APPROXIMATE REASONING BASED ON LINGUISTIC MODIFIERS IN A LEARNING SYSTEM

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Abstract: Approximate reasoning, initially introduced in fuzzy logic context, allows reasoning with imperfect knowledge. We have proposed in a previous work an approximate reasoning based on linguistic modifiers in a symbolic context. To apply such reasoning, a base of rules is needed. We propose in this paper to use a supervised learning system named SUCRAGE, that automatically generates multi-valued classification rules. Our reasoning is used with this rule base to classify new objects. Experimental tests and comparative study with two initial reasoning modes of SUCRAGE are presented. This application of approximate reasoning based on linguistic modifiers gives satisfactory results. Besides, it provides a comfortable linguistic interpretation to the human mind thanks to the use of linguistic modifiers.

1 INTRODUCTION

Most information expressed by human beings is uncertain, vague or imprecise. However, these information is necessary for the realization and the use of intelligent systems. In the literature, several approaches have been proposed for the representation of these types of knowledge, two of which dominate: fuzzy logic (Zadeh, 1965) and multi-valued logic (Akdag et al., 1992). To allow systems manipulating and reasoning with imperfect knowledge, Zadeh (Zadeh, 1975) has introduced approximate reasoning concept in the fuzzy logic context. This reasoning is based on a generalization of Modus Ponens to fuzzy data, known as Generalized Modus Ponens (GMP). It corresponds to the following schema:

$$\frac{\begin{array}{l}\text{If } X \text{ is } \mathcal{A} \text{ then } Y \text{ is } \mathcal{B}\\ X \text{ is } \mathcal{A}'\end{array}}{Y \text{ is } \mathcal{B}'} \quad (1)$$

where X and Y are two variables and $\mathcal{A}$, $\mathcal{A}'$, $\mathcal{B}$ and $\mathcal{B}'$ are predicates. Approximate reasoning can, not only infer with an observation equivalent to the rule premise $\mathcal{A}$, but also with an observation $\mathcal{A}'$ which is approximately equivalent to it.

In a previous work (Kacem et al., 2008; Borgi et al., 2008), we noticed that both fuzzy and multi-valued GMPs are generally based on the concept of similarity (Akdag et al., 1992; Khoukhi, 1996; Bouchon-Meunier et al., 1997). The weakness of this type of reasoning is that it focuses on the modification degree (the degree of similarity between $\mathcal{A}$ and $\mathcal{A}'$) and not to the way $\mathcal{A}$ has been modified to have $\mathcal{A}'$ (weakening, reinforcement, etc.).

We also noted that the concept of linguistic modifiers reflects a form of similarity which can be used in the GMP for the evaluation of the changes made on the premise to lead to the conclusion. The diagram of approximate reasoning based on linguistic modifiers is as follows:

$$\frac{\begin{array}{l}\text{If } X \text{ is } \mathcal{A} \text{ then } Y \text{ is } \mathcal{B}\\ X \text{ is } m(\mathcal{A})\end{array}}{Y \text{ is } m'(\mathcal{B})} \quad (2)$$

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To determine the inference conclusion $\mathcal{B}' = m'(\mathcal{B})$, it is enough to determine the modifier m'. The latter is obtained based on the observed modifier m ($\mathcal{A}' = m(\mathcal{A})$) and the correlation intensity between the premise and the rule's conclusion.

To apply approximate reasoning, it is necessary to have a base of rules. Two ways are possible to obtain such a base. First, it can be directly provided by the expert. Secondly it can be automatically built, through the use of learning systems. In our work, we choose to use the second solution, we used a classification system for automatic generation of rules, called SUCRAGE (Borgi, 1999; Borgi and Akdag, 2001). This system generates classification multi-valued rules, the context of our approximate reasoning (Kacem et al., 2008). A classification rule predicts the class of a new object. For example, a patient is described by a set of attributes such as age, sex, blood pressure, etc, and the class could be a binary attribute concluding or not the illness of the patient by a particular disease.

