A PRACTICAL APPROACH FOR APPLYING NON-LINEAR DYNAMICS TO PARTICLE SYSTEMS

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Abstract: In this paper we present a new method for approximating non-linear dynamics in deformable simulations based on a full cubic polynomial formulation of the inter-particle force. The proposed approach is a non-physical extension of the well known St Venant Kirchhoff force, focusing on practical considerations about the behavior of the deformable model. Therefore it is very useful for simulating and controlling non-linear stretching and compressing properties of deformable models in applications where the computation time comprises a major constraint. The presented force model can be easily implemented by the widely used particle systems and can be parameterized based on a direct relation to spring models. We show that the non linear stretching behavior of the proposed model is required for simulations where the external forces which interact with the models are large in scale, while the performance of the presented method is comparable to simple mass-spring systems.

1 INTRODUCTION

The simulation of deformable objects has become a significant component of computer graphics applications for increasing the level of realism and believability of the visual outcome since textiles and elastic objects are a major component of real environments. An important requirement of modern simulations is the ability to efficiently respond to different scales of external forces and constraints while both the stability of the system and the level of realism remain unaffected (Choi and Ko, 2005; Nealen et al., 2006).

Especially in the case that strong forces are applied to the deformed objects, the material should behave realistically and resist further stretching without collapsing. Linear spring models cannot handle this kind of situations, since they allow overelongation and generally behave realistically only for small deformations (Etzmuss et al., 2003). One solution would be to use springs with high stiffness values. However, this is aggravating for numerical stability and leads to less flexible materials which do not always result to the desired simulation outcome.

The physically "correct" solution to this problem would be to employ non-linear continuum models (Picinbono et al., 2003; Barbič and James, 2005). These models can effectively simulate materials which are able to handle large deformations at the stretch, strain and/or bend direction. However, these models are implemented under the scheme of sophisticated finite element formulations. Even though finite element methods have been well established in the computer graphics literature, they are not yet as fast as particle systems and usually pose a significant degree of implementation difficulty to non experts in the field. Furthermore, they give little intuition about the direct control over the behavior of the modeled object, making them less appealing to animation interfaces (Nealen et al., 2006).

As a consequence, the necessity of employing non-linear models with simpler implementation schemes, such as particle systems, has recently received the attention of researchers (Delingette, 2008; Volino et al., 2009; Kikuuwe et al., 2009). These methods employ continuum formulations of hyperelastic materials, namely the St Venant Kirchhoff materials, in order to provide realistic high performance simulations of deformable solids or cloths.

Our work is largely motivated by this recent direction on the field, although we focus more on practi-

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cal considerations about the behavior of the deformed model and deliberately ignore complex physical formulations for the sake of simplicity and performance.

In particular, the proposed method is based on a full cubic polynomial formulation of the inter-particle force of discrete particle systems. This type of force proves to be very useful in simulations where the external forces take relatively high values and therefore non-linear stretching and compressing properties are necessary to ensure stability and realism, both in cloth and deformable solids simulations. We also present a constraint based parameterization of the force which provides a direct relation to spring models and allows adjustment of the model behavior according to practical considerations, such as the level of allowed stretching or compression.

The paper is organized as follows: Section 2 briefly discusses part of the work in the field focusing on particle systems and methods that connect continuum models to particle systems. Section 3 describes thoroughly the proposed method while section 4 gives the final details of integrating the proposed model to particle systems. The results are presented in section 5 and the final conclusions are drawn in section 6.

