haptic interaction, handling multiple topologies, etc.

There are many researches that work with soft tissue deformation without changing the topology, however, the majority of surgery procedures require incisions and dissections; thus, this paper aims to provide a framework that is easy to implement and suitable for an interactive virtual surgery simulation including topological changes.

This paper is organized as follows: a review of the related work is analyzed in Section 2. After, in Section 3 the design of our framework is explained based on the XFEM and the embedded mapping method that works as mediator between the physical original mesh and the other meshes (visual and collision), also the creation of a specific cutting algorithm that quickly updates the associations of the meshes. Furthermore, the implementation and testing employing SOFA framework in 2D and 3D is showed in Section 4. Finally, in Section 5 the main contributions of our framework are emphasized; moreover, the limitations and the future work are also mentioned.

2 CUTTING APPROACHES

The majority of the applications oriented to medicine, model the tissue using physically-based methods (Vidal et al., 2006); the most popular is the FEM because of its accuracy on the results. An overview of physically based models and applications can be found in (Nealen et al., 2005).

Complex medical simulations require topological changes; but, this is a challenging work to simulate in real time with enough accuracy.

A surgery simulation must show the split of the elements according to the cutting trajectory. A simple manner to cut a tetrahedron is *subdividing* it into more small sub-tetrahedrons (Bielser et al., 2003). Nevertheless, subdividing can cause simulation instability because it generates ill-conditioned elements (slivers); moreover, the subdivision increments the degrees of freedom (DOFs) impacting directly on the simulation performance. Another alternative consists of *removing* the elements in contact with the blade (Forest et al., 2005); despite this method has the advantage that it does not affect the simulation stability, it is physically inaccurate and visually uneven; another option, to avoid the previous methods, is a successive *snapping* of nodes to the cutting trajectory (Serby et al., 2001); unfortunately, this kind of method leads to degenerated elements. Other approaches merge methods like snapping and subdivision (Steinemann et al., 2006), but only non-progressive cutting is enabled.

For fractures simulation, the virtual node algorithm is proposed (Molino et al., 2004); it consists of replicating cut elements assigning a portion of material to each copy, it maintains the initial FEM mesh conditioning, creating a minimal number of elements; however, the slivers are not completely avoided; the limitations of the virtual node algorithm were resolved by (Sifakis et al., 2007), avoiding slivers and allowing arbitrary cutting of tetrahedrons in any number of sub-elements; nonetheless, both previous approaches have been tested in an offline simulation which is undesirable for a surgery simulation.

(Jeřábková and Kuhlen, 2009) employ the XFEM to physically control the mesh when a cut appears, while the user is cutting no new elements are created (in the original mesh); thus the simulation performance is not greatly impacted moreover, the XFEM avoids ill-conditioned elements. Figure 2 shows how the XFEM allows the simulation of the discontinuities maintaining the original mesh.

The XFEM adds local enrichment functions in sub-regions with discontinuities; this method, proposed in 1999 by (Belytschko and Black, 1999), ex-

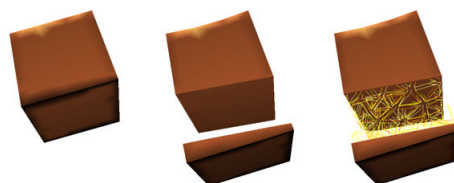


Figure 1: The alternative mesh helps to map the visual elements with its corresponding physical elements, this mesh is displayed on the right side.

ploits the partition of unity property of finite elements (Babuska and Melenk, 1997). The XFEM was initially utilized for fracture mechanics to simulate the crack growth with stiff materials, this method has been improved and can be applied to different domains such as material interfaces, 3D elasto-plastic deformations, fluid mechanics, material-nonmaterial interfaces and topology optimization considering void spaces (Nesme et al., 2009). We explain the basics of the XFEM in the appendix.

