KINEMATIC IDENTIFICATION OF PARALLEL MECHANISMS BY A DIVIDE AND CONQUER STRATEGY

Sebastián Durango[a], David Restrepo[a], Oscar Ruiz[a], John Restrepo-Giraldo[b] and Sofiane Achiche[b]
[a]CAD CAM CAE research laboratory, EAFIT University, Medellín, Colombia
[b]Management Engineering Dept., Technical University of Denmark, Lyngby, Denmark

Keywords: Parallel mechanisms, Kinematic identification, Robot calibration.

Abstract: This paper presents a Divide and Conquer strategy to estimate the kinematic parameters of parallel symmetrical mechanisms. The Divide and Conquer kinematic identification is designed and performed independently for each leg of the mechanism. The estimation of the kinematic parameters is performed using the inverse calibration method. The identification poses are selected optimizing the observability of the kinematic parameters from a Jacobian identification matrix. With respect to traditional identification methods the main advantages of the proposed Divide and Conquer kinematic identification strategy are: (i) reduction of the kinematic identification computational costs, (ii) improvement of the numerical efficiency of the kinematic identification algorithm and, (iii) improvement of the kinematic identification results. The contributions of the paper are: (i) The formalization of the inverse calibration method as the Divide and Conquer strategy for the kinematic identification of parallel symmetrical mechanisms and, (ii) a new kinematic identification protocol based on the Divide and Conquer strategy. As an application of the proposed kinematic identification protocol the identification of a planar 5R symmetrical mechanism is simulated. The performance of the calibrated mechanism is evaluated by updating the kinematic model with the estimated parameters and developing simulations.

1 INTRODUCTION

In mechanisms and manipulators the accuracy of the end-effector critically depends on the knowledge of the kinematic model governing the control model (Zhuang et al., 1998). Therefore, to improve the accuracy of a mechanism its kinematic parameters have to be precisely estimated by means of a kinematic identification procedure (Renaud et al., 2006).

Kinematic identification is an instance of the robot calibration problem. The estimation of rigid-body inertial parameters and the estimation of sensor gain and offset are instances of calibration problems at the same hierarchical level of the kinematic calibration problem (Hollerbach et al., 2008).

This paper is devoted to the kinematic identification of parallel symmetrical mechanisms. Parallel mechanisms are instances of closed-loop mechanisms typically formed by a moving platform connected to a fixed base by several legs. Each leg is a kinematic chain formed by a pattern of links, actuated and passive joints relating the moving platform with the fixed base. If the pattern of joints and links is the same for each leg and each leg is controlled by one actuator, then the parallel mechanism is denoted symmetrical (Tsai, 1999).

For parallel mechanisms the kinematic identification is usually performed minimizing an error between the measured joint variables and their corresponding values calculated from the measured end-effector pose through an inverse kinematic model (Zhuang et al., 1998; Renaud et al., 2006). This method is preferred for the identification of parallel mechanisms because:

1. Inverse kinematics of parallel mechanisms is usually derived analytically avoiding the numerical problems associated with any forward kinematics solution (Zhuang et al., 1998; Renaud et al., 2006).
2. The inverse calibration method is considered to be the most numerically efficient among the identification algorithms for parallel mechanisms (Renaud et al., 2006; Besnard and Khalil, 2001).
3. With respect to forward kinematic identification no scaling is necessary to balance the contribution of position and orientation measurements (Zhuang et al., 1998).

In the case of parallel symmetrical mechanisms

Durango S., Restrepo D., Ruiz O., Restrepo-Giraldo J. and Achiche S. (2010).
KINEMATIC IDENTIFICATION OF PARALLEL MECHANISMS BY A DIVIDE AND CONQUER STRATEGY.
In *Proceedings of the 7th International Conference on Informatics in Control, Automation and Robotics*, pages 167-173
DOI: 10.5220/0002911901670173

the inverse kinematic modeling can be formulated using independent loop-closure equations. Each loop-closure equation relates the end-effector pose, the geometry of a leg, and a fixed reference frame. In consequence, an independent kinematic constraint equation is formulated for each leg forming the mechanism. For the case of parallel symmetrical mechanisms the set of constraint equations is equal to the number of legs and to the number of degrees of freedom of the mechanisms. Each kinematic constraint equation can be used for the independent identification of the parameters of the leg correspondent to the equation.

