A SUBOPTIMAL FAULT-TOLERANT DUAL CONTROLLER IN MULTIPLE MODEL FRAMEWORK

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Keywords: Fault-tolerant control, Fault detection, Optimal control, Dual control, Stochastic systems.

Abstract: The paper focuses on the design of a suboptimal fault-tolerant dual controller for stochastic discrete-time systems. Firstly a general formulation of the active fault detection and control problem that covers several special cases is presented. One of the special cases, a dual control problem, is then considered throughout the rest of the paper. It is stressed that the designed dual controller can be regarded as a fault-tolerant dual controller in the context of fault detection. Due to infeasibility of the optimal fault-tolerant dual controller for general non-linear system, a suboptimal fault-tolerant dual controller based on rolling horizon technique for jump Markov linear Gaussian system is proposed and illustrated by means of a numerical example.

1 INTRODUCTION

Fault detection is an important part of many automatic control systems and it has attracted a lot of attention during recent years because of increasing requirements on safety, reliability and low maintenance costs. An elementary aim of fault detection is early recognition of faults, e.i. undesirable behaviors of an observed system.

The very earliest fault detection methods use additional sensors for detecting faults. These methods are simple and still used in safety-critical systems. A slightly better fault detection methods utilize some basic assumptions on measured signals and therefore they are usually called signal based methods (Isermann, 2005). To further improve fault detection, more complex methods called model based were developed (Basseville and Nikiforov, 1993).

Except for a few situations were the primary objective is the fault detection itself, it usually complements a control system where the quality of control is of main concern. This fact has stimulated research in area of so called fault-tolerant control. Fault-tolerant control methods can be divided into two basic group: passive fault-tolerant control and active fault-tolerant control methods (Blanke et al., 2003). Passive fault-tolerant control methods design a controller that is robust with respect to considered faults and thus an acceptable deterioration of control quality is caused by the considered faults. On the other hand, active fault-tolerant control methods try to estimate faults and reconfigure a controller in order to retain desired closed loop behavior of a system.

The mentioned fault detection methods and fault-tolerant approaches usually use available measurements passively as shown at the top of Fig. 1, where a passive detector uses inputs $\mathbf{u}_k$ and measurements $\mathbf{y}_k$ for generating decisions $\mathbf{d}_k$. In the case of stochastic systems further improvement can be obtained by applying a suitable input signal, see e.g. (Mehra, 1974) for application in parameter estimation problem. This idea leads to so-called active fault detection which is depicted at the bottom of Fig. 1. The active detector and controller generates, in addition to a decision $\mathbf{d}_k$, an input signal $\mathbf{u}_k$ that controls and simultaneously excites the system and thus improves fault detection and control quality. Note, that the terms passive and active have different meaning than in the fault-tolerant control literature.

The active fault detection is a developing area. The first attempt to formulate and solve the active fault detection problem can be found in (Zhang, 1989), where the sequential probability ratio test was used for determining a valid model and an auxiliary input signal was designed to minimize average number of samples. More general formulation of active fault detection was proposed in (Kerestecioğlu, 1993). An active fault detection for systems with deterministic bounded disturbances was introduced in (Campbell and Nikoukhah, 2004). A unified formulation of active fault detection and control for stochastic systems that covers several special cases was proposed

Punčochář I. and Šimandl M. (2010).
A SUBOPTIMAL FAULT-TOLERANT DUAL CONTROLLER IN MULTIPLE MODEL FRAMEWORK.
In *Proceedings of the 7th International Conference on Informatics in Control, Automation and Robotics*, pages 93-98
DOI: 10.5220/0002935300930098

in (Šimandl and Punčochář, 2009). One of these special cases is the optimal dual control problem that has not been elaborated in the context of that general formulation, yet.

Therefore, the aim of this paper is to examine the dual control problem in the context of fault detection problem. The general formulation for the optimal dual control problem is adopted from (Šimandl and Punčochář, 2009) and an optimal fault-tolerant controller that uses idea of active probing for improving the quality of control is designed. Because of infeasibility of the optimal fault-tolerant dual controller for a general nonlinear stochastic system, the systems that can be described using jump linear Gaussian multiple models are considered and the rolling horizon technique is used for obtaining an approximate solution.

