LEARNING DYNAMIC BAYESIAN NETWORKS WITH THE TOM4L PROCESS

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Abstract: This paper addresses the problem of learning a Dynamic Bayesian Network from timed data without prior knowledge to the system. One of the main problems of learning a Dynamic Bayesian Network is building and orienting the edges of the network avoiding loops. The problem is more difficult when data are timed. This paper proposes a new algorithm to learn the structure of a Dynamic Bayesian Network and to orient the edges from the timed data contained in a given timed data base. This algorithm is based on an adequate representation of a set of sequences of timed data and uses an information based measure of the relations between two edges. This algorithm is a part of the Timed Observation Mining for Learning (TOM4L) process that is based on the Theory of the Timed Observations. The paper illustrates the algorithm with a theoretical example before presenting the results on an application on the Apache system of the Arcelor-Mittal Steel Group, a real world knowledge based system that diagnoses a galvanization bath.

1 INTRODUCTION

The theory of Timed Observations is the mathematical framework that defines a Knowledge Engineering methodology called the Timed Observation Modeling for Diagnosis methodology (TOM4D) (Le Goc, 2008) (Figure 1) and a learning process called Timed Observation Mining for Learning (Le Goc, 2006). TOM4D and TOM4L are defined to discover temporal knowledge about a set of functions of the continuous time $x_i(t)$ considered as a dynamic system $X(t)=\{x_i(t)\}$ called a process.

According to TOM4D, a model of a process $X(t)$ is a quadruple $<PM(X(t)), SM(X(t)), BM(X(t)), FM(X(t))>$. The Perception Model $PM(X(t))$ defines the goals of the process $X(t)$. The Structural Model $SM(X(t))$ contains the knowledge about the components and their organization in structures. The Behavioral Model $BM(X(t))$ defines the states and the state transitions that governs the process evolution over time. Finally, the Functional Model $FM(X(t))$ of the process $X(t)$ defines the mathematical functions linking the values of the process variables $x_i(t)$ of $X(t)$. We propose to represent the Functional Model $FM(X(t))$ of a process as a Bayesian Network.

The problem is then to define the learning principles of a Bayesian Network (BN) from a set of sequences of timed data, without prior knowledge to the process. Most of the proposed algorithm deals with un-timed data and faces some difficulties in orienting the edges of the resulting graph and building the conditional probability tables. These two problems are more difficult when data are timed.

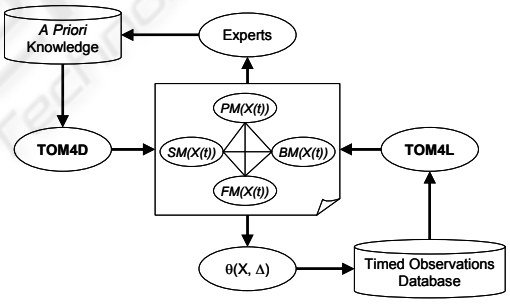

Figure 1: Global Structure of the Project.

The theory of Timed Observations provides the tools to solve these two problems: the BJ-Measure of (Benayadi, 2008) and an adequate representation of a set of sequences of timed data that is called the Stochastic Representation (Le Goc, 2005). The BJ-Measure is an informational measure designed to evaluates the quantity of information flowing in the Stochastic Representation of a set of sequences of

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timed data. This representation facilitates also the building for the CPT tables.

The next section presents a short description of the state of the art techniques concerning the learning of Dynamic Bayesian Networks (DBN). Section 3 introduces the basis of the Stochastic Representation of TOM4L and the BJ-Measure. Section 4 describes the learning principles we define from the properties of the BJ-Measure, the DBN learning algorithm that we proposes is proposed in section 5 and an application to a theoretical example is given in section 6 before showing a real life application of the algorithm in section 7. Our conclusions are presented in section 8.

2 RELATED WORKS

A BN is a couple $<G, \theta>$ where G denotes a Direct Acyclic Graph in which the nodes represent the variables and the edges represent the dependencies between the variables (Pearl, 1988), and θ is the Conditional Probabilities Tables (CP Tables) defining the conditional probability between the values of a variable given the values of the upstream variables of G. BN learning algorithms aims at discovering the couple $<G, \theta>$ from a given data base.

BN learning algorithms fall into two main categories: "search and scoring" and "dependency analysis" algorithms. The "search and scoring" learning algorithms can be used when the knowledge of the edge orientation between the variables of the system is given (Cooper, 1992), (Heckerman, 1997). To avoid this problem, dependency analysis algorithms uses conditional independence tests (Cheng, 1997), (Cheesseman, 1995), (Friedman 1998), (Meyrs et al, 1999). But the number of test exponentially increases the computation time (Chickering, 1994).

$$I(X,Y)=\sum_{i,j} P(X=x_i, Y=y_j)\cdot\log\left(\frac{P(X=x_i, Y=y_j)}{P(X=x_i)\cdot P(Y=y_j)}\right) \quad (1)$$

For example, Cheng's algorithm (Cheng, 1997) for learning a BN from data falls in the dependency analysis category and is representative of most of the proposed algorithms. It is based on the d-separation concept of (Pearl, 1988) to infer the structure G of the Bayesian Network, and the mutual information to detect conditional independency relations. The idea is that the mutual information $I(X, Y)$ (eq. 1) tells when two variables are (1) dependent and (2) how close their relationship is. The algorithm computes the mutual information $I(X, Y)$ between all the pairs of variables (X, Y) producing a list L sorted in descending order: pairs of higher mutual information are supposed to be more related than those having low mutual information values. The List L is then pruned given an arbitrary value of the parameter ε: each pair (X, Y) so that $I(X, Y)<\varepsilon$ is eliminated of L. In real world applications, list L should be as small as possible using the ε parameter. This first step (D*rafting*) creates a structure to start with but it might miss some edges or it might add some incorrect edges.

$$I(X,Y|E)=\sum_{i,j} P(X=x_i, Y=y_j, E)\cdot\log\left(\frac{P(X=x_i, Y=y_j|E)}{P(X=x_i|E)\cdot P(Y=y_j|E)}\right) \quad (2)$$

The second step (*Thickening)* phase tries to separate each pair (X, Y) in L using the conditional mutual information $I(X, Y \mid E)$ (eq. 2) where E is a set of nodes that forms a path between the current tested nodes X and Y from L. When $I(X, Y \mid E)>\varepsilon$, then the edges of the path E should be added between the current nodes X and Y. This phase continues until the end of list L is reached. The last step of the algorithm (*Thinning*) searchs, for each edge in the graph, if there are other paths besides this edge between these two nodes. In that case, the algorithm removes this edge temporarily and tries to separate these two nodes using equation (2). If the two nodes cannot be separated, then the temporarily removed edge will be returned. After building the DBN structure, the orientation of the edges and the CP Tables' computation is to be done. The procedure used by (Cheng, 1997) is based on the idea of searching for the nodes forming a *V-Structure* $X\rightarrow Y\leftarrow Z$ using the conditional mutual information, and then trying to deduce the other edges from the discovered one. This procedure have a very big limitation which is that if a network does not contain a *V-Structure*, no edge can be oriented.