The XFEM was introduced to simulation of surgical cuts by (Vigneron et al., 2004), implementing a simulation of 2D MRI image with small deformations. (Linblad and Turkiyyah, 2007) propose a framework based on the FEM considering predefined boundaries for cutting and suturing; to cut arbitrary surfaces they employ the XFEM; this approach also allows small deformation as a linear discontinuous FEM was used. Later, (Turkiyyah et al., 2009) describe an algorithm for cutting using discontinuous FEM considering large deformations; nevertheless, this approach is described considering only the surface triangular mesh.

3 XFEM FRAMEWORK

The fundamentals of the XFEM are described by (Jeřábková and Kuhlen, 2009), they use the XFEM leading into a stable and accurate simulation. Nevertheless, their approach is very general, omitting how the XFEM must interact with other methods of CG without losing its advantages. Thus, our objective is to create a specific framework that exploits the advantages of the XFEM and, by setting the XFEM as a core of the simulation, create more complex surgery simulations which we know will be stable, accurate and interactive in real-time.

3.1 Corotational XFEM

The co-rotational FEM has been used for some approaches such as (Müller and Gross, 2004), showing that it is physically accurate and suitable for real-

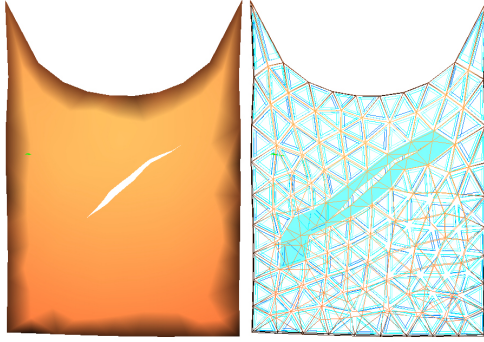


Figure 2: The XFEM adds new DOFs allowing to simulate discontinuities on the elements. This figure shows the original physical mesh composed by tetrahedral elements, this mesh continues even after the elements are split, those original elements must be hidden, showing only the cut of the model.

time applications. This formulation allows us to include large deformations using a Cauchy's strain tensor. We employ the the co-rotational XFEM formulation given by (Jeřábková and Kuhlen, 2009).

The forces in the co-rotational XFEM, must consider that the rotation of a discontinuous element will have different behavior in each part of the cutting plane (above and below). The rotation for the part above of the cut plane is denoted as \mathbf{R}_a and the rotation below as \mathbf{R}_b ; the equations of the deformation forces of the co-rotational XFEM are defined as

$$\begin{aligned} \mathbf{f}_i^X = & \sum_{j=1}^n \mathbf{R}_a \int_{V_a} \mathbf{B}_i^{XT} \mathbf{c} \mathbf{B}_j^X dV (\mathbf{R}_a^T \mathbf{p}_j^X - \mathbf{p}_{0j}^X) \} \text{Above} \\ & + \sum_{j=1}^n \mathbf{R}_b \int_{V_b} \mathbf{B}_i^{XT} \mathbf{c} \mathbf{B}_j^X dV (\mathbf{R}_b^T \mathbf{p}_j^X - \mathbf{p}_{0j}^X) \} \text{Below} \end{aligned} \quad (1)$$

where \mathbf{B}_i^X is the strain matrix defined as $\mathbf{B}^X = [\mathbf{B}_1 \cdots \mathbf{B}_n \ \Psi_1 \mathbf{B}_1 \cdots \Psi_n \mathbf{B}_n]$ where n is the number of element nodes; \mathbf{c} is a matrix of material properties. From the above equation, the equations of the deformation forces of the standard DOFs and the added DOFs can be computed separately as follows

$$\begin{aligned} \mathbf{f}_i = & \frac{V_a}{V} \mathbf{R}_a \sum_{j=1}^n \mathbf{K}_{ij} (\mathbf{R}_a^T \mathbf{p}_{aj} - \mathbf{p}_{0j}) \\ & + \frac{V_b}{V} \mathbf{R}_b \sum_{j=1}^n \mathbf{K}_{ij} (\mathbf{R}_b^T \mathbf{p}_{bj} - \mathbf{p}_{0j}) \end{aligned} \quad (2)$$

$$\begin{aligned} \mathbf{f}_i^a = & \frac{V_a}{V} \mathbf{R}_a \Psi_{ai} \sum_{j=1}^n \mathbf{K}_{ij} (\mathbf{R}_a^T \mathbf{p}_{aj} - \mathbf{p}_{0j}) \\ & + \frac{V_b}{V} \mathbf{R}_b \Psi_{bi} \sum_{j=1}^n \mathbf{K}_{ij} (\mathbf{R}_b^T \mathbf{p}_{bj} - \mathbf{p}_{0j}) \end{aligned} \quad (3)$$