2 RELATED WORK

Since the pioneering work of Terzopoulos (Terzopoulos et al., 1987), deformable object modeling has received significant attention from the computer graphics community and various methods from the theory of classical mechanics have been introduced under the scope of graphics applications (Nealen et al., 2006; Moore and Molloy, 2007). These methods are implemented either as particle systems (Breen et al., 1994; Baraff and Witkin, 1998; Choi and Ko, 2002; Goldenthal et al., 2007; Selle et al., 2008; Selle et al., 2009) or as finite elements (FE) that are based on continuum formulations (Irving et al., 2004; Barbič and James, 2005; Duriez et al., 2006; Irving et al., 2007). In general, FE methods are more accurate and robust while particle systems are simpler to implement and usually faster. Therefore, there is a strong effort in the field for combining the sophisticated properties of continuum dynamics that FE methods facilitate with the computational performance of particle systems (Etzmuss et al., 2003; Lloyd et al., 2007).

More recently, methods for applying non linear dynamics to particle systems have been presented (Delingette, 2008; Volino et al., 2009; Kikuuwe et al., 2009). In particular, these methods do not perform any linearization on the Green-Lagrange tensor in the continuum model by employing St Venant Kirchhoff (StVK) materials. Thus, they can intrinsically describe non-linear properties of deformable models. In order to apply these properties along with particle systems they proposed quite different techniques. (Delingette, 2008) introduced the concept of angular biquadratic springs for simulating membranes, i.e. elastic 2D manifolds. (Volino et al., 2009) calculate the inter-particle forces for cloths directly from the triangle structure of the cloth manifold, instead of the edge-wise approach. (Kikuuwe et al., 2009) derived formulations that describe the inter-particle forces on the edges of a regular tetrahedral based volume by introducing the term of tetrahedron-sharing edge pairs.

We also exploit the same approach to provide a simple yet effective method for simulating and controlling non-linear stretching properties of deformable models with particle systems. In particular, we present a network of non-linear springs, similar to the one of (Kikuuwe et al., 2009), by introducing a full cubic polynomial force which is formulated and parameterized directly from practical considerations about the model behavior. Our work can be considered as an intuitive perspective to non-linear dynamics that can be used directly by existing particle systems to simulate robust and adjustable deformable models at high performance rates.

3 PROPOSED METHOD

Non linear springs have been previously used in particle systems (Breen et al., 1994) and generally they are reported as the natural outcome of describing forces from non-linear strain tensors, such as the Green-Lagrange tensor (Delingette, 2008; Kikuuwe et al., 2009; Volino et al., 2009). In the 1D case, the force $f_{i,j}$ between the particles p_i and p_j that corresponds to this type of material is given by

$$f_{i,j} = \frac{K}{2L^2}\Delta x_{i,j}{}^3 - \frac{K}{2}\Delta x_{i,j} \qquad (1)$$

where $\Delta x_{i,j}$ denotes the 1D distance between the particles, L is the rest length, where the force is zero, and K is the stiffness constant.

The above equation is a cubic polynomial describing the 1D force of StVK materials (Barbič and James, 2005; Delingette, 2008; Kikuuwe et al., 2009; Volino et al., 2009). The extension to 2D manifolds and 3D solids is quite complex since the continuum structure of the materials will introduce more terms in the above force. In particular, the additional terms correspond to the tetrahedron-sharing edge pairs and are reported to be the major difficulty for extending non linear 1D forces to higher dimensions (Kikuuwe et al., 2009).

Even so, we can ignore any additional terms that are induced by the continuum structure of higher dimensions and apply this force directly to 3D. Essentially, we employ a physically based formulation only for the behavior of the inter-particle distance $|\Delta\mathbf{x}_{i,j}|$, which is a 1D problem (Etzmuss et al., 2003). A particle system (or spring network) with this type of nonlinear springs has been described by (Kikuuwe et al., 2009) as a sub-outcome of their work. The authors reported that although this model does not seem to have any physical validity, it produces similar results to the full non-linear model while it is significantly faster. To our knowledge, this discrete formulation is probably the most efficient, in terms of performance and realism, non-linear mass spring system for deformed solids in the literature.