The paper is organized as follows. A general formulation of active fault detection and control is given in Section 2 and the design of a fault-tolerant dual controller is introduced as a special case of the general formulation. The optimal fault-tolerant dual controller obtained using the closed loop information processing strategy is presented in Section 3. Section 4 is devoted to the description of a system using multiple models and the relations for state estimation are given. Finally, a suboptimal fault-tolerant dual controller based on rolling horizon technique is presented in Section 5.

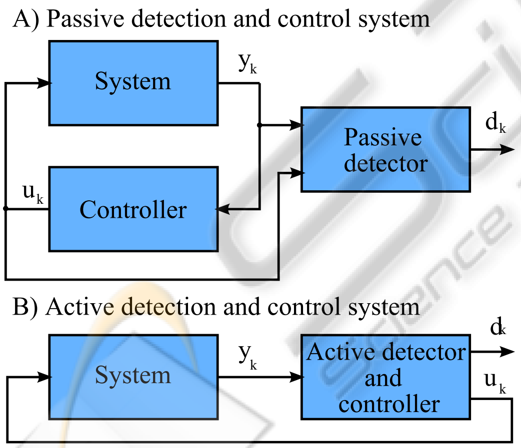

Figure 1: Block diagrams of the passive detection and control system and the active detection and control system.

2 PROBLEM STATEMENT

In this section a general formulation of the active fault detection and control problem is adopted and then a fault-tolerant dual control problem is specified as a special case of the general formulation.

2.1 System

The problem is considered on the finite horizon F. Let an observed system be described at each time $k \in \mathcal{T} = \{0,\dots,F\}$ by the state space discrete-time nonlinear stochastic model

$$\mathbf{x}_{k+1} = \mathbf{f}_k(\mathbf{x}_k, \mu_k, \mathbf{u}_k, \mathbf{w}_k), \quad (1)$$

$$\mu_{k+1} = \mathbf{g}_k(\mathbf{x}_k, \mu_k, \mathbf{u}_k, \mathbf{e}_k), \quad (2)$$

$$\mathbf{y}_k = \mathbf{h}_k(\mathbf{x}_k, \mu_k, \mathbf{v}_k), \quad (3)$$

where nonlinear vector functions $\mathbf{f}_k(\mathbf{x}_k, \mu_k, \mathbf{u}_k, \mathbf{w}_k)$, $\mathbf{g}_k(\mathbf{x}_k, \mu_k, \mathbf{u}_k, \mathbf{e}_k)$ and $\mathbf{h}_k(\mathbf{x}_k, \mu_k, \mathbf{v}_k)$ are known. The input and output of the system are denoted as $\mathbf{u}_k \in \mathcal{U}_k \subseteq \mathbb{R}^{n_u}$ and $\mathbf{y}_k \in \mathbb{R}^{n_y}$, respectively. The subset $\mathcal{U}_k$ can be continuous or discrete and it determines admissible values of the input $\mathbf{u}_k$. The unmeasured state $\bar{\mathbf{x}}_k = [\mathbf{x}_k^T, \mu_k^T]^T$ consists of variables $\mathbf{x}_k \in \mathbb{R}^{n_x}$ and $\mu_k \in \mathcal{M} \subseteq \mathbb{R}^{n_\mu}$. The variable $\mathbf{x}_k$ is the part of the state that should be driven by the input $\mathbf{u}_k$ to a desirable value or region. The variable μ_k carries information about faults. The variable μ_k can be a vector representing fault signals or a scalar that determines the mode of system behavior. The initial state $\bar{\mathbf{x}}_0$ is described by the known probability density function (pdf) $p(\bar{\mathbf{x}}_0) = p(\mathbf{x}_0)\,p(\mu_0)$. The pdfs $p(\mathbf{w}_k)$, $p(\mathbf{e}_k)$ and $p(\mathbf{v}_k)$ of the white noise sequences $\{\mathbf{w}_k\}$, $\{\mathbf{e}_k\}$ and $\{\mathbf{v}_k\}$ are known. The initial state $\bar{\mathbf{x}}_0$ and the noise sequences $\{\mathbf{w}_k\}$, $\{\mathbf{e}_k\}$, $\{\mathbf{v}_k\}$ are mutually independent.