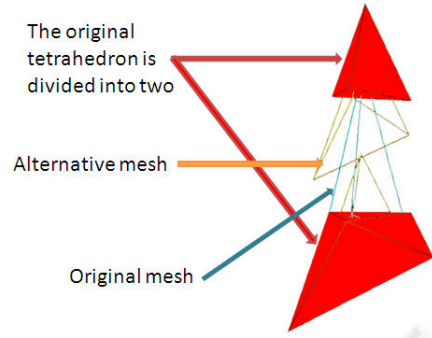


Figure 3: When an element is divided, the alternative mesh is generated with the new virtual elements formed by the added DOFs, this mesh allows to create a visual mapping to show the corresponding volume. Note that in the alternative mesh the virtual elements can overlap, however, this mesh is only required to make the association of topologies easier.

where the positions of the element nodes are calculated

$$\mathbf{p}_{sj} = \mathbf{p}_{0j} + \mathbf{u}_j + \Psi_{sj} \mathbf{a}_j \text{ where } s = a \text{ or } s = b \quad (4)$$

where the subscript s indicates that it can be applicable for the parts above and below the cut plane.

In order to directly obtain the physical position of the DOFs we use the shifted enrichment function (see eq. 9); as a result the deformation forces of the added DOFs yield

$$\mathbf{f}_i^a = \begin{cases} \sum_{j=1}^n -\frac{V_b}{V} \mathbf{R}_b \mathbf{K}_{ij} (\mathbf{R}_b^T \mathbf{p}_{bj} - \mathbf{p}_{0j}) & \text{if } H_i = +1 \\ \sum_{j=1}^n \frac{V_a}{V} \mathbf{R}_a \mathbf{K}_{ij} (\mathbf{R}_a^T \mathbf{p}_{aj} - \mathbf{p}_{0j}) & \text{if } H_i = -1 \end{cases} \quad (5)$$

and the positions of the element nodes

$$\begin{aligned} \mathbf{p}_{aj} = & \begin{cases} \mathbf{p}_{0j} + \mathbf{u}_j & \text{if } H_j = +1 \\ \mathbf{p}_{0j} + \mathbf{u}_j + \mathbf{a}_j & \text{if } H_j = -1 \end{cases} \\ \mathbf{p}_{bj} = & \begin{cases} \mathbf{p}_{0j} + \mathbf{u}_j - \mathbf{a}_j & \text{if } H_j = +1 \\ \mathbf{p}_{0j} + \mathbf{u}_j & \text{if } H_j = -1 \end{cases} \end{aligned} \quad (6)$$

Employing the framework of the co-rotational FEM proposed by (Nesme et al., 2005), thus we never store the whole stiffness matrix, instead, we generate the displacements by computing the dynamic equation of the FEM updating the strain matrix and computing directly the forces as is shown in eq. 1. To obtain the rotation values we use polar decomposition as is explained by (Müller and Gross, 2004).

3.2 Mapping Method

Considering topological changes, the XFEM adds DOFs each time a discontinuity appears remaining the

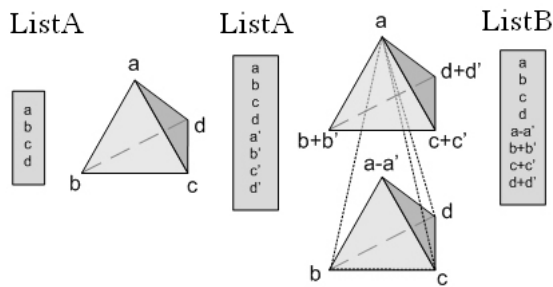


Figure 4: Added DOFs. The original nodal displacements (on the left) of a tetrahedron are four, when the tetrahedron is split new DOFs are added (list on the middle); in order to use the less memory, the alternative mesh stores in the place of the added DOFs the real position of the vertexes of the virtual tetrahedron (on the right).

number of elements of the original mesh; thus, an element is divided using the values of the added DOFs. Then, how should the triangular mesh be associated to the tetrahedrons that do not explicitly exist? If the association of visual points with the physical points (mapping) is done directly to the original mesh (initial FEM mesh), the result will not be correct, because the discontinuous elements do not show the same physical meaning of the original.