In practice, however, there are some concerns about the StVK force. Firstly it does not resist to compression as does with stretching, e.g. the material is softer under compression, while, in the case of deformable solids, there is still the possibility of completely collapsing (Irving et al., 2004). This is a main limitation that made StVK generally less appealing to solid modeling. However, the recent work of (Kikuuwe et al., 2009) showed very promising results for large deformations of arbitrary rotation and the proposed method can been seen as an improvement to this approach regarding the compression problem. Secondly these models are not reported to be easily adjusted to external requirements, e.g. 3D modeling adjustments by artists or forces applied from inside the 3D world of the simulation, and focus mainly on accuracy (Volino et al., 2009; Delingette, 2008) or real time interaction with the deformed objects (Barbič and James, 2005; Volino et al., 2009).

Both drawbacks relate to the simple fact that the StVK force is a cubic polynomial with only two terms. A solution previously applied to this problem was the use of a linear - Neo Hookean biphasic spring force model (Irving et al., 2004). Although this approach solves the compression issue efficiently, the pitfall is that biphasic forces need special treatment for calculating derivatives (needed for the Jacobian) and generally their parameterization is more complex since two different forces are combined. In this work, we present a more intuitive approach to solve these issues utilizing a full cubic force on a discrete particle system and a constraint based approach for calculating the parameters of this force. The presented force has continuous derivatives, is simple to integrate and parameterize and facilitates useful non-linear deformation properties.

3.1 Full Cubic Force

In the rest of the text, the term *force* refers to the vector entity while *force magnitude* refers to the signed magnitude of the force. Let the 3D vector $\mathbf{x}_i$ denote the position of the particle p_i and again $\Delta\mathbf{x}_{i,j} = \mathbf{x}_i - \mathbf{x}_j$. Considering the force magnitude as a full cubic polynomial, we can extend such expression to 3D as

$$\mathbf{f}_{i,j} = \left(a|\Delta\mathbf{x}_{i,j}|^3 + b|\Delta\mathbf{x}_{i,j}|^2 + c|\Delta\mathbf{x}_{i,j}| + d\right)\frac{\Delta\mathbf{x}_{i,j}}{|\Delta\mathbf{x}_{i,j}|} \tag{2}$$

The parameters a, b, c and d that define the force, and consequently the behavior of the cloth, do not have a direct physical interpretation. In the following section, however, we will describe a method to relate these parameters to spring models.

The respective force derivative is given by

$$\frac{\partial\mathbf{f}_i}{\partial\mathbf{x}_j} = \mathbf{I}\left(a|\Delta\mathbf{x}_{i,j}|^2 + b|\Delta\mathbf{x}_{i,j}| + c + \frac{d}{|\Delta\mathbf{x}_{i,j}|}\right) + \Delta\mathbf{x}_{i,j}\Delta\mathbf{x}_{i,j}{}^T\left(2a + \frac{b}{|\Delta\mathbf{x}_{i,j}|} - \frac{d}{|\Delta\mathbf{x}_{i,j}|^3}\right) \tag{3}$$

where $\mathbf{I}$ denotes the $3x3$ identity matrix.

3.2 Constraint based Force Parameterization

The major problem with the intuitive step that we took by employing a full cubic force is the calculation of the respective parameters. In general, a set of parameters can de defined using a corresponding system of equations. Based on this, we can define four constraints on the force magnitude so that the particles will resist to deformation.

Let $g(|\Delta\mathbf{x}|)$ denote the force magnitude as a function of the distance $|\Delta\mathbf{x}|$ between two particles, i.e.

$$\mathbf{f}_{i,j} = g(|\Delta\mathbf{x}|)\frac{\Delta\mathbf{x}}{|\Delta\mathbf{x}|}$$

where particle indexes are omitted for brevity.

The first constraint should “enforce” the rest length L of the particles by setting the force equal to zero, i.e.

$$g(L) = 0 \tag{4}$$

The other constraints should be related to the stiffness induced by the non-linear force. In the case of simple spring forces, the stiffness constant K defines the ratio that the force magnitude changes according to the deformation, i.e. the slope of the force magnitude - deformation curve. Thus, we can define a force magnitude such that the stiffness increases with the distance between the particle in order to resist further elongation.