In this paper, we begin in section 2 by presenting the symbolic multi-valued logic, the context of our work. Then, section 3 deals with the concept of linguistic modifiers. We present in section 4 the SUCRAGE system. Then, in section 5 we explain how to adapt and apply approximate reasoning based on linguistic modifiers on this system. Finally, before concluding this work we present in section 6 a comparative study of experimental test results.

2 SYMBOLIC MULTI-VALUED LOGIC

Multi-valued logic is a generalization of Boolean logic. It provides truth values that are intermediate between True and False. We denote by M the number of truth degrees in multi-valued logic. Akdag et al. (Akdag et al., 1992) have introduced a new generation of multi-valent logic based on the theory of multi-sets.

In symbolic multi-valued logic, each linguistic term (such as *large*) is represented by a multi-set (Akdag et al., 1992). To express the imprecision of a predicate, a qualifier v_α is associated to each multi-set (such as *rather*, *little*, etc). When a speaker uses a statement "X is $v_\alpha A$", v_α is the degree to which X satisfies the predicate A[1]. A truth-degree τ_α must correspond to each adverbial expression v_α so that:

$$\begin{array}{lcl} X \text{ is } v_\alpha A & \Longleftrightarrow & \text{"}X \text{ is } v_\alpha A\text{" is true} \\ & \Longleftrightarrow & \text{"}X \text{ is } A\text{" is } \tau_\alpha\text{-true} \end{array}$$

For example, the statement "John is rather tall" means that John satisfies the predicate *tall* with the degree *rather*.

The set of symbolic truth-degrees forms an ordered list $\mathcal{L}_M = \{\tau_0, \ldots, \tau_i, \ldots, \tau_{M-1}\}$[2] with the total order relation: $\tau_i \leq \tau_j \Leftrightarrow i \leq j$, its smallest element is τ_0 (false) and the greatest is τ_{M-1} (true). In practice, the number of truth-degrees is often close to 7. The expert can even propose his own list of truth-degrees; the only restrictive condition is that they must be ordered.

3 GENERALIZED SYMBOLIC MODIFIERS

A modifier is an operator that builds linguistic terms from a primary linguistic term. This concept was introduced by Zadeh (Zadeh, 1975) in the fuzzy logic framework. We distinguish two types of fuzzy modifiers. First, reinforcing modifiers that reinforce the notion expressed by the term (as *very*). Then weakening modifiers, which weaken the notion expressed by the term (as *more or less*).

As already mentioned, we have used linguistic modifiers in approximate reasoning process in (Kacem et al., 2008; Borgi et al., 2008). Since our work falls in multi-valued framework, we use modifiers defined in this particular context.

A set of linguistic modifiers were proposed in the multi-valued framework by Akdag and al. in (Akdag et al., 2001), they were named *the Generalized Symbolic Modifiers*. A Generalized Symbolic Modifier (GSM) is a semantic triplet of parameters: *radius*, *nature* (i.e dilated, eroded or preserved) and *mode* (i.e reinforcing, weakening or central). The radius is noted ρ with $\rho \in IN^*$.

Definition 1. *Let us consider a symbolic degree τ_i with $i \in IN$ in a scale $\mathcal{L}_M$ of a base $M \in IN^* \setminus \{1\}$, and $i < M$. A GSM m with a radius ρ is denoted m_ρ. The modifier m_ρ is a function which applies a linear transformation to the symbolic degree τ_i to obtain a new degree $\tau_{i'} \in \mathcal{L}_{M'}$ (where $\mathcal{L}_{M'}$ is the linear transformation of $\mathcal{L}_M$) according to a radius ρ such as:*

$$\begin{array}{rccc} m_\rho : & \mathcal{L}_M & \rightarrow & \mathcal{L}_{M'} \\ & \tau_i & \mapsto & \tau_{i'} \end{array}$$

A proportion is associated to each symbolic degree within a base denoted $Prop(\tau_i) = \frac{i}{M-1}$.

[1] Denoted mathematically by "$X \in_\alpha A$": the object X belongs with a degree α to a multi-set A.

[2] With M a positive integer not null.