2.2 Active Fault Detector and Controller

In the general formulation, the goal is to design a dynamic causal deterministic system that uses complete available information to generate a decision about faults and an input to the observed system. Such a system can be described at each time step $k \in \mathcal{T}$ by the following relation

$$\begin{bmatrix} \mathbf{d}_k \\ \mathbf{u}_k \end{bmatrix} = \begin{bmatrix} \sigma_k(\mathbf{I}_0^k) \\ \gamma_k(\mathbf{I}_0^k) \end{bmatrix} = \rho_k(\mathbf{I}_0^k), \quad (4)$$

where $\sigma_k(\mathbf{I}_0^k)$ and $\gamma_k(\mathbf{I}_0^k)$ are some unknown vector functions which should be designed to obtain an active fault detector and controller. The complete available information, which has been received up to the time k, is stored in the information vector $\mathbf{I}_0^k = \left[\mathbf{y}_0^{k\,T}, \mathbf{u}_0^{k-1\,T}, \mathbf{d}_0^{k-1\,T}\right]^T$. The notation $\mathbf{y}_i^j$ represents a sequence of the variables $\mathbf{y}_k$ from the time step i up to the time step j. If $i > j$ then the sequence $\mathbf{y}_i^j$ is empty and the corresponding variable is simply left out from an expression. According to this rule, the information vector for time $k = 0$ is defined as $\mathbf{I}_0^0 = \mathbf{I}_0 = \mathbf{y}_0$.

2.3 Criterion

Analogously to the optimal stochastic control problem (Bar-Shalom and Tse, 1974), the design of the optimal active detector and controller is based on minimization of a criterion. A general criterion that penalizes wrong decisions $\mathbf{d}_k$ and deviations of variables $\mathbf{x}_k$ and $\mathbf{u}_k$ from desired values over the finite horizon is the following

$$J\left(\rho_0^F\right) = \mathrm{E}\left\{L\left(\mathbf{x}_0^F, \mu_0^F, \mathbf{u}_0^F, \mathbf{d}_0^F\right)\right\}, \tag{5}$$

where $\mathrm{E}\{\cdot\}$ is the expectation operator with respect to all included random variables and $L\left(\mathbf{x}_0^F, \mu_0^F, \mathbf{u}_0^F, \mathbf{d}_0^F\right)$ is a non-negative real-valued cost function. Due to practical reasons, the cost function is considered in the following additive form

$$L\left(\mathbf{x}_0^F, \mu_0^F, \mathbf{u}_0^F, \mathbf{d}_0^F\right) = \sum_{k=0}^{F} \alpha_k L_k^{\mathrm{d}}\left(\mathbf{d}_k, \mu_k\right) + \left(1-\alpha_k\right) L_k^{\mathrm{c}}\left(\mathbf{x}_k, \mathbf{u}_k\right), \tag{6}$$

where $L_k^{\mathrm{d}}(\mu_k, \mathbf{d}_k)$ is a non-negative real-valued cost function representing the detection aim, the non-negative real-valued cost function $L_k^{\mathrm{c}}(\mathbf{x}_k, \mathbf{u}_k)$ expresses the control aim, and the coefficient α_k belonging to the closed interval $[0,1]$ weights between these two aims. In order to regard the function $L_k^{\mathrm{d}}(\mu_k, \mathbf{d}_k)$ as a meaningful cost function, it should satisfy the inequality $L_k^{\mathrm{d}}(\mu_k, \mu_k) \leq L_k^{\mathrm{d}}(\mu_k, \mathbf{d}_k)$ for all $\mu_k \in \mathcal{M}$, $\mathbf{d}_k \in \mathcal{M}$, $\mathbf{d}_k \neq \mu_k$ at each time step $k \in \mathcal{T}$, and the strict inequality has to hold at least at one time step. The sequence of the functions $\rho_0^{F*} = [\rho_0^*, \rho_1^*, \ldots, \rho_F^*]$ given by minimization of (5) specifies the optimal active detector and controller. The minimization of the criterion (5) can be solved by using three different information processing strategies (IPS's) (Šimandl and Punčochář, 2009), but only the closed loop (CL) IPS is considered in this paper because of its superiority.