Figure 1 displays the concept. The spring force has constant stiffness and even though this property is good for numerical stability and performance, it does not describe textiles well enough. The StVK materials resist to stretching much better than simple springs but are softer under compression. The proposed approach aims at approximating the expected deformable behavior where the internal forces resist increasingly both to stretch and compression.

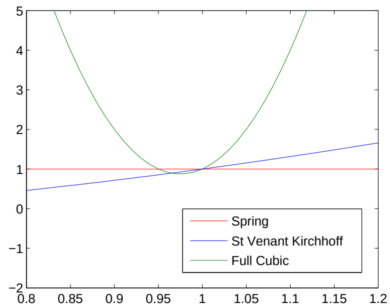

Figure 1: The stiffness ratio expressed as the derivative of the force magnitude for the spring, St Venant Kirchhoff and the proposed (full cubic force) model. The rest length is set to $L = 1$ and the spring constant to $K = 1$. The stiffness of the cubic force increases with the particle distance, significantly resisting to compression and stretching compared to the other two methods.

Based on a given spring model, we can set one stiffness constant for the rest length and the other two for percentages a_1 and a_2 of the rest length

$$g'(L) = K \tag{5}$$

$$g'(a_1 L) = K_1 \tag{6}$$

$$g'(a_2 L) = K_2 \tag{7}$$

where K, K_1 and K_2 are the desired constant stiffness values for different deformations rates and g' denotes the derivative of g. For example, a considerably rational choice would be to set $K_1 > K$ and $K_2 > K$ for $a_1 = 1.1$ and $a_2 = 0.9$ since, in most applications, stretching above 10% should be avoided. In all the parameters can be calculated by solving the following linear system

$$\begin{bmatrix} L^3 & L^2 & L & 1 \\ 3L^2 & 2L & 1 & 0 \\ 3(a_1L)^2 & 2a_1L & 1 & 0 \\ 3(a_2L)^2 & 2a_2L & 1 & 0 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} 0 \\ K \\ K_1 \\ K_2 \end{bmatrix} \tag{8}$$

Figure 2 displays the magnitude of the proposed force for different values of K_1. In the case of stretching (i.e. $|\Delta x| > 1$) the proposed model behaves similar to the StVK force. This is something expected since the StVK force magnitude can be considered as a special case of the proposed force with $b = d = 0$. However, in the case of compression the proposed force increases its stiffness, compared to the StVK force which after a certain value completely collapses. Note also that for low deformation rates (i.e. $|\Delta \mathbf{x}| \approx 1$) the full cubic force behaves similar to the linear spring force. That is a desired feature since we usually want to allow small deformations and resist only to large ones.

In the case of clothes, the expected stretching behavior is practically identical to the StVK force model, as it had been shown by experimental results (Volino et al., 2009; Wu et al., 2003). Since the proposed model is a good approximation of the StVK force, it can describe sufficiently enough the stretching properties of cloths.

The constraints 4, 5, 6 and 7 provide a connection between the parameters of the proposed force model and spring forces. Using for reference a set of spring parameters K and L, we can adjust the parameters a_1, a_2, K_1 and K_2 in order to achieve the desired results, according to practical considerations in cloth simulations. In other words, the introduction of a constraint based force parameterization provides direct manipulation over the behavior of the object.

For example, models that behave like cloths have been previously noted that should not exceed 10% (Provot, 1995) in order to look realistic. This feature is explicitly described by the force constraints of the proposed method making it more suitable for adjusting cloth parameters by someone with none in-depth knowledge of mechanics, like a 3D artist. By increasing or decreasing the desired deformation percentage results in softer or stiffer materials that are, generally, not textiles.