By analogy with fuzzy modifiers, the authors propose a classification of symbolic modifiers according to their behavior: weakening and reinforcing modifiers, and they add the family of central modifiers (Akdag et al., 2001) that neither reinforce nor weaken the concept. The definitions of the reinforcing, weakening and central modifiers are given in table 1[3]. We have studied composition of these modifiers in (Kacem et al., 2009).

4 THE LEARNING SYSTEM SUCRAGE

To apply our approximate reasoning based on linguistic modifiers, a rules base is needed. We used a learning system to obtain this base. SUCRAGE (SUPervised Classification by Rule Automatic GEneration) is a supervised learning system which was proposed by Borgi in (Borgi, 1999; Borgi and Akdag, 2001).

The construction of a classification function in SUCRAGE is done through two phases: the learning phase and the recognition or classification one (Borgi et al., 2003; Borgi, 2006; Borgi et al., 2007).

4.1 Learning Phase

The rule base is generated thanks to a learning base: a set of objects already classified. We denote by B_1,..., B_b the classes defined by the experts, X_1,..., X_t the attributes of the objects. The rules generated by SUCRAGE have the following form:

If X_{e_1} is $v_{\alpha_1}A$ and ... and X_{e_n} is $v_{\alpha_n}A$ then the class is B_i with p

where:

X_{e_j} is $v_{\alpha_j}A$	a proposition which means that the value of X_{e_j} is in $[a,b]$
X_{e_j}	an attribute, $X_{e_j} \in \{X_1,\dots,X_t\}$
$[a,b]$	the sub-interval of index v_{α_i} in the field of X_{e_j}
A	a multi-set for the attributes fields
B_i	the i^{th} class
p	a belief degree representing the uncertainty of the conclusion

The interval $[a,b]$ is a sub-interval of the field of the attribute X_{ej}. It is obtained by the a regular discretization of this field.

[3]We have modified some definitions in order to respect the bounds of M and i.

4.1.1 Construction of the Premise

To construct the rule premise, the first task consists on determining what are the attributes to regroup. In SUCRAGE, the attributes that appear in a same premise are the correlated ones. For that, a correlation matrix C is calculated: $C = (r_{i,j})_{t\times t}$, with $r_{i,j}$ the linear correlation coefficient between X_i and X_j.

Then, one considers that X_i and X_j are correlated if the absolute value of $r_{i,j}$ is greater than a fixed threshold θ.

One must then discretizes the attributes fields. In this work, we retained the regular discretization: it leads to M sub ranges denoted by $v_0, v_1, \dots, v_{M-1}$. Condition parts of rules are then obtained by considering for each correlated components subset, a subinterval (v_i) for each component in all possible combinations. Figure 1 illustrates such a partition in the case of two correlated features with a subdivision size $M = 3$.

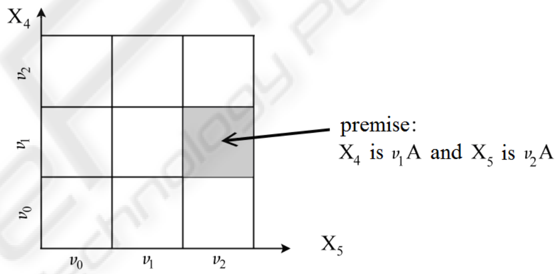

Figure 1: Example of partition of the correlated attributes space.

4.1.2 Construction of the Rule

Each constructed premise according to the method exposed above conducts to the generation of b rules, with b the number of classes. The last stage consists in calculating this belief degree p, which can be represented by the conditional probability to get the conclusion when the premise is verified: $p = proba(premise/conclusion)$. Conditional probabilities are estimated on the training set using a frequentist approach.

4.2 Classification Phase

During the classification phase, the inference engine associates a class to a vector representing an object to classify. Two types of reasoning are used: an exact reasoning and an approximate reasoning. For exact inference, the used method is the classic one. It consists on the use of the Modus Ponens. The approximate inference method applies the Generalized Modus Ponens:

Table 1: Definitions of weakening, reinforcing and central modifiers.