2.4 Fault-tolerant Dual Controller

The introduced general formulation covers several special cases that can be simply derived by choosing a particular weighting coefficient α_k and fixing the function $\sigma_k\left(\mathbf{I}_0^k\right)$ or the function $\gamma_k\left(\mathbf{I}_0^k\right)$ in advance. This paper is focused on the special case where only control aim is considered, i.e. the coefficient α_k is set to zero for all $k \in \mathcal{T}$ and none of the functions $\sigma_k\left(\mathbf{I}_0^k\right)$ and $\gamma_k\left(\mathbf{I}_0^k\right)$ are specified in advance. The cost function $L_k^{\mathrm{c}}(\mathbf{x}_k, \mathbf{u}_k)$ is considered to be a quadratic cost function

$$L_k^{\mathrm{c}}(\mathbf{x}_k, \mathbf{u}_k) = \left[\mathbf{x}_k - \mathbf{r}_k\right]^T \mathbf{Q}_k \left[\mathbf{x}_k - \mathbf{r}_k\right] + \mathbf{u}_k^T \mathbf{R}_k \mathbf{u}_k, \tag{7}$$

where $\mathbf{Q}_k$ is a symmetric positive semidefinite matrix, $\mathbf{R}_k$ is a symmetric positive definite matrix, and $\mathbf{r}_k$ is a reference signal. It is considered that the reference signal $\mathbf{r}_k$ is known for the whole horizon in advance.

Since decisions are no longer penalized in the criterion, the function $\sigma_k\left(\mathbf{I}_0^k\right)$ can not be determined by the minimization and the aim is to find only functions $\gamma_k\left(\mathbf{I}_0^k\right)$ for all k. The resulting controller will steer the system in such a way that the criterion is minimized regardless the faults μ_k. Moreover the controller can exhibit the dual property because the CL IPS is used. Due to these two facts the controller can be denoted as the fault-tolerant dual controller.

3 DESIGN OF FAULT-TOLERANT DUAL CONTROLLER

This section is devoted to the optimal fault-tolerant dual controller design. The minimization of the criterion (5) using the CL IPS can be solved by the dynamic programming where the minimization is solved backward in time (Bertsekas, 1995).

The optimal fault-tolerant dual controller is obtained by solving the following backward recursive equation for time steps $k = F, F-1, \ldots, 0$

$$V_k^*\left(\mathbf{y}_0^k, \mathbf{u}_0^{k-1}\right) = \min_{\mathbf{u}_k \in \mathcal{U}_k} \mathrm{E}\left\{L_k^{\mathrm{c}}(\mathbf{x}_k, \mathbf{u}_k) + V_{k+1}^*\left(\mathbf{y}_0^{k+1}, \mathbf{u}_0^k\right) \Big| \mathbf{y}_0^k, \mathbf{u}_0^k\right\}, \tag{8}$$

where $\mathrm{E}\{\cdot|\cdot\}$ stands for the conditional expectation operator and the Bellman function $V_k^*\left(\mathbf{y}_0^k, \mathbf{u}_0^{k-1}\right)$ is the estimate of the minimal cost incurred from time step k up to the final time step F given the input-output data $\left[\mathbf{y}_0^k, \mathbf{u}_0^k\right]$. The initial condition for the backward recursive equation (8) is $V_{F+1}^* = 0$ and it can be shown that the optimal value of the criterion (5) is $J^* = J\left(\rho_0^{F*}\right) = \mathrm{E}\left\{V_0^*(\mathbf{y}_0)\right\}$. Obviously, the optimal input signal $\mathbf{u}_k^*$ is given as

$$\mathbf{u}_k^* = \gamma_k^*\left(\mathbf{y}_0^k, \mathbf{u}_0^{k-1}\right) = \arg \min_{\mathbf{u}_k \in \mathcal{U}_k} \mathrm{E}\left\{L_k^{\mathrm{c}}(\mathbf{x}_k, \mathbf{u}_k) + V_{k+1}^*\left(\mathbf{y}_0^{k+1}, \mathbf{u}_0^k\right) \Big| \mathbf{y}_0^k, \mathbf{u}_0^k\right\}, \tag{9}$$

where the function $\gamma_k^*(\mathbf{I}_0^k)$ represents the optimal fault-tolerant dual controller. The pdf's $p(\overline{\mathbf{x}}_k|\mathbf{I}_0^k, \mathbf{u}_k, \mathbf{d}_k)$ and $p(\mathbf{y}_{k+1}|\mathbf{I}_0^k, \mathbf{u}_k, \mathbf{d}_k)$ needed for the evaluation of the conditional expectation can be obtained using nonlinear filtering methods. Note that there isn't any closed form solution to equations (8) and (9). Therefore approximate techniques have to be used to get at least a suboptimal solution. The selection of a suitable approximation depends on a particular system description and estimation method.