4 TIME INTEGRATION

Equations 2 and 3 can be applied directly to any discrete model governed by the Newton's second law of motion $\mathbf{f} = m\mathbf{a}$. The proposed force model is employed under an implicit integration scheme as described in (Baraff and Witkin, 1998). The implicit solver is necessary for our approach as non-linear forces generally result in stiffer equations and require more stable solvers than simple linear forces. The scheme is implemented using full evaluation of the Jacobian (Volino et al., 2009). In our test, two itera-

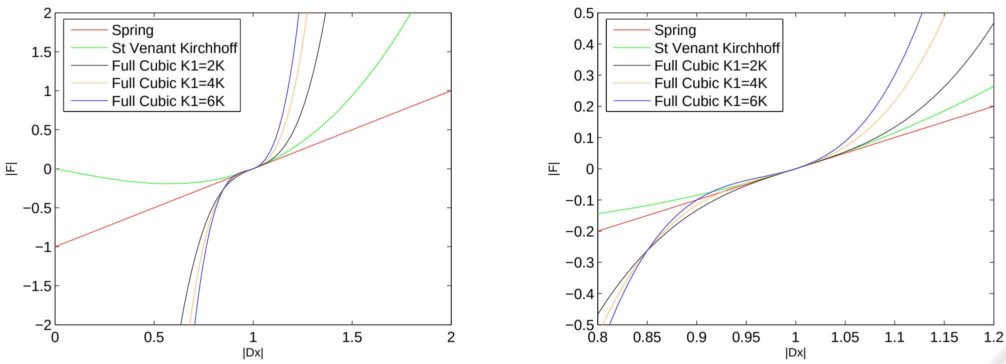

Figure 2: Comparison of the force magnitude between spring, St. Venant Kirchhoff and the proposed (full cubic force) model. Global view (left) and zoom (right). The rest length is $L=1$ and the spring constant is $K=1$. The constraint parameters for calculating the cubic force are set to $a_1=1.1$, $a_2=0.9$, $K_2=2K$ and K_1 varies for each curve ($K_1=2K$ for black, $K_1=4K$ for orange and $K_1=6K$ for blue). The proposed model can increase the resisting force at higher rates than the other models.

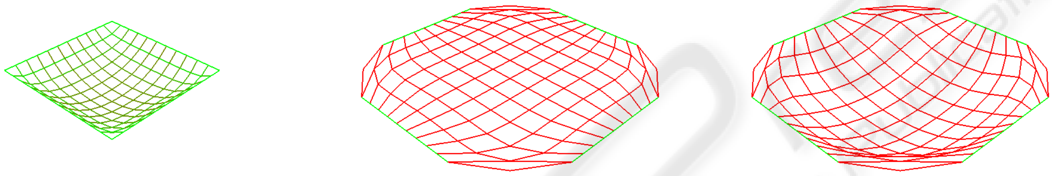

Figure 3: Removing out-of-plane oscillations. A low resolution lattice of particles, shown at the state resting under its own weight(left), is significantly stretched (effectively doubling the 2D area of the manifold) with (middle) and without the proposed damping (right). Green lines denote lower stretching while red lines denote higher stretching. The figure on the right shows a phase of the system oscillations, i.e. the system cannot converge to a stable state. Even though the proposed force model is quite stable in simple draping situations, under very high stretch it is necessary to employ the proposed damping scheme.

tions were enough for the conjugate gradient method to converge to a physically plausible solution, just as (Baraff and Witkin, 1998).