MODE NATURE	Weakening	Reinforcing	Central
Erosion	$\tau_{i'} = \tau_{max(0,i-\rho)}$ $\mathcal{L}_{M'} = \mathcal{L}_{max(2,M-\rho)}$ $\boxed{EW_\rho}$	$\tau_{i'} = \tau_i$ $\mathcal{L}_{M'} = \mathcal{L}_{max(2,i+1,M-\rho)}$ $\boxed{ER_\rho}$ $\tau_{i'} = \tau_{max(0,min(i+\rho,M-\rho-1))}$ $\mathcal{L}_{M'} = \mathcal{L}_{max(2,M-\rho)}$ $\boxed{ER'_\rho}$	$\tau_{i'} = \tau_{max(\lfloor \frac{i}{\rho} \rfloor,1)}$ $\mathcal{L}_{M'} = \mathcal{L}_{max(\lfloor \frac{M}{\rho} \rfloor+1,2)}$ $\boxed{EC_\rho *}$
Dilation	$\tau_{i'} = \tau_i$ $\mathcal{L}_{M'} = \mathcal{L}_{M+\rho}$ $\boxed{DW_\rho}$ $\tau_{i'} = \tau_{max(0,i-\rho)}$ $\mathcal{L}_{M'} = \mathcal{L}_{M+\rho}$ $\boxed{DW'_\rho}$	$\tau_{i'} = \tau_{i+\rho}$ $\mathcal{L}_{M'} = \mathcal{L}_{M+\rho}$ $\boxed{DR_\rho}$	$\tau_{i'} = \tau_{i\rho}$ $\mathcal{L}_{M'} = \mathcal{L}_{M\rho-\rho+1}$ $\boxed{DC_\rho}$
Conservation	$\tau_{i'} = \tau_{max(0,i-\rho)}$ $\mathcal{L}_{M'} = \mathcal{L}_M$ $\boxed{CW_\rho}$	$\tau_{i'} = \tau_{min(i+\rho,M-1)}$ $\mathcal{L}_{M'} = \mathcal{L}_M$ $\boxed{CR_\rho}$	$\tau_{i'} = \tau_i$ $\mathcal{L}_{M'} = \mathcal{L}_M$ $\boxed{CC}$

* $\lfloor . \rfloor$ is the floor function.

If "X_{e_1} is $v_{\alpha_1}A$" and ... and "X_{e_n} is $v_{\alpha_n}A$" **then** the class is B_i with p
"X_{e_1} is $v_{\gamma_1}A$" and ... and "X_{e_n} is $v_{\gamma_n}A$"

The class is B_i with p'

To be more precise in determining the distance between the premise and the observation in approximate reasoning, the attributes values have undergone a discretization finer than exact reasoning, specifically M^2 instead of M.

The approach consists in using a 0+ order inference engine. The engine has to manage the rules' uncertainty and take it into account within the inference dynamic. More precisely, for a new object O to classify, the inference engine allows to obtain a final belief degree associated to each class. The final belief degree is the result of a triangular co-norm applied on the probabilities of the fired rules that conclude to this considered class. Finally, the winner class associated to the new object is the class where the final belief degree is maximal.

5 INTEGRATION OF LINGUISTIC MODIFIERS IN SUCRAGE

The rule of Generalized Modus Ponens with linguistic modifiers that we proposed in (Kacem et al., 2008), and that we intend to use for classifying objects with SUCRAGE, deals with multi-sets. However, the conclusion part of SUCRAGE rules contains a numerical belief degree. To apply our GMP, the probability measure of class B_i must be symbolic. In this section we explain the adaptation made on SUCRAGE in order to use our approximate reasoning based on linguistic modifiers.

5.1 Symbolic Probability

In (Seridi and Akdag, 2001), the authors have defined a symbolic probability theory. This theory is an alternative to the classical theory of probability, in the special case where values of probabilities are symbolic degrees of uncertainty. The authors used this probability in SUCRAGE in (Seridi et al., 2006), and this by replacing the probability measure p of the rules by a symbolic degree of uncertainty. With the specific notation of our work, the rules generated by SUCRAGE become of the form:

If X_{e_1} is $v_{\alpha_1}A_1$ and ... and X_{e_n} is $v_{\alpha_n}A_n$ then the class is $v_\beta B_i$

with B_i a class and τ_β[4] the symbolic belief degree associated with this class. Thus, a degrees scale has been introduced to represent uncertainty $\mathcal{L}_{M_p}$ composed of M_p degrees: $\mathcal{L}_{M_p} = \{\tau_i, i = 0, \ldots, M_p - 1\}$ totally ordered. The boundaries of these sub-intervals are denoted $a_0, a_1, \ldots, a_{M_p}$. Therefore, it is associated with each probability measure p a symbolic degree of uncertainty τ_i.