4 MULTIMODEL APPROACH

In the case of a general nonlinear system the state estimation pose a complex functional problem that has to be solved using approximate techniques. One of the attractive method is based on the assumption that the system exhibits distinct modes of behavior. Such systems can be encountered in various field of interests including maneuvering target tracking (Bar-Shalom et al., 2001), abrupt fault detection (Zhang, 1989) and adaptive control (Athans et al., 2006). In this paper, the multimodel approach is used as one step towards the design of feasible fault-tolerant dual controller.

Henceforth, it is assumed that the variable μ_k is a scalar index from the finite discrete set $\mathcal{M} = \{1, 2, \ldots, N\}$ that determines the model valid at time step k. If the exact behavior modes of the system are not known, the set $\mathcal{M}$ can be determined by using existing techniques, see e.g. (Athans et al., 2006).

It is considered that the system can be described at each time $k \in \mathcal{T}$ as

$$\begin{aligned} \mathbf{x}_{k+1} &= \mathbf{A}_{\mu_k}\mathbf{x}_k + \mathbf{B}_{\mu_k}\mathbf{u}_k + \mathbf{G}_{\mu_k}\mathbf{w}_k, \\ \mathbf{y}_k &= \mathbf{C}_{\mu_k}\mathbf{x}_k + \mathbf{H}_{\mu_k}\mathbf{v}_k \end{aligned} \tag{10}$$

where the meaning of the variables $\mathbf{x}_k$, $\mathbf{y}_k$, $\mathbf{u}_k$, $\mathbf{w}_k$ and $\mathbf{v}_k$ is the same as in (1) to (3). The set $\mathcal{U}_k$ is considered to be discrete. The pdf's of the noises $\mathbf{w}_k$ and $\mathbf{v}_k$ are Gaussian with zero-mean and unit variance. The scalar random variable $\mu_k \in \mathcal{M}$ denotes the index of the correct model at time k. Random model switching from model i to model j is described by the known conditional transition probability $P(\mu_{k+1} = j|\mu_k = i) = P_{ij}$. Obviously, the decision $d_k \in \mathcal{M}$ is now scalar too. Known matrices $\mathbf{A}_{\mu_k}$, $\mathbf{B}_{\mu_k}$, $\mathbf{G}_{\mu_k}$, $\mathbf{C}_{\mu_k}$, and $\mathbf{H}_{\mu_k}$ have appropriate dimensions.

The conditional pdf of the state $\mathbf{x}_k$ is a weighted sum of Gaussian distributions

$$\begin{aligned} &p\left(\mathbf{x}_k|\mathbf{y}_0^k, \mathbf{u}_0^{k-1}\right) = \\ &\sum_{\mu_0^k} p\left(\mathbf{x}_k|\mathbf{y}_0^k, \mathbf{u}_0^{k-1}, \mu_0^k\right) P\left(\mu_0^k|\mathbf{y}_0^k, \mathbf{u}_0^{k-1}\right), \end{aligned} \tag{11}$$

where Gaussian conditional pdf $p(\mathbf{x}_k|\mathbf{y}_0^k, \mathbf{u}_0^{k-1}, \mu_0^k)$ can be computed using a Kalman filter that corresponds to the model sequence μ_0^k. The pdf $P(\mu_0^k|\mathbf{y}_0^k, \mathbf{u}_0^{k-1})$ can be obtained recursively as

$$\begin{aligned} P\left(\mu_0^k|\mathbf{y}_0^k, \mathbf{u}_0^{k-1}\right) &= \frac{p\left(\mathbf{y}_k|\mathbf{y}_0^{k-1}, \mathbf{u}_0^{k-1}, \mu_0^k\right)}{c} \\ &\times P\left(\mu_k|\mu_{k-1}\right) P\left(\mu_0^{k-1}|\mathbf{y}_0^{k-1}, \mathbf{u}_0^{k-2}\right), \end{aligned} \tag{12}$$

where c is a normalization constant. The computation of probability of the terminal model $P(\mu_k|\mathbf{y}_0^k, \mathbf{u}_0^{k-1})$ and the predictive conditional pdf $p(\mathbf{y}_{k+1}|\mathbf{y}_0^k, \mathbf{u}_0^k)$ is straightforward.