The two main limitations of the methods of the dependency analysis category are so the need of defining the ε parameter and the exponential amount of Conditional Independence tests to orient the edges of the graph.

3 TOM4L FRAMEWORK

The Timed Observation Mining for Learning process (TOM4L) proposes a solution to escape from this problem (Le Goc, 2006).

The TOM4L framework defines a message timed at t_k contained in a database as an occurrence $C^i(t_k)$ $C^i(k)$ of an observation class $C^i=\{(x_i, \delta_i)\}$ which is an arbitrary set of couples (x_i, δ_i) where δ_i is one of the discrete values of a variable x_i. An observation class is often a singleton because in that case, two classes $C^i = \{(x_i, \delta_i)\}$ and $C^j = \{(x_j, \delta_j)\}$ are only linked with the variables x_i and x_j when the constants δ_i and δ_j are independent (Le Goc, 2006).

The TOM4L framework represents a sequence $\omega=(\ldots, C^i(k), \ldots)$ of m occurrences $C^i(k)$ defining a set $Cl=\{ C^i \}$ of n timed observations under a specific representation, called the Stochastic Representation, that is made with a set of matrices. The TOM4L framework proposes also the BJ-Measure (Benayadi, 2008) that evaluates the homogeneity of the crisscross of the occurrences of two observation classes C^i and C^j in a sequence. This measure considers two abstract binary variables X and Y linked through a discrete binary memoryless channel of a communication system (Shannon, 1949), where $X(t_k)$ takes a value C^x in $\{C^i ,\neg C^i\}$ and $Y(t_{k+1})$ a value C^y in $\{C^j ,\neg C^j\}$ when reading ω (Figure 2, where $\neg C^a$ denotes any class but C^a). With this model, a sequence ω of m occurrences $C^i(k)$ is a sequence of m-1 instances $r(C^x \rightarrow C^y)$ of a relation $r(X \rightarrow Y)$

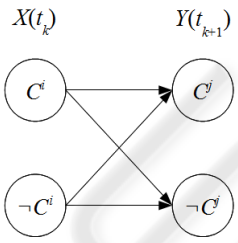


Figure 2: Discrete Binary Memoryless Channel.

The BJ-measure is build on the Kullback-Leibler distance D(P(Y|X=Ci)||P(Y)) that evaluates the relation between the distribution of the conditional probability of Y knowing that X(tk)=Ci and the prior probability distribution of Y. One of the properties of this distance is that D(P(Y|X=Ci)||P(Y))=0 when P(Y| X=Ci)=P(Y) (i.e. P(Y) and P(X) are independent). The BJ-measure decomposes the Kullback-Leibler distance in two terms around the independence point. The BJL-measure BJL($C^i \rightarrow C^j$) of binary relation $r(C^i \rightarrow C^j)$ is the right part of the Kullback-Leibler distance D($P(Y|X=C^i)||P(Y)$) so that:

- $P(Y=C^j|X=C^i)<P(Y=C^j) \Rightarrow BJL(C^i, C^j)=0$
- $P(Y=C^j|X=C^i) \geq P(Y=C^j) \Rightarrow$ $BJL(C^i \rightarrow C^j)= D(P(Y|X=C^i)||P(Y))$

The $BJL(C^i \rightarrow C^j)$ is non-zero when the observation $C^i(k)$ provides some information about the observation $C^j(k)$. Symmetrically, when $BJL(C^i \rightarrow C^j)<0$, the observation $C^i(k)$ provides some information about $\neg C^j(k)$. The BJL-measure $BJL(C^i \rightarrow \neg C^j)$ of a binary relation $r(C^i \rightarrow \neg C^j)$ is then the left part of the Kullback-Leibler distance:

- $P(Y=C^j|X=C^i)<P(Y=C^j) \Rightarrow$ $BJL(C^i \rightarrow \neg C^j)=D(P(Y|X=C^i)||P(Y))$
- $P(Y=C^j|X=C^i) \geq P(Y=C^j) \Rightarrow BJL(C^i \rightarrow \neg C^j)= 0$

Consequently:

$$D(P(Y|X=C^i)||P(Y))=BJL(C^i \rightarrow C^j)+BJL(C^i \rightarrow \neg C^j) \quad (3)$$

Similarly, the BJW-measure evaluates the information distribution between the predecessors ($C^i(k)$ or $\neg C^i(k)$) of an observation $C^j(k+1)$ at time t_{k+1}:

$$D(P(X|Y=C^j)||P(X))=BJW(C^i \rightarrow C^j)+BJW(C^i \rightarrow \neg C^j) \quad (4)$$

Because $(P(C^j|C^i)<P(C^j)) \Leftrightarrow (P(C^i|C^j)<P(C^i))$, the two measures are null at the same independence point and can be combined in a single measure called the BJM-measure which is the norm vector of $BJL(C^i \rightarrow C^j)$ and $BJW(C^i \rightarrow C^j)$:

$$M(C^i, C^j)=\sqrt{BJL(C^i, C^j)^2+BJW(C^i, C^j)^2}- \sqrt{BJL(C^i, \neg C^j)^2+BJW(\neg C^i, C^j)^2} \quad (5)$$

The BJ-Measure is not justifiable when the $\theta_{i,j} = n_i/n_j$ is greater of 4 or less than ¼ (Benayadi, 2008). This property is called the θ property. In most real world cases, when this condition is satisfied, the $M(C^i \rightarrow C^j)$ value is not zero but the eventual relation $r(C^i \rightarrow C^j)$ can not be justified with the BJ-measure.

The main property of the BJ-measure is the following: $M(C^i \rightarrow C^j) > 0$ means that knowing the timed observation distribution of the C^i class brings information about the timed observation distribution of the C^j class. Consequently, when the BJ-measure of a relation $r(C^i \rightarrow C^j) \leq 0$, is negative or null, the relations can not be used to build the structure of a dynamic Bayesian network.

In other words, *considering the positive values only*, the BJ-measure $M(C^i \rightarrow C^j)$ satisfies the three following properties:

1. Dissymmetry: $M(C^i \rightarrow C^j) \neq M(C^j \rightarrow C^i)$ (generally)
2. Positivity: $\forall C^i, C^j, M(C^i \rightarrow C^j) \geq 0$
3. Independence: $M(C^i \rightarrow C^j)=0 \Leftrightarrow C^i$ and C^j are independant (i.e. $P(C^j|C^i)=P(C^j)$)

4. Triangular inequality:
   $M(C^i \rightarrow C^j) < M(C^i \rightarrow C^k) + M(C^k \rightarrow C^j)$

This latter property can be used to reason with the BJ-measure to deduce the structure of a dynamic Bayesian network.

