REMOTE CONTROL OF A MOBILE ROBOT SUBJECT TO A COMMUNICATION DELAY

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Abstract: This paper addresses the remote tracking control of a mobile robot subject to a bilateral time-delay. The delay affects the system since the controller and the robot are linked via a delay inducing communication channel, such as the Internet, and consequently, the performance and stability of the system are compromised. Based on the notion of anticipating synchronization, a state estimator which stabilizes the system when it is affected by a bilateral time-delay is proposed. A stability analysis including the system, tracking controller and estimator is provided, and the applicability of the proposed delay compensation strategy is demonstrated by means of experiments between multi-robot platforms located in Eindhoven, The Netherlands and Tokyo, Japan.

1 INTRODUCTION

In the increasingly fast and diverse technological developments of the last decades the duties and tasks conferred to control systems have become much more complex and decisive. Requirements now encompass flexibility, robustness, ubiquity, transparency and balancing tradeoffs, among others.

Specifically, the study of systems embedded with time-delays and the control methodologies that can be applied to them has become significatively important as a way to undertake remote, dangerous or distributed tasks. As a matter of fact, the remote control, or the control of a system subject to a bilateral time-delay, is part of the underlying problem in two of control engineering's fundamental topics, namely teleoperation strategies and Networked Control Systems (NCS). The problem remains a central issue even though there are many more considerations in addition to this problem when considering teleoperated systems and NCS. Examples of aspects to consider would be transparency and force reflection in teleoperation (Niemeyer et al., 2008); and varying transmission delays and sampling/transmission intervals, packet loss, communication constraints and quantization effects in NCS (Heemels et al., 2010).

Several techniques have been proposed so far in order to cope with bilateral time-delay in this setting, e.g. the use of the scattering transformation, wave variables formulation, and queuing methodologies to name a few. A detailed description of such techniques and many others, together with further references, can be found in (Hokayem and Spong, 2006) and (Tipsuwan and Chow, 2003).

In this work, a control strategy which allows the remote control of a unicycle-type mobile robot is proposed. The bilateral time-delay is compensated by means of a state estimator inspired on a predictor based on synchronization presented in (Oguchi and Nijmeijer, 2005a) and (Oguchi and Nijmeijer, 2005b). The main idea behind the state estimator is to reproduce the system's behavior without time-delay in order to drive an anticipating controller. The problem presents various challenges since the system is nonlinear and subject to a non-holonomic constraint. Additionally, the difficulties faced when implementing the ideas proposed in an experimental setting using the Internet as the communication channel should be taken into account and are also discussed in depth. In (Kojima et al., 2010) a similar state estimator has been

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applied to a mobile robot subject to a communication delay, and the necessary conditions for the estimator's convergence have been derived. In this paper an alternative approach is taken in order to prove the stability of the whole system, including the mobile robot, the tracking controller and the state estimator.

The paper is organized as follows. Section 2 recalls the tracking control of a delay-free mobile robot. In Section 3, a control scheme intended to control a mobile robot subject to a bilateral time-delay is proposed together with its corresponding stability analysis. Section 4 provides an overview of the experimental platform used to validate the control strategies proposed, explains how the most critical implementation issues were addressed, and presents the experimental results. Conclusions and ideas for future work are provided in Section 5.

2 CONTROL OF A UNICYCLE

The tracking control of a unicycle-type mobile robot is presented in this section. To begin with, consider the posture kinematic model of a unicycle,

$$\begin{aligned} \dot{x}(t) &= v(t)\cos\theta(t), \\ \dot{y}(t) &= v(t)\sin\theta(t), \\ \dot{\theta}(t) &= \omega(t), \end{aligned} \tag{1}$$

in which $x(t)$ and $y(t)$ denote the robot's position in the global coordinate frame X-Y, $\theta(t)$ defines its orientation w.r.t to the X axis, and $v(t)$ and $\omega(t)$ describe the robots' translational and rotational velocities respectively, regarded as its control inputs. The system's state is defined as $q(t) = [x(t)\ \ y(t)\ \ \theta(t)]^T$ and the non-slip condition on the unicycle's wheels impose a non-holonomic constraint (Brockett, 1983).