Unfortunately, as the number of model sequences exponentially increases with time, memory and computational demands become unmanageable. To overcome this problem several techniques based on pruning or merging of Gaussian sum have been proposed. A technique that merges model sequences with the same terminal sequence μ_{k-l}^k is used here. The probability of the terminal sequence of models μ_{k-l}^k is

$$P\left(\mu_{k-l}^k|\mathbf{y}_0^k, \mathbf{u}_0^{k-1}\right) = \sum_{\mu_0^{k-l-1}} P\left(\mu_0^k|\mathbf{y}_0^k, \mathbf{u}_0^{k-1}\right) \tag{13}$$

and the filtering density that has the form of a Gaussian sum

$$\begin{aligned} p\left(\mathbf{x}_k|\mathbf{y}_0^k, \mathbf{u}_0^{k-1}, \mu_{k-l}^k\right) &= \sum_{\mu_0^{k-l-1}} \frac{P\left(\mu_0^k|\mathbf{y}_0^k, \mathbf{u}_0^{k-1}\right)}{P\left(\mu_{k-l}^k|\mathbf{y}_0^k, \mathbf{u}_0^{k-1}\right)} \\ &\times p\left(\mathbf{x}_k|\mathbf{y}_0^k, \mathbf{u}_0^{k-1}, \mu_0^k\right) \end{aligned} \tag{14}$$

is replaced by a Gaussian distribution in such a way that the first two moments, i.e. mean value and covariance matrix, of the variable $\mathbf{x}_k$ remain unchanged.

5 FEASIBLE ALGORITHM BASED ON ROLLING HORIZON

Even if the state and output pdfs are known, the backward recursive relation (8) can not be solved analytically because of intractable integrals. A systematic approach to forward solution of the backward recursive relation (8) based on the stochastic approximation method is presented e.g. in (Bayard, 1991). A simple alternative approach is represented by the rolling horizon technique, where the optimization horizon is truncated and terminal cost-to-go of such truncated optimization horizon is replaced by zero. The length $F_o > 0$ of truncated horizon should be as short as possible to save computational demands but on other hand it has to preserve dependence of value of the minimized criterion on the input signal $\mathbf{u}_k$. In this paper the optimization horizon $F_o = 3$ will be considered to simplify computations. The cost-to-go function $V_{k+3}^*\left(\mathbf{y}_0^{k+3}, \mathbf{u}_0^{k+2}\right)$ is replaced by zero value. Then the input $\mathbf{u}_{k+2}^a = 0$ and the cost-to-go function $V_{k+2}^a\left(\mathbf{y}_0^{k+2}, \mathbf{u}_0^{k+1}\right)$ is

$$\begin{aligned} &V_{k+2}^a\left(\mathbf{y}_0^{k+2}, \mathbf{u}_0^{k+1}\right) = \\ &\mathrm{E}\left\{\left[\mathbf{x}_{k+2} - \mathbf{r}_{k+2}\right]^T \mathbf{Q}_{k+2}\left[\mathbf{x}_{k+2} - \mathbf{r}_{k+2}\right]|\mathbf{y}_0^{k+2}, \mathbf{u}_0^{k+1}\right\}. \end{aligned} \tag{15}$$

The input $\mathbf{u}_{k+2}$ is zero because it can not influence the value of the criterion on the optimization horizon and the matrix $\mathbf{R}_{k+2}$ is positive definite. Note, that the value of the cost-to-go function $V^{\mathrm{a}}_{k+2}\left(\mathbf{y}_0^{k+2},\mathbf{u}_0^{k+1}\right)$ can be computed analytically based on the first two moments of the state $\mathbf{x}_{k+2}$ given by the pdf $p\left(\mathbf{x}_{k+2}|\mathbf{y}_0^{k+2},\mathbf{u}_0^{k+1}\right)$. The input $\mathbf{u}^{\mathrm{a}}_{k+1} = -\mathbf{W}^{-1}\mathbf{D}$ and the cost-to-go function at time step $k+1$ is

$$V^{\mathrm{a}}_{k+1}\left(\mathbf{y}_0^{k+1},\mathbf{u}_0^{k+1}\right) = \mathrm{E}\left\{[\mathbf{x}_{k+1}-\mathbf{r}_{k+1}]^T\mathbf{Q}_{k+1}[\mathbf{x}_{k+1}-\mathbf{r}_{k+1}]\,|\mathbf{y}_0^{k+2},\mathbf{u}_0^{k+1}\right\} + K - \mathbf{D}^T\mathbf{W}^{-1}\mathbf{D}, \tag{16}$$