4 LEARNING PRINCIPLES

Let us consider a set $R=\{\ldots, r(C^i \rightarrow C^j), \ldots\}$ of n binary relations. The operation that remove a binary relation $r(C^i \rightarrow C^j)$ from the set R is denoted $Remove(r(C^i \rightarrow C^j))$: $R \leftarrow R - \{ r(C^i \rightarrow C^j) \}$.

The positivity property leads to remove the $r(C^i \rightarrow C^j)$ relations having a negative value of the BJ-measure ("Positivity rule"):

- Rule 1 : $\forall r(C^i \rightarrow C^j) \in R, M(C^i \rightarrow C^j) \leq 0 \Rightarrow Remove(r(C^i \rightarrow C^j))$

The dissymmetry property allows deducing the orientation of a hypothetical relation between two timed observation classes C^i and C^j:

- Rule 2:
  $\forall r(C^i \rightarrow C^j), r(C^j \rightarrow C^i) \in R,$
  $M(C^i \rightarrow C^j) > BJM(C^j \rightarrow C^i) \Rightarrow Remove(r(C^j \rightarrow C^i))$

This rule means that when $M(C^i \rightarrow C^j) > M(C^j \rightarrow C^i)$, the C^i class brings more information about the C^j class than the reverse. The relation $r(C^j \rightarrow C^i)$ can then be removed from the set R without any consequence. This rule is so called the "orientation rule".

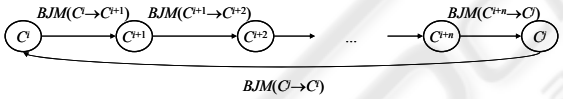


Figure 3: Loops.

Now, let us consider a set $R=\{r(C^i \rightarrow C^{i+1}), r(C^{i+1} \rightarrow C^{i+2}), \ldots, r(C^{i+n} \rightarrow C^j), r(C^j \rightarrow C^i)\}$ of $n+2$ binary relations defining a loop (Figure 2) where:

- $\forall r(C^x \rightarrow C^y) \in R, M(C^x \rightarrow C^y) > 0$

The problem of the set R is that computing the distribution of a class C^x requires knowing its distribution: loops must then be avoided. In other words, a relation $r(C^i \rightarrow C^j)$ must be removed from R to break the loop. To solve this problem, the idea is to used the monotonous property of the BJ-measure: finding two of class C^i and C^j so that the BJ-measure of the relation $r(C^i \rightarrow C^j)$ is the lowest of the loop ("Loop Rule"):

- Rule 3: $\forall r(C^x \rightarrow C^y) \in R, \exists r(C^i \rightarrow C^j) \in R, x \neq i, y \neq j,$
  $M(C^x \rightarrow C^y) > BJM(C^i \rightarrow C^j) \Rightarrow Remove(r(C^i \rightarrow C^j))$

When $M(C^x \rightarrow C^y) = M(C^i \rightarrow C^j))$, any of the relations can be removed. The extreme case of loop can be find in a set R containing a reflexive relation $r(C^i \rightarrow C^i)$ where $M(C^i \rightarrow C^i) > 0$. Rule 3 must then be adapted to this extreme (but frequent) case ("Reflexivity rule"):

- Rule 4: $\forall r(C^i \rightarrow C^i) \in R, BJM(C^i \rightarrow C^i) > 0 \Rightarrow Remove(r(C^i \rightarrow C^i))$

Finally, to build naïve Bayesian Networks, the algorithm must avoid the multiple paths leading to a same C^i class (Figure 3). To avoid this problem, as for loops, the idea is to use the monotonous property of the BJ-measure: finding two of class C^i and C^j so that the BJ-measure of the relation $r(C^i \rightarrow C^j)$ is the lowest of the paths. To use this idea, all the paths leading to a particular C^i class must be find in R. Let us suppose that R contains n paths $R_1 \subseteq R$, $R_2 \subseteq R$, …, $R_n \subseteq R$ leading to the C^i class (i.e. each R_i is of the form $R_i=\{r(C^i \rightarrow C^{k-n}), r(C^{k-n} \rightarrow C^{k-n+1}), \ldots, r(C^k \rightarrow C^j), r(C^i \rightarrow C^{l-n}), r(C^{l-n} \rightarrow C^{l-n+1}), r(C^l \rightarrow C^j)\}$. The algorithm must find the $r(C^i \rightarrow C^j)$ relation with the lowest BJ-measure to remove it in R ("Transitivity rule"):

- Rule 5: $\forall r(C^x \rightarrow C^y) \in R_1 \cup R_2 \cup \ldots \cup R_n,$
  $\exists r(C^i \rightarrow C^j) \in R_1 \cup R_2 \cup \ldots \cup R_n, x \neq i, y \neq j,$
  $M(C^x \rightarrow C^y) > M(C^i \rightarrow C^j) \Rightarrow Remove(r(C^i \rightarrow C^j))$

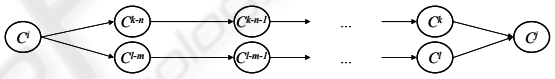


Figure 4: Multiple Paths.

Again, in case of equality (i.e. $BJM(C^x \rightarrow C^y) = BJM(C^i \rightarrow C^j)$), any relation can be removed.

These five rules are necessary (but not sufficient) to design an algorithm that builds naïve Bayesian Networks from timed data, but its efficiency depends mainly of the number of relation in the initial set R. The TOM4L framework provides the mathematical tools to remove the relations that can not play a significant role in the building of a naïve Bayesian Network.

Given the set $R=\{\ldots, r(C^i \rightarrow C^j), \ldots\}$ of n binary relations that can be build from a sequence ω of timed observation $C^i(k)$ defining a set $C=\{C^x\}$ of $N(C)$ classes C^x. The size of the Stochastic Representation matrix of the TOM4L framework is then $N(C) \cdot N(C) = N(C)^2$. This provides two ways to eliminate a relation $r(C^i \rightarrow C^j)$ having no interest for building a naïve Bayesian Network:

- Test 1: $P(C^j|C^i) \cdot P(C^i, C^j) \leq 1/N(C)^3$
  $\Rightarrow Remove(r(C^i \rightarrow C^j))$

This first test compares $b_{ij} \equiv P(C^j|C^i) \cdot P(C^i, C^j)$ with the "absolute" hazard according to the discrete binary memoryless chanel (Figure 2): because ω defines $N(C)$ classes, when supposing that all the classes are independent and have the same Poisson

rate of occurrences, the probability of having an occurrence $C^i(k)$ of a C^i class followed by an occurrence $C^j(k+1)$ of the C^j class is simply $P(C^j|C^i)=1/N(C)$ and the probability of reading a couple $(C^i(k), C^j(k+1))$ in ω is $P(C^i, C^j)= 1 / N(C)^2$. So each b_{ij} value can be compared with the "absolute" hazard $1 / (N(C)\, N(C)^2)$.