The control objective for the robot is to track the reference trajectory generated by the exosystem,

$$\begin{aligned} \dot{x}_r(t) &= v_r(t)\cos\theta_r(t), \\ \dot{y}_r(t) &= v_r(t)\sin\theta_r(t), \\ \dot{\theta}_r(t) &= \omega_r(t), \end{aligned} \tag{2}$$

with state $q_r(t) = [x_r(t)\ \ y_r(t)\ \ \theta_r(t)]^T$. The exosystem's reference velocities $v_r(t)$ and $\omega_r(t)$ are defined in terms of its Cartesian velocities $\dot{x}_r(t)$, $\dot{y}_r(t)$ and accelerations $\ddot{x}_r(t)$, $\ddot{y}_r(t)$, i.e.,

$$\begin{aligned} v_r(t) &= \sqrt{\dot{x}_r^2(t) + \dot{y}_r^2(t)}, \\ \omega_r(t) &= \frac{\dot{x}_r(t)\ddot{y}_r(t) - \ddot{x}_r(t)\dot{y}_r(t)}{\dot{x}_r^2(t) + \dot{y}_r^2(t)}. \end{aligned} \tag{3}$$

The difference between the exosystem's and the system's states may be expressed w.r.t the system's

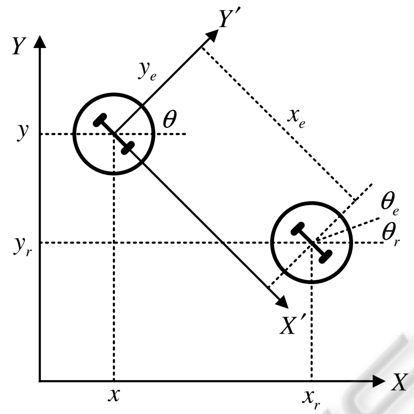

Figure 1: Mobile robot, reference exosystem, and error coordinates.

local coordinate frame X'–Y' in order to define the error coordinates $q_e(t) = [x_e(t)\ \ y_e(t)\ \ \theta_e(t)]^T$, as proposed by (Kanayama et al., 1990) and shown in Figure (1). These coordinates are given by the clockwise rotation of the position differences between $q_r(t)$ and $q(t)$, resulting in,

$$\begin{bmatrix} x_e(t) \\ y_e(t) \\ \theta_e(t) \end{bmatrix} = \begin{bmatrix} \cos\theta(t) & \sin\theta(t) & 0 \\ -\sin\theta(t) & \cos\theta(t) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_r(t) - x(t) \\ y_r(t) - y(t) \\ \theta_r(t) - \theta(t) \end{bmatrix}. \tag{4}$$

Differentiating (4) w.r.t. time yields the following error dynamics,

$$\begin{aligned} \dot{x}_e(t) &= \omega(t)y_e(t) + v_r(t)\cos\theta_e(t) - v(t), \\ \dot{y}_e(t) &= -\omega(t)x_e(t) + v_r(t)\sin\theta_e(t), \\ \dot{\theta}_e(t) &= \omega_r(t) - \omega(t). \end{aligned} \tag{5}$$

A tracking controller which results in closed-loop error dynamics which have a cascaded structure has been proposed in (Jakubiak et al., 2002), (Panteley et al., 1998), and is given by,

$$\begin{aligned} v(t) &= v_r(t) + c_2 x_e(t) - c_3\omega_r(t)y_e(t), \\ \omega(t) &= \omega_r(t) + c_1\sin\theta_e(t), \end{aligned} \tag{6}$$

with c_1, $c_2 > 0$ and $c_3 > -1$ ensuring stability.

3 BILATERAL TIME-DELAY COMPENSATION

In this section, a state estimator with a predictor-like structure similar to the one proposed in (Kojima et al., 2010) is applied to a unicycle-type mobile robot subject to a bilateral time-delay. The origin of this type of predictor can be traced back to the appearance of the notion of anticipating synchronization in coupled

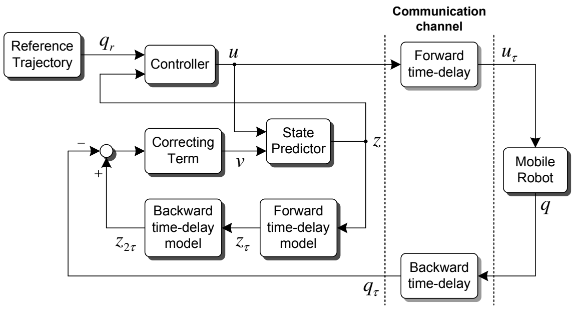

Figure 2: Bilateral time-delay compensation scheme block diagram representation.

chaotic systems, which was first observed by (Voss, 2000). After the same behavior was observed in certain physical systems such as specific electronic circuits and lasers, it was studied for more general systems in (Oguchi and Nijmeijer, 2006). As a result of this generalization, a state predictor based on synchronization for nonlinear systems with input time-delay was proposed in (Oguchi and Nijmeijer, 2005a). The same concept, which can be seen as a state estimator with a predictor-like structure, is proposed for a mobile robot subject to a bilateral time-delay in the block diagram in Figure 2.