The discretization of probability can be regular or irregular. Seridi et al. (Seridi et al., 2006) chose to use an irregular discretization to obtain a scale L_7 of 7 sub-intervals. The numerical probability is subdivided as follows:

[4]Let's remind that τ_β is the symbolic degree associated to the linguistic expression v_β.

$p = 0$	$\Leftrightarrow$	τ_0	$\Leftrightarrow$	Impossible
$p \in]0, 0.5[$	$\Leftrightarrow$	τ_1	$\Leftrightarrow$	Very little possible
$p \in [0.5, 0.7[$	$\Leftrightarrow$	τ_2	$\Leftrightarrow$	Little possible
$p \in [0.7, 0.8[$	$\Leftrightarrow$	τ_3	$\Leftrightarrow$	Possible
$p \in [0.8, 0.9[$	$\Leftrightarrow$	τ_4	$\Leftrightarrow$	Enough possible
$p \in [0.9, 0.95[$	$\Leftrightarrow$	τ_5	$\Leftrightarrow$	Very possible
$p \in [0.95, 1]$	$\Leftrightarrow$	τ_6	$\Leftrightarrow$	Certain

In this work, we use the irregular discretization, as used by Seridi and al. (Seridi et al., 2006). In (Seridi et al., 2006) symbolic probabilities are used in SUCRAGE with exact inference, unlike our work where we are interested in approximate reasoning.

5.2 Conclusion Deduction

After building the rules, we have to exploit them. An inference must be applied in order to classify new objects.

5.2.1 Simple Premise

Let us start with a simple case where the rule has a simple premise. The corresponding rule of Generalized Modus Ponens based on linguistic modifiers is as follows:

Rule :	If "X_e is $v_\alpha A$" then "Y is $v_\beta B$"	
Fact :	"X_e is $m(v_\alpha A)$"	
Conclusion :		"Y is $m'(v_\beta B)$"

The modifier m' to apply to the conclusion is obtained according to the modifier m. The first step is to determine the modifier m. As in the case of the original approximate reasoning used in SUCRAGE, the attributes subdivision cardinal in the rules premises is M, while the one in the observations is M^2. The proposed solution is to first find the decomposition of the modifier m with the dilating central DC operator. Indeed, the observation base is a multiple of the premise base, so it undergoes a dilatation. Let τ_γ the membership degree of the observation in the base $\mathcal{L}_{M^2}$, the decomposition of the modifier m is given as follows:

$$m = m_{\rho_1} \circ DC_{\rho_2} \text{ with: } \begin{array}{l} \rho_1 = |\gamma - \alpha\rho_2| \\ \rho_2 = \frac{M^2-1}{M-1} = M+1 \end{array} \quad (3)$$

with m_{ρ_1} an elementary modification operator of radius ρ_1. This operator m_{ρ_1} may be either CW, CR or CC, since it acts only on the truth degree[5]. Its choice depends on the sign of $(\gamma - \alpha.\rho_2)$. Thus:

$$m_{\rho_1} = \begin{cases} CW_{\rho_1}, & \text{if } (\gamma - \alpha.\rho_2) < 0; \\ CR_{\rho_1}, & \text{if } (\gamma - \alpha.\rho_2) > 0; \\ CC, & \text{else.} \end{cases} \quad (4)$$

[5]The base is already dilated by the central operator DC.