- Test 2: $P(C^j|C^i)\cdot P(C^i, C^j)\leq(1/(N(C)\cdot P(C^i)\cdot P(C^j)) \Rightarrow Remove(r(C^i\rightarrow C^j))$

This second test defines the "relative" hazard when supposing that C^i and C^j classes are independent. In that case, the probability $P((C^j(k), C^i(k+1))$ in ω of having a couple $(C^i(k), C^j(k+1))$ in ω is $P(C^i)\cdot P(C^j)$ and having an occurrence $C^i(k)$ of a C^i class, the hazard is to read any occurrence $C^j(k+1)$ of the C^j class: $P(C^j|C^i)=1/N(C)$. So each b_{ij} value can be compared with the "relative" hazard $1/(N(C)\cdot P(C^i)\cdot P(C^j))$.

The θ property of the BJ-measure complete these two tests to eliminate the relation having no meaning according to the BJ-measure:

- Test 3: $\theta_{i,j}>4 \vee \theta_{i,j}<1/4 \Rightarrow Remove(r(C^i\rightarrow C^j))$

Within the TOM4L framework, these tree tests are implemented in the $F0/1=[f_{ij}]$ matrix:

- $(b_{ij}>1/N(C)^3) \wedge (b_{ij}>(1/(N(C)\cdot P(C^i)\cdot P(C^j)) \wedge (1/4 \leq\theta_{i,j}\leq 4) \Leftrightarrow f_{ij} = 1$

So this lead to the rule number 6:

- Rule 6: $\forall r(C^i\rightarrow C^j)\in R$, $f_{ij} = 0 \Rightarrow Remove(\, r(C^i\rightarrow C^j)\,)$

These six rules are used by the algorithm inspired from Cheng's method to build a naïve Bayesian Network from timed data.

5 THE BJM4BN ALGORITHM

The proposed algorithm is called "BJM4BN" for "BJ-Measure for Bayesian Networks". This algorithm takes as inputs a sequence ω of *m* timed observation $C^i(k)$ defining a set $Cl=\{C^x\}$ of $N(Cl)$ classes C^x and an output C^j class that is the class for which the DBN is computed. It produces a set $G=\{\ldots, r(C^i\rightarrow C^j), \ldots\}$ of *n* binary relations that form the structure of a naïve Bayesian Network (G, θ).

The "BJM4BN" algorithm contains 5 stages. The first stage computes the Stochastic Representation of ω to produce the initial $M=[m_{ij}]$ matrix containing the BJ-measure values m_{ij} of the $N(Cl)^2$ binary relations $r(C^i\rightarrow C^j))$ defined by ω (line 1). Next, the $F0/1=[f_{ij}]$ matrix is computed using test 4 (line 3) so rule 6 is applied (line 4). Finally, the M matrix is normalized using rules 1 (line 5.1) and 4 (line 5.2).

Stage 2 computes the list *L* from the normalized matrix *M*. Stage 3 builds recursively the initial *G* graph from the C^j class. This stage uses a recursive function called "Build(*G*, C^x)" where C^x is the class the graph of which is to build.

Stage 4 finds and removes the loops in *G* with Rule 3. This stage finds all the loops R_i in *G* of the form $R_i\equiv\{r(C^i\rightarrow C^{i+1}), r(C^{i+1}\rightarrow C^{i+2}), \ldots, r(C^{i+n}\rightarrow C^j), r(C^j\rightarrow C^i)\}$ and put them in a set *R* (line 12). Next, a new list L_1 is build containing all the relation $r(C^x\rightarrow C^y)$ in *R* with its associated m_{xy} BJ-measure value (line 13). All loops R_i in *R* are then removed using Rule 3 (line 14). At the end of this stage, *G* contains no more loops. Note that the L_1 list being global (i.e. containing all the relations $r(C^x\rightarrow C^y)$ participating in a loop), it is guaranty that the set of removed relation $r(C^x\rightarrow C^y)$ is optimal: it is minimal and the removed relations are the smallest of the *G* graph.

Similarly, stage 5 removes the multiple paths in the *G* graph with Rule 5, but the *R* set contains only paths R_i of the form $R_i\equiv\{r(C^i\rightarrow C^{k-n}), r(C^{k-n}\rightarrow C^{k-n+1}), \ldots, r(C^k\rightarrow C^j), r(C^i\rightarrow C^{l-n}), r(C^{l-n}\rightarrow C^{l-n+1}), \ldots, r(C^l\rightarrow C^j)\}$ (line 16). It is guaranty that the set of removed relation $r(C^x\rightarrow C^y)$ is optimal.

// Stage 1

1. Compute the $M=[m_{ij}]$ matrix
2. $\forall i=0\ldots N(Cl)$, $\forall j=0\ldots N(Cl)$, $f_{ij}=0$
3. $\forall i=0\ldots N(Cl)$, $\forall j=0\ldots N(Cl)$, $(b_{ij}>1/N(C)^3)\wedge(b_{ij}>(1/(N(C)\cdot P(C^i)\cdot P(C^j))\wedge(1/4\leq\theta_{i,j}\leq 4) \Rightarrow f_{ij}=1$
4. $M=M\cdot F0/1$
5. $\forall i=0\ldots N(Cl)$, $\forall j=0\ldots N(Cl)$,
   5.1. $m_{ij}\leq 0 \Rightarrow m_{ij}=0$ // rule 1
   5.2. i=j $\Rightarrow m_{ij}=0$ // rule 4

// Stage 2

6. $L=\{\phi\}$
7. $\forall i=0\ldots N(Cl)$, $\forall j=0\ldots N(Cl)$, $m_{ii}>0, \Rightarrow L=L+\{(r(C^i\rightarrow C^j), m_{ii})\}$

// Stage 3

8. $C^x=C^j$, $G=\{\phi\}$
9. $\forall r(C^y\rightarrow C^x))\in L \Rightarrow G=G+\{r(C^y\rightarrow C^x)\}$
10. Build(G, C^x){
    $\forall r(C^y\rightarrow C^x))\in G$, $\forall r(C^z\rightarrow C^y))\in L$,
    $G=G+\{r(C^z\rightarrow C^y)\}$
    Build(G, C^y)
    }// End Build Function

// Stage 4

11. $R=\{\phi\}$
12. $\forall R_i\subseteq G$, $R_i\equiv\{r(C^i\rightarrow C^{i+1}), r(C^{i+1}\rightarrow C^{i+2}), \ldots, r(C^{i+n}\rightarrow C^j), r(C^j\rightarrow C^i)\} \Rightarrow R=R+\{R_i\}$
13. $\forall R_i\in R$, $\forall r(C^x\rightarrow C^y)\in R_i$, $r(C^x\rightarrow C^y)\notin L_1 \Rightarrow L_1=L_1+\{(r(C^x\rightarrow C^y), m_{xy})\}$