4.1 Damping

Damping plays a significant role in the stability of the numerical system (Nealen et al., 2006; Baraff and Witkin, 1998; Schmedding et al., 2009). In general, damping derives from both the intrinsic properties of the modeled material and the artificial damping induced by the numerical integration scheme. A simple and widely used approach is to apply a damping force along the direction of the relative velocity between two particles (Choi and Ko, 2002; Nealen et al., 2006; Schmedding et al., 2009). We also employ this method in our work. However, the non-linear properties of the full cubic force magnitude result in high stiffness and therefore the system is sensitive to yield undesired out-of-plane oscillations that the relative velocity damping is not guaranteed to resolve. In particular, if we express the derivative of the force as

$$\frac{\partial \mathbf{f}_i}{\partial \mathbf{x}_j} = \mathbf{I}c + a|\Delta\mathbf{x}_{i,j}|^2\left(\mathbf{I}+2\frac{\Delta\mathbf{x}_{i,j}\Delta\mathbf{x}_{i,j}^T}{\Delta\mathbf{x}_{i,j}^T\Delta\mathbf{x}_{i,j}}\right)+ b|\Delta\mathbf{x}_{i,j}|\left(\mathbf{I}+\frac{\Delta\mathbf{x}_{i,j}\Delta\mathbf{x}_{i,j}^T}{\Delta\mathbf{x}_{i,j}^T\Delta\mathbf{x}_{i,j}}\right)+ \frac{d}{|\Delta\mathbf{x}_{i,j}|}\left(\mathbf{I}-\frac{\Delta\mathbf{x}_{i,j}\Delta\mathbf{x}_{i,j}^T}{\Delta\mathbf{x}_{i,j}^T\Delta\mathbf{x}_{i,j}}\right) \quad (9)$$

we can distinguish between the effect of each parameter a, b, c and d to the Jacobian. In general, parameters with negative values may introduce negative eigenvalues in the system that will affect stability.

Solving the linear system (8) for the four parameters, b and d take negative values (considering that $K<K_1$, $K<K_2$ and L, a_1, a_2 take positive values). Especially in the case of parameter d, the term $\left(\mathbf{I}-\frac{\Delta\mathbf{x}_{i,j}\Delta\mathbf{x}_{i,j}^T}{\Delta\mathbf{x}_{i,j}^T\Delta\mathbf{x}_{i,j}}\right)$ expresses stiffness orthogonal to the interaction direction, i.e. out-of-plane direction, which is something that can cause problems in simulations (Choi and Ko, 2002).

In previous approaches, terms that end up to negative eigenvalues (such as the two last terms of the right part of equation 9) were either disregarded (Volino and Thalmann, 2000; Choi and Ko, 2002) or their effect was reduced using specially designed damping forces (Baraff and Witkin, 1998). Completely disregarding these terms resulted in excessively resisting to bending in our simulations. Instead of this, we reduced, by a small percentage, the absolute values of the parameters b and d only in the calculation of the Jacobian (effectively "damping" specific terms of the Jacobian). This way we can avoid adding more forces in our scheme and effectively reduce out-of-plane oscillations.

Figure 3 shows an example of a particle system with and without the proposed damping. The system is fixed on the edges and is left draping under its own weight. By moving further the fixed edges, the system is stretched reaching to a final position that is almost twice the the size of the textile at its rest state. This excessive stretching generates high valued interparticle forces that also "stretch" the numerical solver (in the draping state, i.e. low stretch, the force values are close to the ones of a simple linear spring and no safe conclusions can be drawn about the stability of the proposed force). When the proposed damping scheme is not used, the system cannot converge to a stable state and oscillates in the out-of-plane direction (Figure 3 right). If we reduce the parameters b and d to 99% of their absolutes values in the calculation of the Jacobian then the system performs robustly and converges to a stable stretched state (Figure 3 middle).

Note that a consequence of this kind of damping is that it artificially dissipates energy from the system and resists to bending (Volino and Thalmann, 2000; Choi and Ko, 2002; Baraff and Witkin, 1998). Even so, it is still possible to ignore it in applications that high stretching is not directly applied to the particle system without affecting the stability of the system.

5 EXPERIMENTAL RESULTS

We have tested the proposed method in technical simulations, both with solids and textiles, that involve high scales of forces applied to the deformed materials, yet these forces are generated directly from conditions of the virtual worlds. We also evaluated the effect of the constraint parameters (section 3.2) to the behavior of the cloth under simple draping. The whole experimental setup was implemented using C++ on a Core2 6600 2,4GHz CPU PC with 2GB of RAM and a GeForce 7600 GS Graphics Card.