The operator m_{ρ_1} represents the real modification made on the premise. Indeed, the DC operator is central, so it acts as a zoom on the base and has no effect on the proportion of degrees. For this reason, the operator m_{ρ_1} which we denote by $m^\diamond$ is the one that we consider in determining the modifier m' to be deduced. The problem in that this operator is compatible with the observations base $\mathcal{L}_{M^2}$. It can not be directly applied to the symbolic probability, given that its base is different from the base of the symbolic probability. Thus, it is necessary to convert the modifier $m^\diamond$ to be compatible with the conclusion base $\mathcal{L}_{M_p}$. We propose to keep the same type of operator and change only the radius ρ_1.

The conversion of the modifier $m^\diamond$ when the symbolic probability is irregular is a complex task. Indeed, the amplitudes of the probability sub-intervals are different. For this reason, we propose a solution that takes into account these amplitudes. We associate with each symbolic probability degree $\tau_\alpha = \{\alpha \in [0..M-1]\}$ a value called $weight(\tau_\alpha)$, which is equal to the amplitude of the sub-interval number α. The weight value is given by the following function:

$$weight: \quad [0..M-1] \rightarrow [0,1], \qquad \alpha \mapsto a_{\alpha+1} - a_\alpha$$

with a_i the discretization bounds. Then, the new radius ρ_1 of formula (3) is obtained by the algorithm *conversion_mod* above. The principle of this algorithm is to find the radius which causes the same modification percentage in the probability base $\mathcal{L}_{M_p}$ that is caused by the radius ρ_1 in the basis $\mathcal{L}_{M^2}$.

Algorithm 1. *Begin of algorithm conversion_mod.*

- *Input values:*
  - *The radius ρ_1 of the modifier to translate;*
  - *The symbolic probability degree to modify;*
  - *The size of the base M^2.*
- *The values to initialize:*
  - *A real* proportion $\leftarrow \frac{\rho_1}{M^2}$ *which represents the proportion of the radius ρ_1 in the corresponding base $\mathcal{L}_{M^2}$;*
  - *A real* compteurPoids $\leftarrow 0$, *a weight counter.*
- *loop through the symbolic probability degrees with a decrement, beginning with the degree to modify. Until compteurPoids $\leq$ proportion, increase compteurPoids by the weight of the current degree.*
- *The new radius is equal to the degree to modify minus the current degree minus 1.*

End of algorithm.

Thus, with this algorithm we determine the radius ρ'_1 to use to modify the probability degree, ie: $m' = m_{\rho'_1}$.

5.2.2 Composed Premise

SUCRAGE generates rules whose premise consists of a conjunction of propositions. Thus, the inference process can be achieved through the Generalized Modus Ponens based on generalized linguistic modifiers in the case of conjunctive rules. It is as follows:

$$\frac{\begin{array}{l}\text{If } "X_{e_1} \text{ is } v_{\alpha_1}A_1" \text{ and } \dots \text{ and } "X_{e_n} \text{ is } v_{\alpha_n}A_n" \text{ then } "Y \text{ is } v_\beta B" \\ "X_{e_1} \text{ is } m_1(v_{\alpha_1}A_1)" \text{ and } \dots \text{ and } "X_{e_n} \text{ is } m_n(v_{\alpha_n}A_n)"\end{array}}{"Y \text{ is } m'(v_\beta B)"}$$

The determination of m_i is performed by the modifier determination method of simple rule, case that we have described above. Thus, they must be operators of type CR, CW or CC. The deduced modifier m' is determined by aggregating the modifiers m_i. We define for that an operator that aggregate modifiers. In this application framework, the conclusion uncertainty degree weakens when the observation moves away from the premise. For this reason, we defined an aggregator which is adaptable to the rules in the SUCRAGE system:

$$A_S(m_\alpha, m'_\beta) = CW_\gamma \text{ so that } \tau_\gamma = S(\tau_\alpha, \tau_\beta) \qquad (5)$$

with:
m and m': modifiers of type CR, CW or CC;
τ_α, τ_β and τ_γ: symbolic degrees belonging to $\mathcal{L}_M$;
S: a T-conorm such as the Lukasiewicz T-conorm.

As in the original SUCRAGE, the rules triggered are grouped by class. Then, the final symbolic belief degree of each class is calculated, and that by aggregating the belief degrees by the max T-conorm. Finally, the selected class is whose final belief degree is the greater one.