We propose a FEM-XFEM mapping that allows us to easily control the discontinuous elements; it consists in creating an alternative tetrahedral mesh that inserts elements each time the element is subdivided as shown in Figure 1 and Figure 3. The alternative mesh will always have the same number of DOFs as the original mesh, storing only the real values of the vertexes of the new tetrahedrons (i.e. *virtual elements*) as is shown in Figure 4, which are classified according to the side of the cut plane (i.e. above or below). Note that if an element is split, the mapping method creates two new virtual elements in the same place, for that reason, it is necessary to identify them in order to avoid mapping errors.

Another important advantage of the alternative topology is its speed that takes to reassign the visual or collision mesh nodes to their corresponding physical tetrahedron when a cut occurs by recording the points associated to the tetrahedron of the alternative mesh as is shown in Figure 5. The alternative mesh is controlled directly from the physical topology; in this manner, all the changes in the alternative mesh are quickly performed and also the search of associated elements is fairly straightforward. The vertices of the alternative topology are not DOFs; thereby, the alternative mesh does not harm the performance of the simulation.

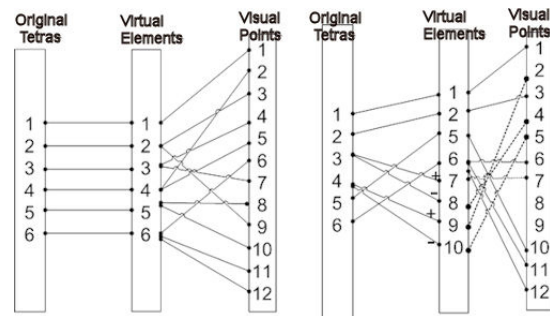


Figure 5: Initially the association of points is direct, and is updated when an element is cut. For instance the elements 3 and 4 are cut, then, two new virtual tetrahedrons are created for each element (4 in total); after, the visual points (only those connected with the discontinuous elements) must be re-associated to its tetrahedrons considering the cut plane (above + or below -).

3.3 Cutting Algorithm

A progressive cut generates many unnecessary faces of very small sizes, instead we split the tetrahedron until the tool crosses the whole element obtaining a semi-progressive cut because there is a delay before showing the incision. For a well refined mesh the delay in the semi progressive cut can be considered suitable in surgical simulations as is shown in Sec.4.3. When the interactive tool touches a tetrahedron (indicated by the collision detection algorithm), the nearest tetrahedron to the collision point is sought, right after, all the neighbors of the tetrahedron are stored in a list which is used to find the next tetrahedron touched by the cutting tool. Employing the collision detection and the assignation of points, it is possible to quickly extract the list of neighbors for the collided region.

If an element has three crossed edges, it can be cut; however, it is needed to ensure that the three intersections must be in different edges, the error may happen if the tool trembles while cutting.

```

L1,L2,L3 //elements with 1,2,3 intersections
collisionEvent (collisionPoint)
{
  if the cut starts
    FirstPoint = collisionPoint
    T=nearest(Mesh, FirstPoint)//nearest tetra
  else //while cutting
    listNT = neighbors(T); //neighbors of T
    Tc = nearest (listNT, fistPoint)
    SecondPoint = CollisionPoint
    semiProgressiveCut (Tc)
}

semiProgressiveCut (T0)
{
  cq=generateQuad(Tool)//collision quad