The independent identification of the kinematic parameters of each leg in parallel mechanisms allows to improve:

1. The numerical efficiency of the identification algorithm (Zhuang et al., 1998).

2. The kinematic calibration performance by the design of independent experiments optimized for the identification of each leg.

The independent identification of leg parameters in parallel mechanisms was sketched in (Zhuang et al., 1998) and developed for the specific case of Gough platforms in (Daney et al., 2002; Daney et al., 2005). However, the idea of the independence in the kinematic identification of each leg in a parallel mechanism is not completely formalized.

This article presents a contribution to the improvement of the pose accuracy in parallel symmetrical mechanisms by a kinematic calibration protocol based on inverse kinematic modeling and a divide and conquer strategy. The proposed divide and conquer strategy takes advantage of the independent kinematic identification of each leg in a parallel mechanism not only from a numerical stand point but also from the selection of the optimal measurement set of poses that improves the kinematic identification of the parameters of the leg itself.

The layout for the rest of the document is as follows: section 2 develops a literature review on the inverse calibration of parallel mechanisms method, section 3 presents the divide and conquer identification of parallel mechanisms strategy, section 4 develops a kinematic identification of parallel mechanisms protocol, section 5 presents the simulated kinematic identification of a planar 5R symmetrical mechanism using the identification protocol, finally, in section 6 the conclusions are developed.

2 LITERATURE REVIEW

The modeling of mechanical systems include the design, analysis and control of mechanical devices. An accurate identification of the model parameters is required in the case of control tasks (Hollerbach et al., 2008). Instances of models of mechanical systems includes kinematic, dynamic, sensor, actuators and flexibility models. For parallel mechanisms updating the kinematic models with accurately estimated parameters is essential to achieve precise motion at high-speed rates. This is the case when parallel mechanisms are used in machining applications (Renaud et al., 2006).

The inverse calibration method is accepted as the natural (Renaud et al., 2006; Zhuang et al., 1998) and most numerically efficient (Besnard and Khalil, 2001) among the identification algorithms for parallel mechanisms. The inverse calibration method is based on inverse kinematic modeling and a external metrological system. The calibration is developed minimizing an error residual between the measured joint variables and its estimated values from the end-effector pose though the inverse kinematic model. The derivation of the inverse kinematic model of parallel mechanisms is usually straightforward obtained (Merlet, 2006).

For our Divide and Conquer kinematic calibration strategy we adopt the inverse calibration method. The method takes advantage of an intrinsic characteristic of parallel mechanisms: the straightforward calculation of the inverse kinematics. However, not all the intrinsic characteristics of parallel mechanisms are exploited. Specifically, (Zhuang et al., 1998; Ryu and Rauf, 2001) reported that for parallel mechanism, methods based on inverse kinematics allow to identify error parameters of each leg of the mechanism independently. The independent parameter identification of each leg is reported to improve the numerical efficiency of the kinematic identification algorithm, (Zhuang et al., 1998). However, it is not reported a general kinematic identification strategy based on the independent identification of the legs and its advantages with respect to traditional identification methods.

This article presents a contribution to the kinematic calibration of parallel mechanisms developing a kinematic identification protocol based on the inverse calibration method and on the independent identification of the parameters of each leg (Divide and Conquer strategy).

With respect to traditional identification methods, our Divide and Conquer strategy has the following advantages:

1. The identification poses can be optimized to the identification of reduced sets of parameters (the sets corresponding to each leg).

2. The independent identification of the parameters

of each leg improves the numerical efficiency of the identification algorithms.
3. By 1. and 2. the identified set of parameters is closer to the real (unknown) set of parameters than sets identified by other traditional calibration methods.

The divide and Conquer strategy for the independent kinematic identification of the parameters of each leg in a parallel symmetrical mechanism is presented in section 3.

3 DIVIDE AND CONQUER IDENTIFICATION STRATEGY

Parallel symmetrical mechanisms satisfy (Tsai, 1999):
1. The number of legs is equal to the number of degrees of freedom of the end-effector.
2. All the legs have an identical structure. This is, each leg has the same number of active and passive joints and the joints are arranged in an identical pattern.