For comparison we have also implemented: 1) a simple linear model, 2) the StVK triangle based method described in (Volino et al., 2009) (we refer

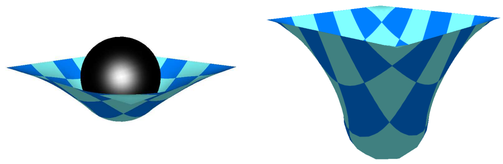

Figure 4: A heavy ball falling into a cloth. The weight of the ball is 10 kg, effectively 10 times the total weight of the cloth. The particle system consists of 1024 and the parameters of the system are: $L = 2cm$, $a_1 = 1.04$, $a_2 = 0.96$, $K = 1000N/m$, $K_1 = 6K$, $K_2 = 2K$. The proposed method can robustly handle the heavy ball (left) while a mass-spring system set to the same base stiffness K allows unrealistic over-elongation of the cloth (right). Testing larger values of K led to partially or totally unstable simulations.

Figure 5: A heavy cube hanging from a cloth. The weight of the cube is 5 kg, effectively 5 times the weight of the cloth. The rest of the parameters are set to $L = 2cm$, $K = 500N/m$, $a_1 = 1.1$, $a_2 = 0.9$, $K_2 = 2K$, $K_1 = 6K$. Again, testing larger values of K led to partially or totally unstable simulations.

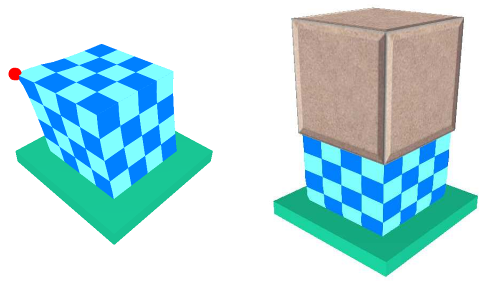

Figure 6: Deformable cube simulation composed of 729 particles. The cube can react robustly both under low deformation (left), such as linear models do, but can also resist to high compression (right). In this test (right) neither the linear or the StVK models managed to resist compression and completely collapsed, resulting in unacceptable visual outcome. The cube weight was 5kg which is almost 7 times the mass of the deformed cube. The rest of the parameters are: $L = 2cm$, $K = 1000N/m$, $a_1 = 1.1$, $a_2 = 0.9$, $K_2 = 10K$, $K_1 = 6K$.

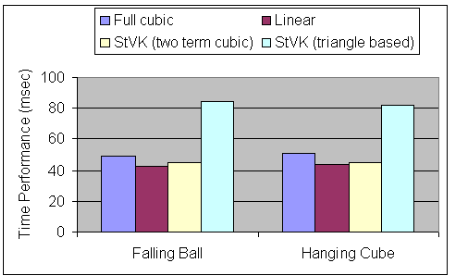

Figure 7: Timing Performance of the compared methods. The StVK (two term cubic) model refers to method of (Kikuuwe et al., 2009) while the StVK (triangle based) refers to the one of (Volino et al., 2009). The proposed method is insignificantly slower (10%) than the linear and the two term cubic models while it is considerably faster (40%) than the triangle based StVK.

to it as *triangle based* because the calculations require knowledge of the triangle structure of the mesh) and 3) the spring network of (Kikuuwe et al., 2009), which essentially employs a cubic force with two terms. Methods 1 and 3 were tested in all simulations while method 2 was tested only in cloth simulations since it is specialy described for these purposes. We have not compared with any full FEM method since recent results of (Kikuuwe et al., 2009) showed that method 3 is much faster than full FE implementations. Note also that this comparison is performed under the scope of a general evaluation of the results, since none of these works specifically aimed at simulating adaptive models that resist high value forces. To our knowledge, they are not any methods in the literature that have dealt with this particular problem.