where

$$\mathbf{W} = \mathbf{R}_{k+1} + \sum_{\mu_{k+1}}\mathbf{B}^T_{\mu_{k+1}}\mathbf{Q}_{k+2}\mathbf{B}_{\mu_{k+1}}P\left(\mu_{k+1}|\mathbf{y}_0^{k+1},\mathbf{u}_0^{k}\right), \tag{17}$$

$$\mathbf{D} = \sum_{\mu_{k+1}}\mathbf{B}^T_{\mu_{k+1}}\mathbf{Q}_{k+2}\left[\mathbf{A}_{\mu_{k+1}}\hat{\mathbf{x}}_{k+1}(\mu_{k+1})-\mathbf{r}_{k+2}\right] \times P\left(\mu_{k+1}|\mathbf{y}_0^{k+1},\mathbf{u}_0^{k}\right), \tag{18}$$

$$K = \sum_{\mu_{k+1}}\Big\{\left[\mathbf{A}_{\mu_{k+1}}\hat{\mathbf{x}}_{k+1}(\mu_{k+1})-\mathbf{r}_{k+2}\right]^T\mathbf{Q}_{k+2} \times\left[\mathbf{A}_{\mu_{k+1}}\hat{\mathbf{x}}_{k+1}(\mu_{k+1})-\mathbf{r}_{k+2}\right] + \mathrm{Tr}\left(\mathbf{Q}_{k+2}(\mathbf{A}_{\mu_{k+1}}\mathbf{P}_{k+1}(\mu_{k+1})\mathbf{A}^T_{\mu_{k+1}}+\mathbf{G}_{\mu_{k+1}}\mathbf{G}^T_{\mu_{k+1}})\right)\Big\} \times P\left(\mu_{k+1}|\mathbf{y}_0^{k+1},\mathbf{u}_0^{k}\right). \tag{19}$$

The mean $\hat{\mathbf{x}}_{k+1}(\mu_{k+1}) = \mathrm{E}\left\{\mathbf{x}_{k+1}|\mathbf{y}_0^{k+1},\mathbf{u}_0^{k},\mu_{k+1}\right\}$ and the corresponding covariance matrix $\mathbf{P}_{k+1}(\mu_{k+1})$ can be obtained from estimation algorithm. If the input $\mathbf{u}_{k+1}$ was used at time step $k+1$ the resulting controller would be cautious because it would respect uncertainty. The input at time step k is given as

$$\mathbf{u}^{\mathrm{a}}_k = \min_{\mathbf{u}_k\in\mathcal{U}_k}\mathrm{E}\left\{L^{\mathrm{c}}_k(\mathbf{x}_k,\mathbf{u}_k)+V^{\mathrm{a}}_{k+1}\left(\mathbf{y}_0^{k+1},\mathbf{u}_0^{k}\right)\mathbf{y}_0^{k},\mathbf{u}_0^{k}\right\}.$$

The expectation of the cost function $V^{\mathrm{a}}_{k+1}\left(\mathbf{y}_0^{k+1},\mathbf{u}_0^{k}\right)$ with respect to $\mathbf{y}_{k+1}$ seems to be computationally intractable. Therefore the expectation and subsequent minimization over discrete set $\mathcal{U}_k$ are performed numerically.

6 NUMERICAL EXAMPLE

The proposed fault-tolerant dual controller is compared with a cautious (CA) controller and a heuristic certainty equivalence (HCE) controller. The CA controller is obtained when just one-step look ahead policy is used and it takes uncertainties into account but lacks probing. The HCE controller is based on the assumption that the certainty equivalence principle holds even it is not true and inputs are determined as solutions to the problem where all uncertain quantities were fixed at some typical values.

Although the relative performance of three suboptimal controllers can differ in dependence on a particular system, the dual controller should outperform HCE and CA controllers in problems where uncertainty plays a major role. This numerical example illustrates a well known issue of pure CA controllers called 'turn-off' phenomenon, where the CA controller refuses to control a system because of large uncertainty. The initial uncertainty is quite high, but once it is reduced through measurements the problem becomes almost certainty equivalent. It is the reason why the HCE controller performs quite well in this particular example.