In a practical way, the definition of parallel symmetrical mechanism covers most of the industrial parallel structures. For parallel symmetrical mechanisms the kinematic identification by inverse kinematics and a divide and conquer strategy is stated for each leg κ independently, $\kappa = 1, 2, \ldots, n_{limbs}$.

Given:
1. A set of nominal kinematic parameters of the κth leg (φ_κ). n_κ parameters are assumed to be identified:

$$\varphi_\kappa = \begin{bmatrix}\varphi_{\kappa,1} & \ldots\varphi_{\kappa,n_\kappa}\end{bmatrix}^T. \tag{1}$$

2. An inverse kinematic function g_κ relating the κth active joint variable ($\mathbf{q}_\kappa$) with the end-effector pose ($\mathbf{r}$). For the jth pose of the mechanism the inverse function of the κth leg is defined to be:

$$\begin{aligned} g_\kappa^j : \varphi_\kappa \times \mathbf{r}^j \rightarrow \mathbf{q}_\kappa^j, \\ j = 1, 2, \ldots, N. \end{aligned} \tag{2}$$

3. A set of N end-effector measured configurations ($\hat{\mathbf{R}}_\kappa$) for the identification of the κth leg:

$$\hat{\mathbf{R}}_\kappa = \begin{bmatrix}\hat{\mathbf{r}}_\kappa^1 & \cdots & \hat{\mathbf{r}}_\kappa^N\end{bmatrix}^T. \tag{3}$$

4. A set of measured input variables ($\hat{\mathbf{Q}}_\kappa$) corresponding to the set of end-effector measurements ($\hat{\mathbf{R}}_\kappa$):

$$\hat{\mathbf{Q}}_\kappa = \begin{bmatrix}\hat{\mathbf{q}}_\kappa^1 & \cdots & \hat{\mathbf{q}}_\kappa^N\end{bmatrix}^T. \tag{4}$$

Goal. To find the set of unknown (real) kinematic parameters ($\bar{\varphi}_\kappa$) that minimizes an error between the measured joint variables ($\hat{\mathbf{Q}}_\kappa$) and their corresponding values ($\bar{\mathbf{Q}}_\kappa$) estimated from the measured end-effector poses by the inverse kinematic model g_κ. The problem can be formally stated as the following non-linear minimization problem:

$$\begin{aligned} \bar{\varphi}_\kappa : \sum_{j=1}^{N} \left\|\hat{\mathbf{Q}}_\kappa - \bar{\mathbf{Q}}_\kappa\left(\hat{\mathbf{R}}_\kappa, \varphi_\kappa\right)\right\|^2 \text{is minimum,} \\ \text{subject to} : \mathbf{R}_\kappa \subset \mathbf{W_R}, \\ \mathbf{W_R} \text{ is the usable end} - \text{effector workspace.} \end{aligned} \tag{5}$$

The optimization problem is constrained by the useful workspace (a workspace without singularities) of the mechanism.

A kinematic identification of parallel symmetrical mechanisms protocol based on the Divide and Conquer identification strategy is developed in section 4.

4 KINEMATIC IDENTIFICATION PROTOCOL

Based on the Divide and Conquer strategy for the kinematic identification of parallel symmetrical mechanisms (section 3) the following kinematic identification protocol (Figure 1) is proposed.
1. Given the nominal parameters of the κth leg (φ_κ, Eq. 1) and the correspondent inverse kinematic function (g_κ, Eq. 2) to calculate the κth Jacobian identification matrix of a representative set of postures of the usable workspace:

$$C_\kappa(\mathbf{W_R}, \varphi_\kappa) = \frac{\partial g_\kappa(\mathbf{W_R}, \varphi_\kappa)}{\partial \varphi_\kappa^T}. \tag{6}$$