6 EXPERIMENTAL STUDY

In this section, we first describe the extension done on SUCRAGE, in order to integrate approximate reasoning with linguistic modifiers. Then, we present experimental results obtained with this application.

As we noted earlier, to implement our approach, we need to use symbolic probability as belief degree of the rules. For this, we integrated into SUCRAGE, in addition to numerical probability, a new type of belief degrees: irregular symbolic probability.

To perform a comparative study on the classification results, we used the learning set Iris. These data are available on the server of the Irvine University of California[6]. This database consists of 150 examples represented by 4 numerical attributes (sepal lenth, sepal width, petal lenth, petal width). The examples are divided over 3 classes: Iris setosa, Versicolor and Virginica. The tests are made by 10-folds cross-validation. We use different thresholds for correlation and for discretization cardinals of the attributes. We tested our approximate reasoning based on linguistic modifiers with irregular symbolic probabilities in Table 2.

[6]//ftp.ics.uci.edu/pub/machine-learning-databases/

Table 2: Iris, comparative study of approximate reasoning with irregular symbolic probability.

Subdivision size		**3**	**5**	**7**
threshold	**Reasoning**			
0.95	AR_{is}	**98**	93.33	96
	AR_{is}/ER_{is}	1	1	1.09
	AR_{is}/ER_n	1	1.01	1.08
	AR_{is}/AR_n	1	1.03	1.08
0.9	AR_{is}	**98**	93.33	96
	AR_{is}/ER_{is}	1	1	1.09
	AR_{is}/ER_n	1	1.01	1.08
	AR_{is}/AR_n	1	1.03	1.08
0.8	AR_{is}	97.33	95.33	**98**
	AR_{is}/ER_{is}	1	1.05	1.11
	AR_{is}/ER_n	1.01	1.05	1.13
	AR_{is}/AR_n	1.01	1	1.05
0.5	AR_{is}	97.33	95.33	**98**
	AR_{is}/ER_{is}	1	1.05	1.11
	AR_{is}/ER_n	1.01	1.05	1.13
	AR_{is}/AR_n	1.01	1	1.05

The ratio rows AR_{is}/ER_{is} gives the division of the classification rate of approximative reasoning AR_{is} by whose of exact reasoning ER_{is} with irregular symbolic probability (corresponding to the subscript *is*). We note that the approximate reasoning based on linguistic modifiers gives a best results in all cases. Moreover, it improves the results of exact reasoning with irregular symbolic probability when the subdivision cardinal increases. This is because the variation of this parameter introduces imperfections (Borgi and Akdag, 2001). Indeed, approximate reasoning helps to limit borders problems of the discretization: the imperfections due to a high subdivision size are corrected. The rate reaches at 98% for a subdivision of 7 which represents the best rates obtained by the SUCRAGE system.

We also compare our approximate reasoning to the original exact and approximate reasoning of SUCRAGE. The ratio rows AR_{is}/ER_n and AR_{is}/AR_n present a comparison of our approximate reasoning with respectively the original exact reasoning ER_n and the original approximate reasoning AR_n of SUCRAGE that is based on numerical probability (corresponding to the subscript *n*). We note through this table that in all cases, the new version gives better results than the original version.

Approximate reasoning with linguistic modifiers gives satisfactory results, moreover it presents a great advantage with regard to the numerical approach. Indeed, approximate reasoning with linguistic modifiers can refine the interpretation of classification results. The original version of SUCRAGE is a numerical approach, the results of objects assignments to classes are provided through numerical probabilities. On the other side, approximate reasoning with linguistic modifiers is a linguistic approach which provides a linguistic interpretation of the results, allowing readability and easy interpretation by the human mind. Moreover, the use of approximate reasoning is more advantageous when the data provided by the experts are imprecise.