14. While $R \neq \{\phi\}$ repeat
    . $\exists r(C^x \rightarrow C^y) \in L_1$, $m_{xy} = \text{Min}(m_{ij}, L_1)$
    $\forall R_i \in R$, $\exists r(C^x \rightarrow C^y) \in R_i \Rightarrow$
    $R = R - \{R_i\}$
    $G = G - \{r(C^x \rightarrow C^y)\}$
    $L_1 = L_1 - \{(r(C^x \rightarrow C^y), m_{xy})\}$

//Stage 5

15. $R = \{\phi\}$
16. $\forall R_i \subseteq G$,
    $R_i \equiv \{r(C^i \rightarrow C^{k-n}), r(C^{k-n} \rightarrow C^{k-n+1}), \ldots,$
    $r(C^k \rightarrow C^j), r(C^i \rightarrow C^{l-n}), r(C^{l-n} \rightarrow C^{l-n+1}),$
    $\ldots, r(C^l \rightarrow C^j)\} \Rightarrow R = R + \{R_i\}$
17. $\forall R_i \in R$,
    $\forall r(C^x \rightarrow C^y) \in R_i$, $r(C^x \rightarrow C^y) \notin L_1 \Rightarrow$
    $L_1 = L_1 + \{(r(C^x \rightarrow C^y), m_{xy})\}$
18. While $R \neq \{\phi\}$ repeat
    $\exists r(C^x \rightarrow C^y) \in L_1$, $m_{xy} = \text{Min}(m_{ij}, L_1)$
    $\forall R_i \in R$, $\exists r(C^x \rightarrow C^y) \in R_i \Rightarrow$
    $R = R - \{R_i\}$
    $G = G - \{r(C^x \rightarrow C^y)\}$
    $L_1 = L_1 - \{(r(C^x \rightarrow C^y), m_{xy})\}$

Stage 6 computes the conditional probabilities tables for G and finalizes the algorithm. The computing of the Conditional Probabilities Tables (θ CP Tables) is based on the numbering table $N=[n_{ij}]$ of the Stochastic Representation of the ω sequence. The following property is a consequence of the model of the discrete memoryless communication channel (Figure 2):

- $P(Y=C^o \mid X=C^i) + P(Y=\neg C^o \mid X=C^i) = 1$.

The computing of θ uses this property (for simplicity, $P(C^y|C^x)$ is rewritten $P(y|x)$). For a root node C^x:

- $P(x)=(\Sigma_j n_{xj}) / \Sigma_i \Sigma_j n_{ij}$

For a single relation $r(C^x \rightarrow C^y)$:

$P(y|x) = n_{xy} / (\Sigma_j n_{yj})$

$P(y|\neg x) = ((\Sigma_i n_{iy}) - n_{xy}) / (\Sigma_i \Sigma_j n_{ij} - (\Sigma_j n_{xj}))$

For a set $R=\{r(C^x \rightarrow^{jy}), r(C^e \rightarrow C^y)\}$ of two relations converging to the same C^y class:

- $P(y|x,z) = (n_{xy}+n_{zy}) / (\Sigma_j n_{xj} + \Sigma_j n_{zj})$
- $P(y|\neg x,z) = (\Sigma_i n_{iy} - n_{xy}) / (\Sigma_i \Sigma_j n_{ij} - \Sigma_j n_{xj})$
- $P(y|x,\neg z) = (\Sigma_i n_{iy} - n_{zy}) / (\Sigma_i \Sigma_j n_{ij} - \Sigma_j n_{zj})$
- $P(y|\neg x,\neg z) = (\Sigma_i n_{iy} - n_{xy} - n_{zy}) / (\Sigma_i \Sigma_j n_{ij} - \Sigma_j n_{xj} - \Sigma_j n_{zj})$

6 A THEORETICAL EXAMPLE

This section illustrates the usage of the proposed algorithm on the theoretical car example of (Le Goc, 2007). This example is inspired from the (simple) car technical diagnosis knowledge base of (Schreiber, 2000) (Figure 5).

Figure 5 shows a knowledge base of 9 rules that can be used to diagnose a (very simplified) car. These rules provide the reasons that might affect the car to stop functioning: a car might "stops" or "does not start" if the fuse is blown or the battery is low or the fuel tank is empty.

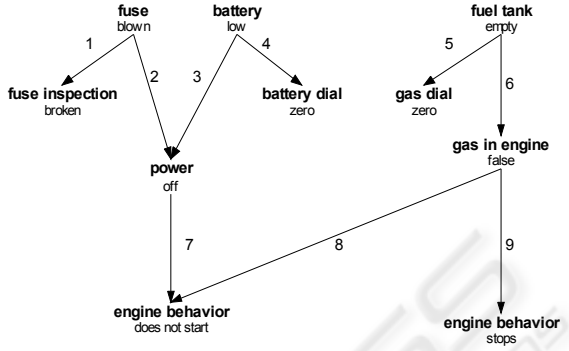


Figure 5: Car Diagnosis Knowledge Base.

Using the TOM4D methodology, the underlying structural model of the system considered in this knowledge base is provided in Figure 6. This figure shows a set of connected components c_i and defines a set of variables x_i. The evolution of variable x_i denoted with functions of time $x_i(t)$.

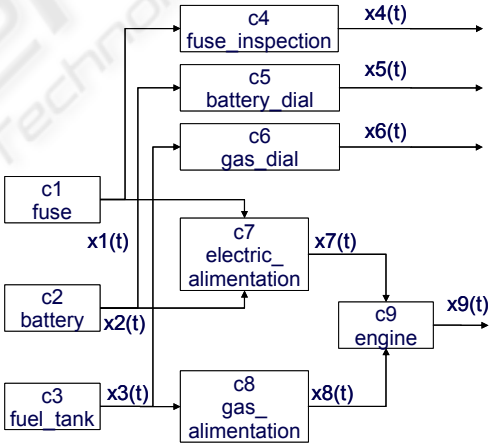
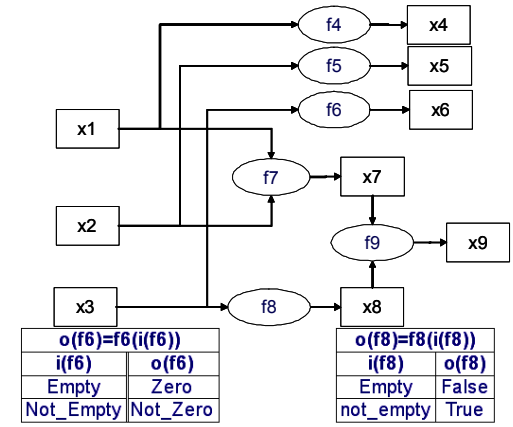


Figure 6: Structural Model of the Car Example.