2. Given the Jacobian identification matrix calculated in the first step to select an optimal set of postures ($\mathbf{R}_\kappa(C_\kappa)$) for the kinematic identification of the κth leg. The set of postures is selected searching the improvement of the observability of the set of parameters φ_κ. To select the poses we adopt the active calibration algorithm developed by (Sun and Hollerbach, 2008) that reduces the complexity of computing an observability index reducing computational time for finding optimal poses. The optimized identification set of postures is then defined in the following manner:

$$\begin{aligned} \mathbf{R}_\kappa : O_1(\mathbf{R}_\kappa) \text{ is maximal,} \\ O_1(\mathbf{R}_\kappa) = \frac{\sqrt[n_\kappa]{s_1\ s_2 \cdots s_{n_\kappa}}}{n_\kappa}, \\ \mathbf{R}_\kappa \subset \mathbf{W_R}, \end{aligned} \tag{7}$$

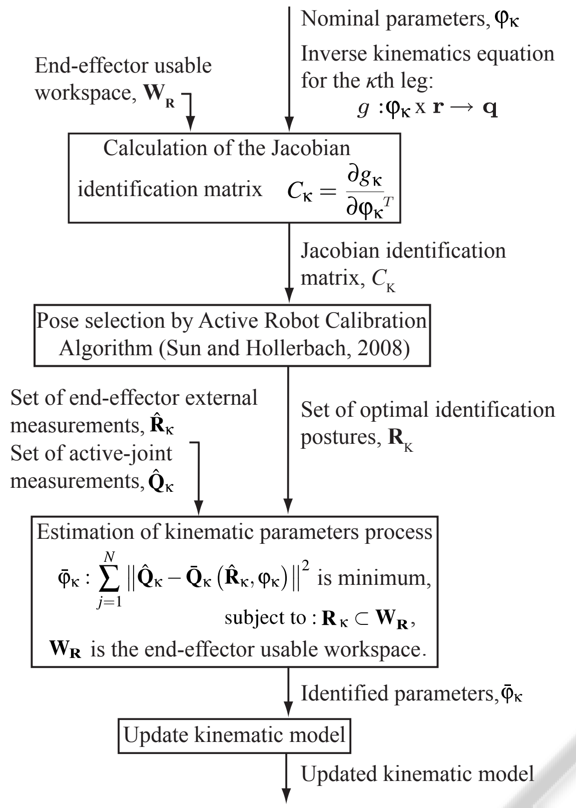


Figure 1: Kinematic identification of parallel symmetrical mechanisms protocol.

were O_1 is an observability index of the identification matrix ($C_\kappa(\mathbf{R}_\kappa, \varphi_\kappa)$) of the κth leg, n_κ is the number of parameters to be identified in the κth leg, and $s_1,\ s_2\ \ldots, s_{n_\kappa}$ are the singular values of the identification matrix C_κ. As a rule of thumb, in order to suppress the influence of measurement noise, the number of identification poses should be two or three times larger than the number of parameters to be estimated (Jang et al., 2001).

3. Given the optimized set of identification postures obtained in the second step and the correspondent sets of active joint ($\hat{\mathbf{Q}}_\kappa$) and end-effector ($\hat{\mathbf{R}}_\kappa$) measurements to solve the optimization problem defined on Eq. 5 for the identification of the kinematic parameters (φ_κ) of the κth leg.

4. Given the identified set of parameters of the κth leg obtained in the third step to update the kinematic model of the parallel mechanism.

The protocol is repeated until all the legs in the mechanism are identified.

With respect to traditional identification algorithms for the kinematic identification of parallel mechanism (Renaud et al., 2006; Zhuang et al., 1998) the proposed kinematic identification protocol has the following advantages:

1. Reduction of the kinematic identification computational costs. If a linear least-squares estimation of the kinematic parameters is used to solve the identification problem (Eq. 5), then the correction to be applied to the kinematic parameters ($\Delta\varphi$) can be estimated iteratively as (Hollerbach and Wampler, 1996):

$$\Delta\varphi = \left(C^T C\right)^{-1} C^T \Delta\mathbf{Q}. \qquad (8)$$

The computational cost of the matrix inversion $(C^T C)^{-1}$ is reduced proportionally to the square of the number of legs of the mechanism, Table 1.

2. Improvement of the numerical efficiency of the kinematic identification algorithm by the independent identification of the parameters of each leg.

3. Improvement of the kinematic identification by the design of independent experiments optimized for the identification of each leg.

Table 1: Computational and measurement costs of kinematic identification.