7 CONCLUSIONS

In this work, we have presented an application of approximate reasoning with linguistic modifiers that we have defined in (Kacem et al., 2008; Borgi et al., 2008). For this purpose, we have used a rule base generated by a supervised learning system: SUCRAGE (Borgi, 1999). Some adaptations have been made to this system in order to infer with our approximate reasoning. More precisely, we have included the use of symbolic probability (Seridi and Akdag, 2001) as belief degree of the generated rules. Moreover, we have defined an aggregator of modifiers in order to aggregate the modifiers that transform the observation elements to the premise elements. We have noticed that classification results were improved by using our approximate reasoning based of linguistic modifiers. This improvement was noticed in comparison with the exact reasoning, as well as with the approximate reasoning introduced in (Borgi and Akdag, 2001). In addition, our approach provides a linguistic interpretation through the use of linguistic modifiers. It would be interesting to complete the validation tests with other data, and more generally to consider an application of our approximate reasoning on a base of rules resulting from expert knowledge acquisition.

REFERENCES

Akdag, H., Glas, M. D., and Pacholczyk, D. (1992). A qualitative theory of uncertainty. *Fundam. Inform.*, 17(4):333–362.

Akdag, H., Truck, I., Borgi, A., and Mellouli, N. (2001). Linguistic modifiers in a symbolic framework. *International Journal of Uncertainty, Fuzziness and Knowledge-Based Systems*, 9(Supplement):49–61.

Borgi, A. (1999). *Apprentissage supervisé par génération de règles : le système SUCRAGE*. PhD thesis, Université Paris 6, France.

Borgi, A. (2006). Approximate reasoning to learn classification rules. In *First International Conference on Software and Data Technologies (ICSOFT)*, pages 203–210.

Borgi, A. and Akdag, H. (2001). Knowledge based supervised fuzzy-classification: An application to image processing. *Annals of Mathematics and Artificial Intelligence*, 32:67–86.

Borgi, A., Akdag, H., and Ghedjati, F. (2003). Using genetic algorithms to optimize the number of classification rules in sucrage. *ACS/IEEE International Conference on Computer Systems and Applications, Tunis*, pages 110–116.

Borgi, A., Kacem, S. B. H., and Ghédira, K. (2008). Approximate reasoning in a symbolic multi-valued framework. In *Computer and Information Science*, volume 131 of *Studies in Computational Intelligence*, pages 203–217. Springer.

Borgi, A., Lahbib, D., and Ghédira, K. (2007). Optimizing the number of rules in a knowledge based classification system. In *International Conference on Artificial Intelligence and Pattern Recognition, AIPR-07, Orlando, Florida, USA*, pages 185–192.

Bouchon-Meunier, B., Delechamp, J., Marsala, C., and Rifqi, M. (1997). Several forms of fuzzy analogical reasoning. In *IEEE International Conference on Fuzzy Systems*, pages 45–50, Barcelone, Spain.

Kacem, S. B. H., Borgi, A., and Ghédira, K. (2008). Generalized modus ponens based on linguistic modifiers in a symbolic multi-valued framework. In *Proceeding of the 38th IEEE International Symposium on Multiple-Valued Logic*, pages 150–155, Dallas, USA.

Kacem, S. B. H., Borgi, A., and Tagina, M. (2009). On some properties of generalized symbolic modifiers and their role in symbolic approximate reasoning. In *ICIC - Emerging Intelligent Computing Technology and Applications. Lecture Notes in Computer Science*, volume 5755, pages 190–208.

Khoukhi, F. (1996). *Approche logico-symbolique dans le traitement des connaissances incertaines et imprécises dans les systèmes à base de connaissances*. PhD thesis, Université de Reims, France.

Seridi, H. and Akdag, H. (2001). Approximate reasoning for processing uncertainty. *Journal of Advanced Computational Intelligence and Intelligent Informatics*, 5(2):110–118.

Seridi, H., Akdag, H., Mansouri, R., and Nemissi, M. (2006). Approximate reasoning in supervised classification systems. *Journal of Advanced Computational Intelligence and Intelligent Informatics*, 10(4):586–593.

Zadeh, L. A. (1965). Fuzzy sets. *Information and Control*, 8(3):338–353.

Zadeh, L. A. (1975). The concept of a linguistic variable and its application to approximate reasoning - i - ii - iii. *Information Sciences*, pages 8:199–249, 8:301–357, 9:43–80.