TOM4D methodology considers that the variables x_4, x_5, and x_6 are associated to the sensor components c_4, c_5 and c_6 and these components never failed. So this figure defines a set $X=\{x_1, x_2, x_3, x_7, x_8, x_9\}$ of 6 variables. The values of these variables are a set of constants: $\Delta=\{$ $\Delta x1=\{$Blown, Not_Blown$\}$, $\Delta x2=\{$Low, Not_Low$\}$, $\Delta x3=\{$Empty, Not_Empty$\}$, $\Delta x7=\{$On, Off$\}$, $\Delta x8=\{$True, False$\}$, $\Delta x9=\{$Start, Does_Not_Start$\}$ $\}$. It is to note that (Le Goc, 2007) eliminates the constant "Stops": in the TOM4D

framework, this corresponds to rewrite the constant "Stops" as "Does_Not_Start".

Figure 7 shows the functional model of the car according to the TOM4D methodology.



o(f6)=f6(i(f6))	
i(f6)	o(f6)
Empty	Zero
Not_Empty	Not_Zero

o(f8)=f8(i(f8))	
i(f8)	o(f8)
Empty	False
not_empty	True

o(f7)= f7(i1(f7), i2(f7))		
i1(c7)	i2(f7)	o(f7)
not_blown	not_low	On
not_blown	Low	Off
Blown	not_low	Off
Blown	Low	Off

o(f9)= f9(i1(f9), i2(f9))		
i1(f9)	i2(f9)	o(f9)
On	True	Start
On	False	Does_not_start
Off	True	Does_not_start
Off	False	Does_not_start

Figure 7: Functional Model of the Car Example.

A functional model is an organized set of logical relations between the possible values the variables can take over time. The functions are denoted at the right of Figure 7 and are specified at the left (function f_4 and f_5 being similar to function f_6, they are not specified in the figure).

The role of the Bayesian Network <G, θ >to learn is to make a link between the probabilities of the value of a set X of variables. The discovered Bayesian network must then be compatible with the functional model of Figure 7.

Because the "BJM4BN" algorithm works on timed data, a sequence ω must be built according to rules 2, 3, 6, 7 and 8 of the knowledge base of Figure 5. To this aim, let us suppose that the car is monitored, the abnormal behaviour of the car can be defined with a set $\Omega=\{\omega_1, \omega_2, \omega_3\}$ of three models of sequences:

- ω_1 = {$x_1(t_1)$=Blown, $x_7(t_1+\Delta t_7)$=Off, $x_9(t_1+\Delta t_7+\Delta t_9)$=Does_Not_Start}
- ω_2 = {$x_2(t_2)$=Low, $x_7(t_2+\Delta t_7)$=Off, $x_9(t_2+\Delta t_7+\Delta t_9)$=Does_Not_Start}
- ω_3 = {$x_3(t_3)$=Empty, $x_8(t_3+\Delta t_8)$=False, $x_9(t_3+\Delta t_8+\Delta t_9)$=Does_Not_Start}

These sequences define a set $Cl=\{C^i\}$ of 6 observation classes, each being a singleton: $Cl=\{C^1 = \{(x_1, \text{Blown})\}, C^2 = \{(x_2, \text{Low})\}, C^3 = \{(x_3, \text{Empty})\}$ $C^7 = \{(x_7, \text{Off})\}, C^8 = \{(x_8, \text{false})\}, C^9 = \{(x_9, \text{Does\_Not\_Start})\}\}$

Consequently, each constant δ_i of Δ being linked with a unique variable x_i of X, there is a bijection between a class C^i and a variable x_i. The three sequences of the set Ω can be rewritten in terms of the class occurrences:

- $\omega_1 = \{C^1(t_1), C^7(t_1+\Delta t_7), C^9(t_1+\Delta t_7+\Delta t_9)\}$
- $\omega_2 = \{C^2(t_2), C^7(t_2+\Delta t_7), C^9(t_2+\Delta t_7+\Delta t_9)\}$
- $\omega_3 = \{C^3(t_3), C^8(t_3+\Delta t_8), C^9(t_3+\Delta t_8+\Delta t_9)\}$

These sequences will be used to produce a theoretical sequence according to the method described in [Bouché, 2008]. To this aim, let us assign hand probabilities to the occurrence of each observation classes with the following principle: the observations of the C^1 class ($x_1(t_1)$=Blown) are less probable to happen than the observations of the C^2 class ($x_2(t_2)$=Low), while the occurrences of the C^3 class ($x_3(t_3)$=Empty) are more frequent (with carefree driver for example). This lead for example to the probabilities of Table 1:

Table 1: Prior Probabilities of the car example.

$P(C^1)$	$P(C^2)$	$P(C^3)$	$P(C^7)$	$P(C^8)$	$P(C^9)$
0.05	0.15	0.3	0.2	0.2	0.1

According to the method of [Bouché, 2008], these probabilities and the three models of sequence of Ω allow building a sequence ω of 100 occurrences (Figure 8) that satisfies the probabilities of Table 1.

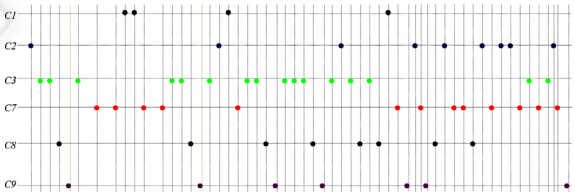


Figure 8: Theoretical ω Sequence (beginning).

Table 2: The numbering table N between variables.

N	x1	x2	x3	x7	x8	x9	TOTAL
x1	1	0	0	4	0	0	5
x2	1	1	2	9	1	0	14
x3	1	6	8	4	11	0	30
x7	1	2	8	2	2	5	20
x8	1	3	8	0	3	5	20
x9	0	2	4	1	3	0	10
Total	5	14	30	20	20	10	99

The numbering matrix N computed from the sequence ω is given in Table 2. Next, the M matrix (Table 3), the B matrix (Table 4, where each cells is multiplied by 1000 to have readable values) and the θ matrix (Table 5) are computed to produce the F0/1 matrix (Table 6) that implements the rule 6.

The $M{\cdot}F0/1$ matrix is then computed and normalized (Table 7) using rule 1 (pink cells) and rule 4

Table 3: The M Matrix for the Car Example.