	Traditional kinematic identification	Divide and conquer identification
Regressor	$C^T C(N\, n_{limbs} \times N\, n_{limbs})$	$C_\kappa^T C_\kappa(N \times N)$
Computational cost (Matrix inversion)	$\propto N^3\, n_{limbs}{}^3$	$\propto N^3\, n_{limbs}$

The kinematic identification of parallel mechanisms protocol is used in the simulated identification of a planar 5R symmetrical mechanism in section 5.

5 RESULTS

The results on kinematic identification of parallel mechanisms by a Divide and Conquer strategy are presented using a case study: the simulated kinematic identification of the planar 5R symmetrical parallel mechanism.

The planar 5R symmetrical mechanism (Figure 2) has two degrees-of-freedom (DOF) that allows it to position the end-effector point (P) in the plane that contains the mechanism. The mechanism is formed by two driving links (l_1 and l_2) and a conducted dyad (L_1 and L_2), Figure 2. The planar 5R symmetrical mechanism is an instance of the parallel symmetrical mechanisms defined in section 3. A complete characterization of the assembly configurations (Cervantes-Sánchez et al., 2000), kinematic design (Cervantes-Sánchez et al., 2001; Liu et al., 2006a; Liu et al., 2006b; Liu et al., 2006c), workspace (Cervantes-Sánchez et al., 2001; Cervantes-Sánchez et al., 2000; Liu et al., 2006a), singularities (Cervantes-Sánchez et al., 2001; Cervantes-Sánchez et al., 2000; Liu et al., 2006a) and performance atlases (Liu et al., 2006b) are reported. However, no research is reported on kinematic identification.

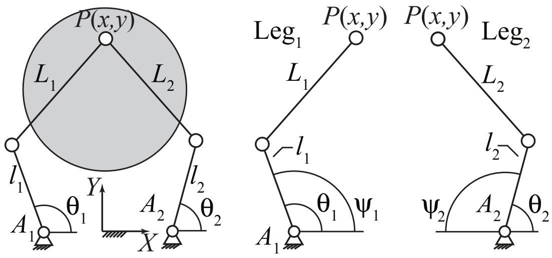


Figure 2: Planar 5R symmetrical mechanism.

The kinematic identification of the planar 5R symmetrical mechanism is simulated using the kinematic identification of parallel symmetrical mechanisms protocol (section 4) under the following conditions:

1. A linear model is assumed for the active joint A_κ:

$$\theta_\kappa = k\psi_\kappa + \gamma_\kappa, \tag{9}$$

where the k_κ represent the joint gain, γ_κ is the joint offset, ψ_κ is the measured active joint angle and θ_κ is the active joint angle, $\kappa = 1, 2$.

2. In parallel mechanisms the principal source of error in positioning is due to limited knowledge of the joint centers, leg lengths and active joint parameters (Daney et al., 2002). In consequence, the parameters to be estimated are the attachment points (A_κ), the leg lengths (l_κ, L_κ), and the joint gain and offset (k_κ, γ_κ), $\kappa = 1, 2$:

$$\varphi_\kappa = [l_\kappa \; L_\kappa \; A_{\kappa x} \; A_{\kappa y} \; k_\kappa \; \gamma_\kappa]^T . \tag{10}$$

3. The external parameters associated with the measuring device will not be identified. For the external measuring system this implies that its position is known and coincident with the reference frame $X-Y$ and the measurement target is coincident with the end-effector point.

4. The nominal kinematic parameters of the mechanism are disturbed adding a random error with normal distribution and a standard deviation σ. The nominal and disturbed parameters are shown in Table 2.

5. The constrain equation of the inverse kinematics is defined independently for the κth leg (Liu et al., 2006a), $\kappa = 1, 2$:

$$L_\kappa{}^2 = (x - l_\kappa \cos\theta_\kappa - A_{\kappa x})^2 + (y - l_\kappa \sin\theta_\kappa - A_{\kappa y})^2. \tag{11}$$

6. The end-effector and joint workspace are limited by the maximal inscribed workspace (MIW),

Table 2: Planar 5R symmetrical mechanism. Nominal and real (disturbed) parameters.