```

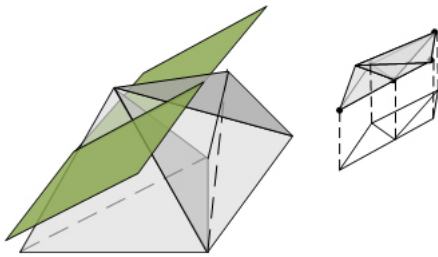


Figure 6: The collision quad generated by the cutting tool crosses multiple tetrahedrons in one time, the internal mesh is created by connecting intersected points on the element edges.

```

lT = neighbors(T0)
for each T in lT
  for each edgeT in edges(T)
    if Intersects(edgeT, cq, intersectionPoint)
      if exists T in list L2
        L3.add(L2.p1, L2.p2, intersectionPoint)
        L2.delete(T)
      else
        if = exists T in list L1
          L2.add(L1.p, intersectionPoint)
          L1.delete(T)
        else
          L1.add(T, intersectionPoint)
          L1.add(T, intersectionPoint)

for = each tetra j in L3
  createXFEM(j)

L3.clear
FirstPoint = SecondPoint
}
    
```

The cut is achieved by the intersection of a *collision quad* that specifies the trajectory and deepness of the tool; this quad is updated if the size is higher of a value specified by a constant, this size helps to avoid to re-compute the execution of the algorithm if the tool continues on the same point. We obtain better responses if the size of this quad is approximated to the average sizes of the elements. The cutting algorithm is presented in Figure 14.

When an element is divided into two new virtual elements, the visual and collision mapping methods are called to be updated. Therefore, the visual and collision meshes must also divide its element according to the cut plane and be assigned to its corresponding virtual element.

Note that if the quad of collision is long, in a well refined mesh the number of tetrahedrons to split is higher and there is possibly at least one that is not a neighbor to the reference tetrahedron; consequently, we can search in a neighborhood on more levels in order to obtain the higher list of tetrahedron to compare, this level must be specified depending on the

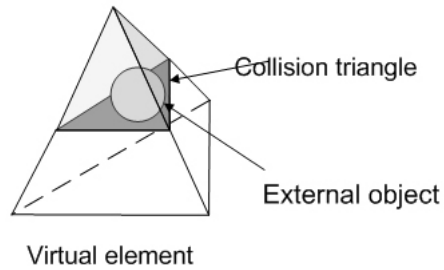


Figure 7: The collision model is mapped to the alternative mesh, not necessarily on the surface of the elements; it can be in the corresponding part of volume. The external object can overlap the elements, colliding in the collision triangle.

maximum size of the collision quad (defined by the tool blade). The precision of the trajectory of the tool blade will be better as a smaller quad of collision is chosen.

The physical, visual and collision topologies are different from each other, commonly the visual mesh is more refined to obtain a better look of the model, however, it is very possible that the triangular edges (visual topology) do not coincide with the edges of the tetrahedral elements at the time of cutting; given these facts, we execute the algorithm for the physical and visual topologies in an independent manner.

3.4 Visual Mapping

The barycentric mapping is used to connect the alternative mapping and the visual mesh; this mapping is only applied to show the portions of material associated to the virtual elements. In order to show how the object is opening while cutting, we use subdivision, despite its creation of points in each cut; these points do not strongly impact the simulation performance.

In 3D environments the deepness must be considered, therefore, when a cut happens an internal mesh must also be created and mapped. To this end, the new mesh is easily created by connecting the intersection points of the edges of the virtual tetrahedron as shows the Figure 6. Note that if the material is very thin, it is possible that the tool crosses both sides of the body, requiring the subdivision and the remapping of both sides.

The connection of visual points give place to two different possibilities, searching if a point with the same value and in the same side of the cut plane already exists, if this point is found it is used to generate the new triangular faces; if no point is found then it is required to search in the list of future points to be created, avoiding the generation of two points in the same edge and place. The list of future points is generated if the collision quad cuts more than one element at the same time (considering also the deepness).

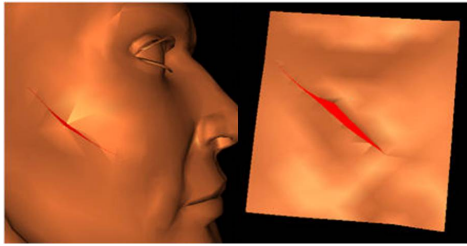


Figure 8: Cutting the skin in a triangular mesh (2D).