The quality of control is evaluated by M Monte Carlo runs. The value of the cost L for particular Monte Carlo simulation is denoted L_i and the value of the criterion J is estimated as $\hat{J} = 1/M\sum_{i=1}^{M}L_i$. Variability among Monte Carlo simulations is expressed by $\mathrm{var}\{L\} = 1/(M-1)\sum_{i=1}(L_i-\hat{J})^2$ and the quality of the criterion estimate $\hat{J}$ is expressed by $\mathrm{var}\{\hat{J}\}$ which is computed using bootstrap technique.

The detection horizon $F = 30$ is considered and the parameters of a single input single output scalar system are given in Table 1. The initial probabilities are $P(\mu_0 = 1) = P(\mu_0 = 2) = 0.5$, the transition probabilities are $P_{1,1} = P_{2,2} = 0.9$, $P_{1,2} = P_{2,1} = 0.1$, and parameters of Gaussian distribution are $\hat{x}'_0 = 1$ and $\mathbf{P}'_{x,0} = 0.01$. The discrete set of admissible values of input u_k is chosen to be $\mathcal{U}_k = \{-3,-2.9,\ldots,2.9,3\}$ for all $k\in\mathcal{T}$. The reference signal is the square wave with peaks of ±0.4 and the period 13 steps and the weighting matrices in the cost function are chosen to be $Q_k = 1$ and $R_k = 0.001$ for all time steps.

Table 1: Parameters of the controlled system.

μ_k	$a_k(\mu_k)$	$b_k(\mu_k)$	$g_k(\mu_k)$	$c_k(\mu_k)$	$h_k(\mu_k)$
1	0.9	0.1	0.01	1	0.05
2	0.9	-0.098	0.01	1	0.05

An example of the typical state trajectories for all three controllers is given in Fig. 2. It can be seen that the CA controller does not control the system at the beginning of the control horizon at all. The criterion value estimates $\hat{J}$, the accuracies of these estimates $\mathrm{var}\{\hat{J}\}$, and the variability of Monte Carlo simulations $\mathrm{var}\{L\}$, that were computed using $M = 200$ Monte Carlo simulations, are given in Table 2. In comparison with the CA controller, the quality of control is improved by 55% in the case of the HCE controller and by 68% in the case of the dual controller.

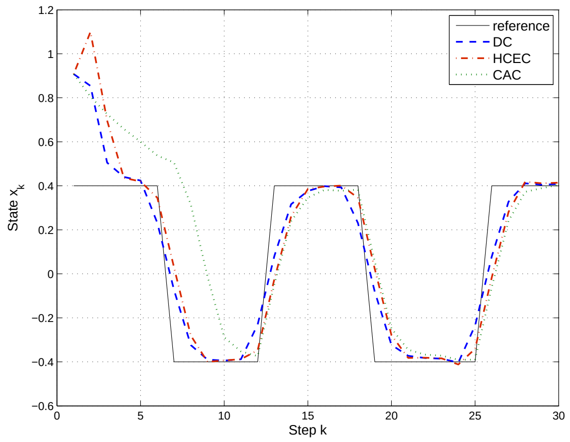

Figure 2: State trajectories for dual controller (DC), heuristic certainty equivalence controller (HCEC) and cautious controller (CAC).

Table 2: Criterion value estimates for particular controllers.

Controller	$\hat{J}$	var$\{\hat{J}\}$	var$\{L\}$
HCEC	3.2126	0.0164	3.2885
CAC	7.2186	0.0068	1.3194
DC	2.3131	0.0109	2.0889

7 CONCLUSIONS

The optimal fault-tolerant dual controller has been obtained as a special case of the general formulation. Since the optimal fault-tolerant controller is computationally infeasible the multimodel approach and rolling horizon techniques were used to obtain a suboptimal fault-tolerant dual controller. The performance of the proposed controller was compared with a heuristic certainty equivalence controller and cautious controller in a numerical example. Although all controllers were able to control the system even a fault occurred, the fault-tolerant dual controller exhibits the best performance.

ACKNOWLEDGEMENTS

This work was supported by the Ministry of Education, Youth and Sports of the Czech Republic, project No. 1M0572, and by the Czech Science Foundation, project No. GA102/08/0442.

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