	Nominal value	Real value		Nominal value	Real value
A_{1x} [m]	-0.5000	-0.4988	A_{2x} [m]	0.5000	0.4961
A_{1y} [m]	0.0000	0.0028	A_{2y} [m]	0.0000	0.0066
k_1	1.0000	1.004	k_2	-1.0000	-0.9984
γ_1 [rad]	0.0000	0.0048	γ_2 [rad]	3.1416	3.1418
l_1 [m]	0.7500	0.7507	l_2 [m]	0.7500	0.7559
L_1 [m]	1.1000	1.0995	L_2 [m]	1.1000	1.0959

Figure 2. The MIW corresponds to the maximum singularity-free-end-effector workspace limited by a circle (Liu et al., 2006c).

7. Each leg is identified using a set of 18 postures of the mechanism to measure the end-effector position and the corresponding active joint variable. The designed sets of identification postures in the end-effector workspace are presented in Figure 4b.

8. The set of end-effector measurements ($\hat{\mathbf{R}}_\kappa$) and its corresponding active joint measurements ($\hat{\mathbf{Q}}_\kappa$) are simulated using forward kinematics and adding random disturbances with normal distribution and standard deviation $\sigma = 1 \cdot 10^{-4}$.

9. A linearization of the inverse kinematics is used for iteratively solving the non-linear optimization problem (Eq. 5), then, for the jth identification pose the identification problem of the κth leg is in the following form:

$$\begin{aligned} \Delta\mathbf{q}_\kappa^j = \frac{\partial g_\kappa^j}{\partial\varphi_\kappa}\Delta\varphi_\kappa &= C_\kappa^j \Delta\varphi, \\ \Delta\mathbf{q}_\kappa^j &= \hat{\mathbf{q}}_\kappa^j - \bar{\mathbf{q}}_\kappa^j, \\ \Delta\varphi_\kappa &= \bar{\varphi}_\kappa - \varphi_\kappa. \end{aligned} \tag{12}$$

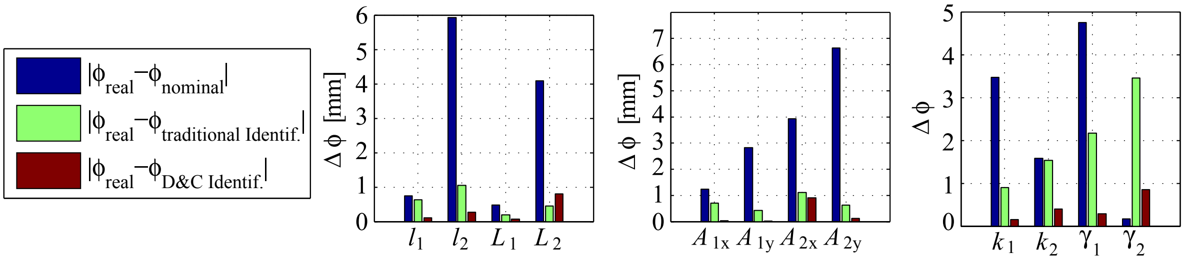
Using $N = 18$ measurements to identify the set of parameters φ_κ the identification problem is stated in the following manner:

$$\begin{aligned} \Delta\mathbf{Q}_\kappa &= C_\kappa \Delta\varphi_\kappa, \\ C_\kappa &= \left[C_\kappa^1 \cdots C_\kappa^N\right]^T, \\ \Delta\mathbf{Q}_\kappa &= \left[\Delta\mathbf{q}_\kappa^1 \cdots \Delta\mathbf{q}_\kappa^N\right]^T, \end{aligned} \tag{13}$$

were C_κ is the identification matrix of the κth leg. The parameters of the κth leg can be updated using a linear least-squares solution of Eq. 13, (Hollerbach and Wampler, 1996):