Figure 4 shows a heavy ball falling into the surface of a cloth. The proposed force model allows small deformations of the cloth so that it looks loose (and not stiff) while can also resist to the weight of the ball. A linear force which allows the same level of small deformation is unable to realistically hold the ball. Figure 5 illustrates a similar example where one edge of a cloth is connected to a heavy cube while the other one is fixed. Again the proposed approach manages to resist over-elongation compared to a linear model with the same parameters.

Figure 6 shows an example of a deformable solid cube modeled by a particle system and the proposed force. The proposed method is able to react both to small deformations, like the interactive stretching displayed in the left image, and also resist the compression caused by the heavy cube. The linear model resulted in similar behavior for small deformation but completely collapsed under compression. Likewise, while in both cloth applications the StVK methods produced similar results with the proposed method

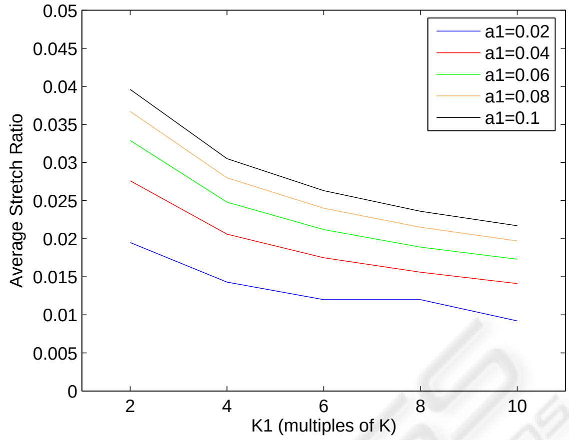

Figure 8: Deformation rates for different constraint parameters. For testing stretching, we measured the average deformation ratio of a cloth with one edge fixed draping under its own weight. The results show that we can achieve similar results with a different combination of the parameters, thus the proposed constraint based parameterization is very adaptive to application requirements.

(allowing a bit more elongation which did not affect much the level of realism however), we could not produce a visually acceptable result using the two-term StVK force of (Kikuuwe et al., 2009) (the triangle-based StVK does not apply directly to solids) since StVK materials are softer under compression and eventually collapse due to the weight of the heavy cube.

In figure 7 we display the comparative timing results for the two cloth applications. The timing calculations include the calculation of the force values, the calculation of the Jacobian and the iterations of the Conjugate Gradient. The proposed method is approximately 5-10% slower than the linear model and (Kikuuwe et al., 2009) which is something expected since these models employ computationally simpler forces. However, note that the linear method failed in all simulations, (Kikuuwe et al., 2009) failed in the compression of the deformed solid and the time difference is not so significant. Compared to the continuum based StVK of (Volino et al., 2009), the proposed method is approximately 40% faster.

In the last figure 8 we display the average deformation rate of a cloth according to different constraint parameters. The cloth was simply left draping under its own weight until it stopped. The diagram shows that the proposed model can effectively produce similar deformation ratios with various combinations of the constraints.

This allows the user to adjust the parameters according to the restrictions imposed from different parts of the application. For example, if large values of K are not desired but the average stretch ratio should remain at a low level (e.g. solids that practically keep their volume but have soft surface), then a lower value for a_1 can achieve the desired result without this increase in stiffness K.

6 CONCLUSIONS

In this paper we presented a particle system that utilizes a full cubic formulation of the inter-particle force to apply non-linear deformation resistance to simulation conditions of high scale. We also introduced a constraint based force parameterization for adjusting the proposed force and described the details regarding the necessary damping that the presented non-linear scheme requires in order to ensure numerical stability. The proposed method is conceptually simple to implement and parameterize according to practical considerations. We showed that the presented method can robustly handle deformable models that are subject to environment forces of high scale, both for stretching and compression. Since the presented force is implemented with particle systems, it has very efficient timing performance, comparable to simple linear models.

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