M	x1	x2	x3	x7	x8	x9
x1	0.3112	-0.7698	-1.4340	0.3319	-1.0257	-0.5980
x2	0.0217	-0.0629	-0.0851	0.1386	-0.1434	-0.7709
x3	-0.0386	0.0156	-0.0016	-0.0214	0.0343	-1.2864
x7	0.0000	-0.0168	0.0081	-0.0639	-0.0639	0.1087
x8	0.0000	0.0005	0.0081	-0.9955	-0.0112	0.1087
x9	-0.5980	0.0177	0.0124	-0.0749	0.0231	-0.6683

(the yellow cells defines the orientation of a relation $r(x_i \rightarrow x_j)$). This achieves the first stage of the BJT4BN algorithm.

Table 4: The B(*1000) matrix for the car Example.

B*1000	x1	x2	x3	x7	x8	x9
x1	20.20	0.00	0.00	323.23	0.00	0.00
x2	7.22	7.22	28.86	584.42	7.22	0.00
x3	3.37	121.21	215.49	53.87	407.41	0.00
x7	5.05	20.20	323.23	20.20	20.20	126.26
x8	5.05	45.45	323.23	0.00	45.45	126.26
x9	0.00	40.40	161.62	10.10	90.91	0.00

Table 5: The Matrix for the car Example.

T	x1	x2	x3	x7	x8	x9
x1	1	0.35714	0.16667	0.25	0.25	0.5
x2	2.8	1	0.46667	0.7	0.7	1.4
x3	6	2.14286	1	1.5	1.5	3
x7	4	1.42857	0.66667	1	1	2
x8	4	1.42857	0.66667	1	1	2
x9	2	0.71429	0.33333	0.5	0.5	1

Table 6: The $F0/1$ Matrix for the car Example.

F0/1	x1	x2	x3	x7	x8	x9
x1	1	0	0	1	0	0
x2	0	0	0	1	0	0
x3	0	1	0	0	1	0
x7	0	0	1	0	0	1
x8	0	0	1	0	0	1
x9	0	1	1	0	1	0

Table 7: Normalized M Matrix for the Car Example.

Norm M	x1	x2	x3	x7	x8	x9
x1	0.0000	0.0000	0.0000	0.3319	0.0000	0.0000
x2	0.0000	0.0000	0.0000	0.1386	0.0000	0.0000
x3	0.0000	0.0156	0.0000	0.0000	0.0343	0.0000
x7	0.0000	0.0000	0.0081	0.0000	0.0000	0.1087
x8	0.0000	0.0000	0.0081	0.0000	0.0000	0.1087
x9	0.0000	0.0177	0.0124	0.0000	0.0231	0.0000

The second stage of the algorithm computes the L list that contains only the yellow cells of the normalized M matrix. The Table 8 provides the L list when sorted with decreasing values of the BJ-measure $m_{ij} = M(x_i \rightarrow x_j)$ of the corresponding relation $r(x_i \rightarrow x_j)$. The third stage transforms the normalized M matrix in the initial G graph with a depth first algorithm (Figure 9).

Stage 4 is dedicated to find and remove the eventualloops in the initial G graph.

Table 8: The L list of the Car Example.

L = {r(i, j), mij}		
i	j	mij
x1	x7	0.3319
x2	x7	0.1386
x7	x9	0.1087
x8	x9	0.1087
x3	x8	0.0343
x9	x2	0.0177
x3	x2	0.0156
x9	x3	0.0124
x7	x3	0.0081

The initial G graph of the car example contains a lot of loops: all the relations participates at least one loop except the $r(x_1 \rightarrow x_7)$ relation. The loops are suppressed with the iterative removing of the $r(x_i \rightarrow x_j)$ relations with the minimal BJ-measure m_{ij}. To this aim, the algorithm duplicates the L list without the $r(x_1 \rightarrow x_7)$ relation to constitute the L_1 list. It is then easy to see that the firstly removed relation is $r(x_7 \rightarrow x_3)$, and that the algorithm will successively remove the $r(x_9 \rightarrow x_3)$ and the $r(x_3 \rightarrow x_2)$ relations (Figure 10).

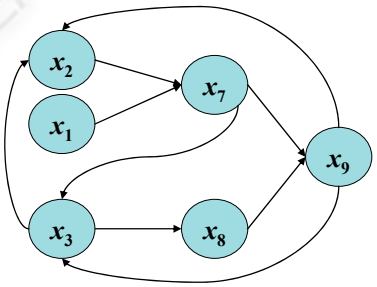


Figure 9: Initial G Graph for the Car Example.

The resultant G graph having no multiple paths, the stage 5 modifies nothing. The final stage of the algorithm is dedicated to the computing of the CP Tables from the N matrix (Table 2) with the equations provided in section 5. For the two root nodes x_1, x_2 and x_3:

- $P(x_1) = 5 / 99 \approx 0.050$
- $P(x_2) = 14 / 99 \approx 0.141$
- $P(x_3) = 30 / 99 \approx 0.303$

For the single relation $r(x_3 \rightarrow x_8)$:

- $P(x_8|x_3) = 11 / 30 \approx 0.366$
- $P(x_8|\neg x_3) = (20\text{-}11) / (99\text{-}30) \approx 0.130$

For the converging node x_7 corresponding to the set $R_7 = \{r(x_1 \rightarrow x_7), r(x_2 \rightarrow x_7\}$:

- $P(x_7|x_1,x_2) = (4+9) / (5+14) \approx 0.684$
- $P(x_7|\neg x_1,x_2) = (20-4) / (99-5) \approx 0.170$
- $P(x_7|x_1,\neg x_2) = (20-9) / (99-14) \approx 0.129$
- $P(x_7|\neg x_1,\neg x_2) = (20-4-9) / (99-5-14) \approx 0.087$

For the converging node x_9 corresponding to the set $R_9=\{r(x_7\rightarrow x_9), r(x_8\rightarrow x_9\}$:

- $P(x_9|x_7,x_8) = (5+5) / (20+20) \approx 0.250$
- $P(x_9|\neg x_7,x_8) = (10-5) / (99-20) \approx 0.063$
- $P(x_7|x_1,\neg x_2) = (10-5) / (99-20) \approx 0.063$
- $P(x_7|\neg x_1,\neg x_2) = (10-5-5) / (99-20-20) =0$

This leads to the final naïve Bayesian Network for the car example (Figure 10), which is compatible with the functional model of Figure 7.

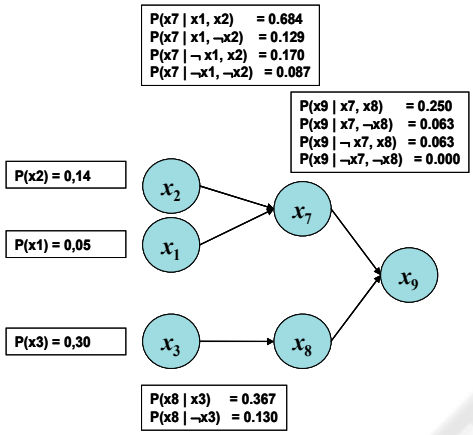


Figure 10: Bayesian Network for the Car Example.