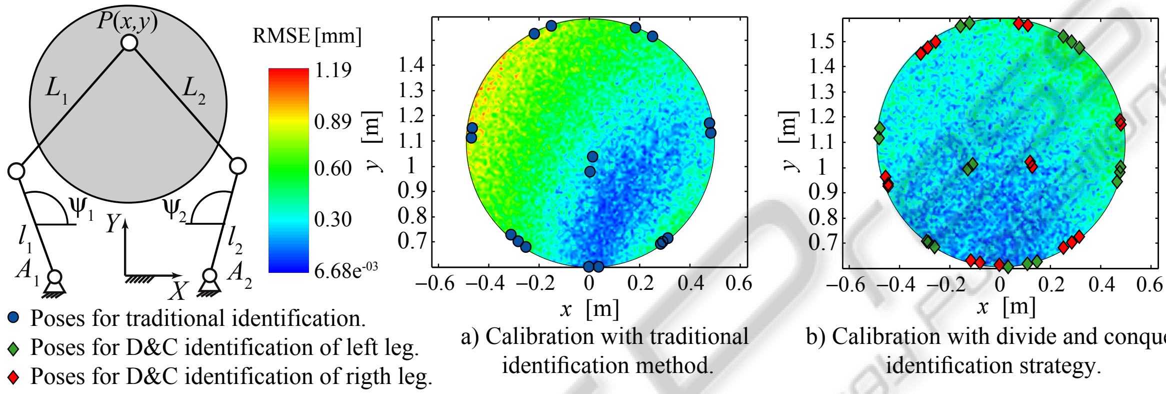
$$\Delta\varphi_\kappa = (C_\kappa^T C_\kappa)^{-1} C_\kappa^T \Delta\mathbf{Q}_\kappa. \tag{14}$$

10. An alternative traditional kinematic identification by inverse kinematic modeling is calculated and used as a comparison with respect to the proposed kinematic identification protocol. The traditional identification is performed by means of a set of 36 optimized postures selected in order to maximize the observability of the total identification matrix. The observability



a. Link lengths b. Position of fixed joints c. Active joint parameters

Figure 3: Planar 5R mechanism. Residual errors in the kinematic parameters before and after calibration.



a) Calibration with traditional identification method.

b) Calibration with divide and conquer identification strategy.

Figure 4: Planar 5R mechanism. Estimated end-effector local root mean square error for the maximal inscribed workspace (MIW) after calibration.

was defined as the Eq. 7. The designed set of identification postures is presented in Figure 4a.

The results of the kinematic identification under these conditions are presented in Figure 3 (residual errors in kinematic parameters before and after calibration). The residual errors are calculated as the difference between the real (virtually disturbed) parameters and the estimated parameters. Finally, Figure 4 presents the estimated local root mean square error for the MIW after calibration and the selected postures for kinematic identification. Additionally the computational and measurement identification costs are estimated for the identification of the planar 5R parallel mechanism, Table 3. The measurement costs of the Divide and Conquer strategy are incremented with respect to a traditional identification method. The increment of the measurements is required because each leg requires an independent set of end-effector measurements. In the case of a traditional identification the set of end-effector measurements is common to all the legs. In despite of the measurement increment the Divide and Conquer identification results in a superior estimation with respect to a traditional kinematic identification methods (Renaud et al., 2006; Zhuang et al., 1998). The conclusions of the paper are proposed in section 6.

Table 3: Planar 5R symmetrical mechanism. Computational and measurement costs of kinematic identification.

	Traditional kinematic identification	Divide and conquer identification
Regressor	$C^T C(36 \times 36)$	$C_\kappa^T C_\kappa(18 \times 18)$
Computational cost (Matrix inversion)	$\propto 18^3 \cdot 2^3$	$\propto 18^3 \cdot 2$
Measurement cost	$2 \cdot 18 \cdot 2 = 72$	$18 \cdot 2(2+1) = 108$

6 CONCLUSIONS

This article presents a new (Divide and Conquer) strategy for the kinematic identification of parallel symmetrical mechanisms. The new strategy develops a formalization of the inverse calibration method proposed by (Zhuang et al., 1998). The identification strategy (section 3) is based on the independent identification of the kinematic parameters of each leg of the parallel mechanism by minimizing an error between the measured active joint variable of the identified leg and their corresponding value, estimated through an inverse kinematic model. With respect to traditional identification methods the Divide and Conquer strategy presents the following advantages:

1. Reduction of the kinematic identification computa-

tional costs,
2. Improvement of the numerical efficiency of the kinematic identification algorithm and,
3. Improvement of the kinematic identification results.