Tetrahedra can be split in different manners as is shown by (Steinemann et al., 2006), in the case the element is split with a face of four points, then two new triangles must be generated using the Delaunay triangulation.

3.5 Collision Detection

The collision detection algorithm must be mapped to the alternative mesh; then, if a collision appears, the values of the forces can be sent directly to the corresponding virtual tetrahedrons and the alternative mesh will propagate the forces to the original elements, following with the XFEM equations.

The added DOFs of a discontinuous element that shares an edge of non-discontinuous element, is set to zero, fixing the point on the corresponding edge, this means, that the added DOF works only when the neighbors (of edge) is also discontinuous.

A virtual element is able to collide only on the corresponding part of volume, this means that the collision method only manages the forces of that specific part and the virtual element helps to obtain the barycentric coordinates associated to collision elements in order to be deformed correctly (see Figure 7).

4 IMPLEMENTATION

In order to evaluate the approach in a medical context, the responses of the approach is tested in an open surgery. The approach can be analyzed specifically by simulating the cut of the skin; thus, it is possible to observe the responses of the XFEM exposed to many elements that are small and thin (i.e. low volume), and thereby, the robustness of the approach can be confirmed by the avoidance of sliver. The material parameters of the skin used to test the approach considers a Young's modulus as $1 \times 10^4 Pa$, Poisson ratio of 0.3 and the density of $1000 kg/m^3$.

SOFA framework is used to implement the approach (Allard et al., 2007). In SOFA, a single object

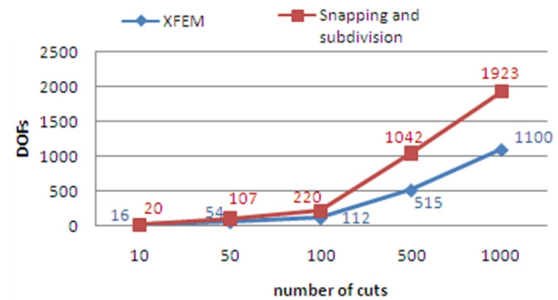


Figure 9: Generation of DOFs (2D): comparing XFEM and Snapping and Subdivision. While the second one impacts the performance, the XFEM simulates an approximated accuracy improving the performance of the simulation.

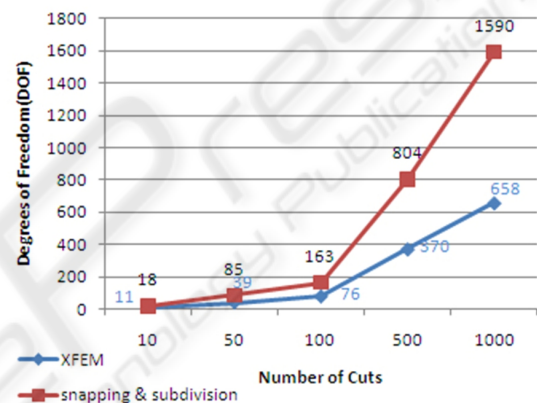


Figure 10: Generation of DOFs(3D): comparing XFEM and Snapping and Subdivision. This figure shows the increase of DOFs when the blade cuts tetrahedral elements. The test has been performed using continuous cuts.

can be represented by multiple geometrical models that are connected through mappings (e.g. barycentric mapping).

4.1 2D: Human Face Skin

A human face model, represented with triangular elements, is compared with a method already included in SOFA (i.e. the snapping and subdivision). The Figure 9 shows that the XFEM generates less DOFs than snapping and subdivision; this difference increases as the number of cuts increase. Therefore, the XFEM allows to simulate physically the discontinuities without harming the performance and the simulation stability; some images of the simulation are shown in Figure 8

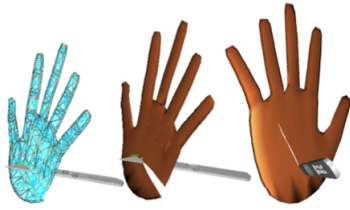


Figure 11: The tetrahedral model of the hand can be cut interactively.