Naturally, this result comes from the way the ω sequence has been made: the Figure 11 shows the signature tree of the C^9 class as provided by the "BJT4S" algorithm of the TOM4L tools. This tree allows recognizing the 10 observations of the C9 class in the ω sequence (i.e. the cover rate is equal to 100%): 5 observations of the C^9 class are recognized by the $\{r(C^2, C^7, [0, 6s]), r(C^1, C^7, [0, 4s]), r(C^7, C^9, [0, 6s])\}$ signature, the other 5 observations being recognized by the $\{r(C^3, C^8, [0, 6s]), r(C^8, C^9, [0, 12s])\}$ signature.

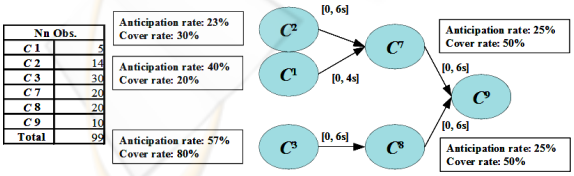


Figure 11: Signature tree of the C^9 class.

Despite of the simplicity of the knowledge base of the car example (Figure 5), this shows *a posteriori* the difficulty to compute the car example Bayesian network from the observations of the ω sequence. The Bayesian Network of Figure 11 allows the building of the functional model for the car example of Figure 12: this functional model is identical to the functional model Figure 7, but it adds probabilities to the functions f_7, f_8 and f_9. These probabilities provide some confidence about the existence of the corresponding functions. This example show the way the TOM4D methodology and the TOM4L process complete together.

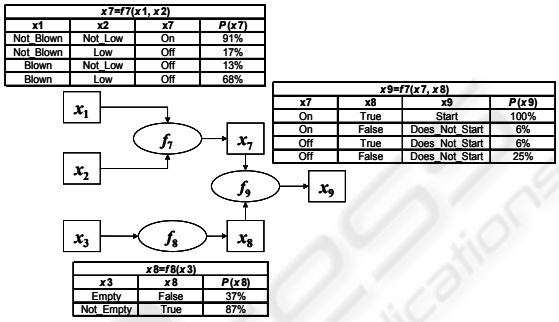


Figure 12: Functional Model for the Car Example.

This example shows the simplicity and the efficiency of the BJM4BN algorithm. The complexity of the algorithm is proportional with the number of timed data in the given set of sequences ω and the number of class $N(C)$ in ω. This is to be compared with the exponential complexity of the methods of the dependency analysis category.

The next section shows the use of the "Transitivity rule" and the ease of use of the proposed algorithm in an industrial environment.

7 REAL WORLD APPLICATION

The Apache system is a clone of Sachem, the knowledge based systems that The Arcelor Group, one of the most important steal companies in the world, has developed to monitor and diagnose its production tools (Le Goc, 2004). Apache aims at controlling a zinc bath, a hot bath containing a liquid mixture of aluminum and zinc continuously fed with aluminum and zinc ingots in which a hot steel strip is immerged. Apache monitors and diagnoses around 11 variables and is able to detect around 24 types of alarms. The analyzed sequence ω contains 687 events of 13 classes for 11 discrete variables. The counting matrix N contains then 156 cells n_{ij} (Bouché, 2005), (Le Goc, 2005).

The node of interest being 1006, the initial G graph resulting from stage 3 of the "BJM4BN" algorithm is given in Figure 13. This graph having no loops,

the stage 4 modify noting and stage 5 builds the final *G* graph of Figure 14.

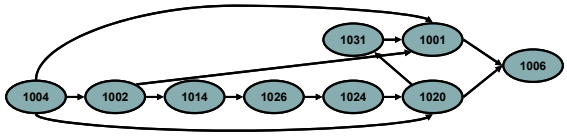


Figure 13: Initial *G* graph.

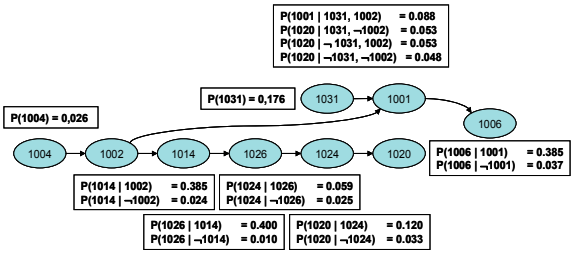


Figure 14: Final *G* graph.

Figure 15 shows a handmade sketch of the functional model of the galvanization bath problem produced by the experts of the Arcelor Group in 2003. The dotted lines in the graph indicate the expert's relations that are not included in the final *G* graph of Figure 14: the *G* graph is all contained in the expert's graph. But the expert's graph does not contain the 1024 class: corresponding to an operator query for a chemical analysis, this class has been removed by experts.

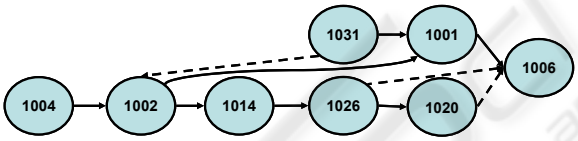


Figure 15: Handmade graph of experts.

Finally, the Figure 16 shows the graph made with the signatures of the 1006 class that can be found in (Bouché, 2005) and (Le Goc, 2005). The signatures are produced with specific Timed Data Mining techniques. This graph is identical to the initial G graph of Figure 13.

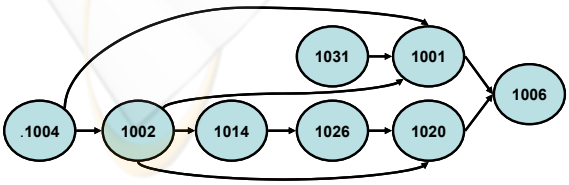


Figure 17: Handmade graph from 1006 signatures.

The main problem is the disappearance of the *r*(1020, 1006) relation in the final *G* graph. The first analysis seem to lead to the conclusion that the 1024 class is responsible of the *r*(1020, 1006) removing.

8 CONCLUSIONS

This paper proposes a new algorithm called the "BJT4BN" algorithm to learn a Bayesian Network from timed data.

The originality of the algorithm comes from the fact that it is designed in the framework of the Timed Observation Theory. This theory represents a set of sequences of timed data in a structure, the Stochastic Representation that allows the definition of a new information measure called the BJ-Measure. This paper defines the principles of a learning algorithm that are based on the BJ-Measure.

The BJT4BN algorithm is efficient both in terms of pertinence and simplicity. These properties come from the BJ-measure that provides an operational way to orient the edges of a Bayesian Network without the exponential CI Tests of the methods of the dependency analysis category.

Our current works are concerned with the combination of the Timed Data Mining techniques of the TOM4L framework with the "BJT4BN" algorithm to define a global validation of the TOM4L learning process.

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