Based on the Divide and Conquer strategy, a new protocol for the kinematic identification of parallel symmetrical mechanisms is proposed (section 4, Figure 1). For the selection of optimal identification postures the protocol adopts the active robot calibration algorithm of (Sun and Hollerbach, 2008). The main advantage of the active robot calibration algorithm is the reduction of the complexity of computing an observability index for the kinematic identification, allowing to afford more candidate poses in the optimal pose selection search. The kinematic identification protocol summarizes the advantages of the Divide and Conquer identification strategy and the advantages of the active robot calibration algorithm.

The kinematic identification protocol is demonstrated with the simulated identification of a planar 5R symmetrical mechanism (section 5). The performance of our identification protocol is compared with a traditional identification method obtaining an improvement of the identification results (Figs. 3, 4).

ACKNOWLEDGEMENTS

The authors wish to acknowledge the financial support for this research by the Colombian Administrative Department of Sciences, Technology and Innovation (COLCIENCIAS), grant 1216-479-22001.

REFERENCES

Besnard, S. and Khalil, W. (2001). Identifiable parameters for parallel robots kinematic calibration. In *IEEE International Conference on Robotics and Automation, 2001. Proceedings 2001 ICRA*, volume 3.

Cervantes-Sánchez, J. J., Hernández-Rodríguez, J. C., and Angeles, J. (2001). On the kinematic design of the 5r planar, symmetric manipulator. *Mechanism and Machine Theory*, 36(11-12):1301 – 1313.

Cervantes-Sánchez, J. J., Hernández-Rodríguez, J. C., and Rendón-Sánchez, J. G. (2000). On the workspace, assembly configurations and singularity curves of the rrrrr-type planar manipulator. *Mechanism and Machine Theory*, 35(8):1117 – 1139.

Daney, D., Lorraine, I., and LORIA, V. (2002). Optimal measurement configurations for Gough platform calibration. In *IEEE International Conference on Robotics and Automation, 2002. Proceedings. ICRA'02*, volume 1.

Daney, D., Papegay, Y., and Madeline, B. (2005). Choosing measurement poses for robot calibration with the local convergence method and Tabu search. *The International Journal of Robotics Research*, 24(6):501.

Hollerbach, J., Khalil, W., and Gautier, M. (2008). *Springer Handbook of Robotics, Model Identification*. Springer.

Hollerbach, J. and Wampler, C. (1996). The calibration index and taxonomy for robot kinematic calibration methods. *The International Journal of Robotics Research*.

Jang, J., Kim, S., and Kwak, Y. (2001). Calibration of geometric and non-geometric errors of an industrial robot. *Robotica*, 19(03):311–321.

Liu, X., Wang, J., and Pritschow, G. (2006a). Kinematics, singularity and workspace of planar 5r symmetrical parallel mechanisms. *Mechanism and Machine Theory*, 41(2):145 – 169.

Liu, X., Wang, J., and Pritschow, G. (2006b). Performance atlases and optimum design of planar 5r symmetrical parallel mechanisms. *Mechanism and Machine Theory*, 41(2):119 – 144.

Liu, X., Wang, J., and Zheng, H. (2006c). Optimum design of the 5r symmetrical parallel manipulator with a surrounded and good-condition workspace. *Robotics and Autonomous Systems*, 54(3):221 – 233.

Merlet, J. (2006). *Parallel Robots*. Springer Netherlands, second edition.

Renaud, P., Vivas, A., Andreff, N., Poignet, P., Martinet, P., Pierrot, F., and Company, O. (2006). Kinematic and dynamic identification of parallel mechanisms. *Control Engineering Practice*, 14(9):1099–1109.

Ryu, J. and Rauf, A. (2001). A new method for fully autonomous calibration of parallel manipulators using a constraint link. In *2001 IEEE/ASME International Conference on Advanced Intelligent Mechatronics, 2001. Proceedings*, volume 1.

Sun, Y. and Hollerbach, J. (2008). Active robot calibration algorithm. In *Robotics and Automation, 2008. ICRA 2008. IEEE International Conference on*, pages 1276–1281.

Tsai, L. (1999). *Robot analysis: the mechanics of serial and parallel manipulators*. Wiley-Interscience.

Zhuang, H., Yan, J., and Masory, O. (1998). Calibration of Stewart platforms and other parallel manipulators by minimizing inverse kinematic residuals. *Journal of Robotic Systems*, 15(7).