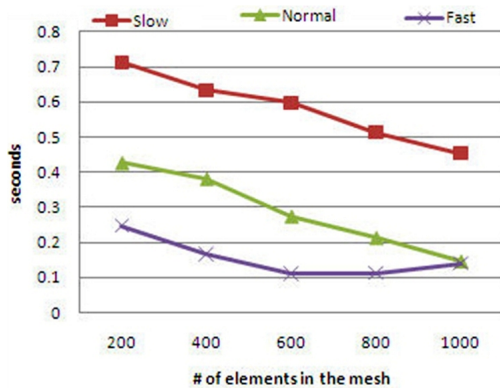


Figure 12: The delay of crossing an element considering different speeds and meshes. If the number of cuts increases the processing time also increases, impacting the performance of the simulation.

4.2 3D: Hand Skin

The test consists of cutting the skin of the dorsal part of hand; this procedure can be applicable in different surgeries (e.g. lipoma removal). The thickness of the skin of the hand is from 0.5 mm to 2 mm in soft regions (Schmidt, 2003), we choose to use 2 mm as thickness of the skin to observe the deepness of the object.

In 3D, the physical topology is represented by a tetrahedral mesh; the visual and collision topologies are represented by a triangular surface mesh. The hand model is tetrahedralized employing the mesh generator called *tetgen*; the force feedback is obtained using the phantom omni. The comparative with snapping and subdivision is presented in Figure 10.

4.3 Other Remarks

In order to generate a real time simulation the time step will be dynamically adapted according to the processing charge. Using an implicit method ensures the stability for all the supported simulation time steps. The resultant images of the simulation of the skin deformed and dissected are shown in Figure 11 and Figure 13. Moreover, the skin can be used together with

other models to simulate an open surgery as is expressed in Figure 15.

As we are using a semi-progressive cutting we analyze the Framework in a very conventional PC with a processor Intel Core 2Duo 2GHz, RAM memory of 2G, the delay depends basically on the refinement of the physical mesh, the speed of the user while cuts and the speed of the execution of the algorithms, as is shown in Figure 12.

5 CONCLUSIONS

In this paper, a framework focused on the efficient application of the XFEM has been designed; this framework includes the embedded design of an alternative mapping method, that allows associating, in an easy and direct manner, the diverse topologies when the cut of an element appears. The mapping also stores the association of point-tetra, which makes the update of the assignation of visual or collision points faster (i.e. vertices), with its corresponding virtual element while the user is cutting; all these, considering the side of the cut plane. Also, a semi progressive cutting algorithm has been created. Additionally, a variety of meshes in 2D and 3D were tested obtaining considerable differences in the creation of nodal DOFs, in consequence, it is proved that the XFEM does not strongly impact the simulation performance, allowing real-time simulations.



Figure 13: Interactive surgery simulation considering topological changes in real time.

However, there are many important aspects to deal with that have not been considered such as: self-collisions of discontinuous elements, multi-resolution techniques, textures in the internal mesh, creation of a fully-progressive cutting algorithm and parallelism of the methods. All these aspects are left as future work.

This work was completely focused on the XFEM, obtaining a simulation that fulfill with stability, accuracy, interactivity and real time; which are the properties required for a virtual surgery simulation.

The inclusion of this framework will make the generation of more complex simulations possible, in which the interaction of diverse models (organs) that act together can be possible and, in this manner, design simulations with major impact in the medical area, such as the extraction of a tumor or the fully physical modeling of one part of the body.

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APPENDIX A. EXTENDED FINITE ELEMENT METHOD (XFEM)

The main idea of exploiting the partition of unity property is to construct basis functions through products of classical shape functions and a local enriched basis; allowing to generate discontinuous elements. Hence, the equation of the displacements can be calculated as

$$u(x) = \underbrace{\sum_{i=1}^n \Phi_i(x) \mathbf{u}_i}_{\text{classical}} + \underbrace{\sum_{j=1}^n \Phi_j(x) \psi_j(x) \mathbf{a}_j}_{\text{enrichment}} \quad (7)$$

where $\Phi_i(x)$ are the classical shape functions; the discontinuous enrichment functions are denoted by $\psi_j(x)$, and the new nodal DOFs as \mathbf{a}_j . The enrichment function $\psi(x)$ can be any discontinuous function; commonly, it is the Heaviside function (Eq. 8), another option is the shifted function defined in Eq. 9.

$$\Psi(x) = H(x) = \begin{cases} +1 & \text{above the crack} \\ -1 & \text{below the crack} \end{cases} \quad (8)$$

$$\psi_i(x) = \frac{1}{2}(H(x) - H_i) \quad (9)$$

where H_i is the value of Heaviside function at the i -th node. The shifted function consists in using the enrichment contribution only inside of the discontinuous element and ignores the contribution on the borders and outside of the element.

To keep the delta property, the displacements of the enriched nodes has to be computed as the sum of the components $\mathbf{u}_i + \Psi_i \mathbf{a}_i$. The shifted function in contrast with the Heaviside, directly stores the values of the displacement in \mathbf{u}_i and the added DOFs \mathbf{a}_i are only required to establish the displacements of an enriched element.

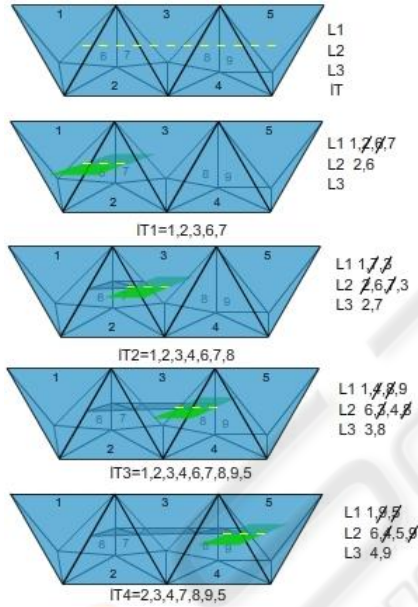


Figure 14: Cutting algorithm: elements 1-5 are frontal elements and 6-9 internal elements. When the cut starts, the first element found is tetrahedron 1, its neighbors are obtained and stored in list IT (list of tetrahedrons), the neighbors are obtained considering all tetrahedrons that share at least one vertex. For all neighbors, we search those whose collide with the collision quad. If an element has one, two or three edges intersecting are stored respectively in lists L1,L2,L3. After crossing an element, a new internal mesh is created considering the intersection points. If an element is recorded in list L2 or L3 must be deleted from the previous list. In order to ensure that the element is fully crossed, the crossed edges of the element must be different from the others stored in list.

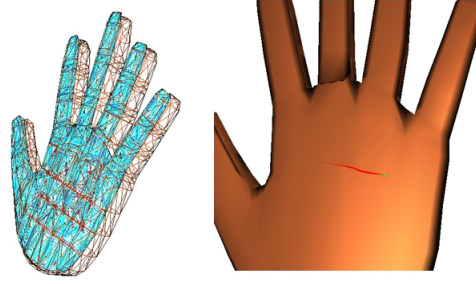


Figure 15: The skin of the hand can be used to simulate an open surgery.

APPENDIX B. CO-ROTATIONAL FEM

The corotational method is based on the linear FEM using Cauchy's strain tensor; this method stores a reference state of the elements. To compute the deformation forces, these have to be translated to the reference state and rotated back to the current state. Therefore, the deformation forces are expressed as follows

$$\mathbf{f}_i = \mathbf{R} \sum_{j=1}^n \mathbf{K}_{ij} (\mathbf{R}^T \mathbf{p}_j - \mathbf{p}_{0j}) \quad (10)$$

where \mathbf{R} is the rotation matrix is required to translate from the current to initial state, \mathbf{p} are the positions of the element nodes in the current deformed state defined as $\mathbf{p} = \mathbf{p}_0 + \mathbf{u}$, in which \mathbf{p}_0 indicates the initial positions of the element nodes.