3.1 Controller Structure

When considering a bilateral time-delay the system's output is also delayed. In this case the mobile robot is subject to a forward τ_f and backward τ_b time-delay, as denoted in (Hokayem and Spong, 2006). Given the mobile robot (1) subject to a bilateral time-delay, the robot's posture kinematic model after the forward delay is given by,

$$\begin{aligned}\dot{x}(t) &= v(t-\tau_f)\cos\theta(t),\\ \dot{y}(t) &= v(t-\tau_f)\sin\theta(t),\\ \dot{\theta}(t) &= \omega(t-\tau_f).\end{aligned} \tag{7}$$

From the controller's side point of view, the robot's kinematic model is also affected by the backward time-delay τ_b, resulting in the following model,

$$\begin{aligned}\dot{x}(t-\tau_b) &= v(t-\tau_f-\tau_b)\cos\theta(t-\tau_b),\\ \dot{y}(t-\tau_b) &= v(t-\tau_f-\tau_b)\sin\theta(t-\tau_b),\\ \dot{\theta}(t-\tau_b) &= \omega(t-\tau_f-\tau_b).\end{aligned} \tag{8}$$

To improve the robot's performance when subject to the time-delay, the dynamics of the estimator, with state $z(t) = [z_1(t)\ z_2(t)\ z_3(t)]^T$, are proposed as,

$$\begin{aligned}\dot{z}_1(t) &= v(t)\cos z_3(t) + \nu_x(t),\\ \dot{z}_2(t) &= v(t)\sin z_3(t) + \nu_y(t),\\ \dot{z}_3(t) &= \omega(t) + \nu_\theta(t),\end{aligned} \tag{9}$$

with $\nu(t) = [\nu_x(t)\ \nu_y(t)\ \nu_\theta(t)]^T$ defining a correcting term relating the estimator's and the system's states.

The correcting term $\nu(t)$ is intended to bring the estimator's and the system's states closer, since the robot's initial conditions are assumed to be unknown. There is complete freedom in the design of this term, with the simplest choice being,

$$\begin{aligned}\nu_x(t) &= -K_x(z_1(t-\tilde{\tau}_f-\tilde{\tau}_b) - x(t-\tilde{\tau}_b)),\\ \nu_y(t) &= -K_y(z_2(t-\tilde{\tau}_f-\tilde{\tau}_b) - y(t-\tilde{\tau}_b)),\\ \nu_\theta(t) &= -K_x(z_3(t-\tilde{\tau}_f-\tilde{\tau}_b) - \theta(t-\tilde{\tau}_b)),\end{aligned} \tag{10}$$

where $\tilde{\tau}_f$ and $\tilde{\tau}_b$ model the robot's forward and backward delays. Hereinafter the forward and backward time-delays are assumed to be equal and constant, i.e. $\tau_b = \tau_f = \tau$, and modeled perfectly, i.e. $\tilde{\tau}_f = \tilde{\tau}_b = \tau$. The feasibility of this assumption will be discussed before the experimental results are presented.

In order to ease the computations in the stability analysis, a different correcting term will be proposed. Two new sets of error coordinates are defined, namely $z_e(t)$ and $p_e(t)$. The first coordinates relate the estimator's state with the reference trajectory, i.e.

$$\begin{bmatrix} z_{1_e}(t)\\ z_{2_e}(t)\\ z_{3_e}(t)\end{bmatrix} = \begin{bmatrix}\cos z_3(t) & \sin z_3(t) & 0\\ -\sin z_3(t) & \cos z_3(t) & 0\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix} x_r(t)-z_1(t)\\ y_r(t)-z_2(t)\\ \theta_r(t)-z_3(t)\end{bmatrix}. \tag{11}$$

while the second set relates the delayed estimator's state with the current system's state, i.e.,

$$\begin{bmatrix} p_{1_e}(t)\\ p_{2_e}(t)\\ p_{3_e}(t)\end{bmatrix} = \begin{bmatrix} p_{11} & p_{12} & 0\\ p_{21} & p_{22} & 0\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix} x(t-\tau)-z_1(t-2\tau)\\ y(t-\tau)-z_2(t-2\tau)\\ \theta(t-\tau)-z_3(t-2\tau)\end{bmatrix}, \tag{12}$$

where $p_{11} = p_{22} = \cos z_3(t-2\tau)$ and $p_{12} = -p_{21} = \sin z_3(t-2\tau)$.

Given the error coordinates (11) and (12), the following correcting term is proposed,

$$\begin{aligned}\nu_x(t) &= -K_x p_{1_e}(t)\cos z_3(t) + K_y p_{2_e}(t)\sin z_3(t),\\ \nu_y(t) &= -K_x p_{1_e}(t)\sin z_3(t) - K_y p_{2_e}(t)\cos z_3(t),\\ \nu_\theta(t) &= -K_\theta \sin p_{3_e}(t).\end{aligned} \tag{13}$$

Since the control scheme of Figure 2 uses the state estimator's output as the controller's input, a new control law is proposed based on the tracking control (6),

$$\begin{aligned} v(t) &= v_r(t) + c_2 z_{1_e}(t) - c_3\omega_r(t) z_{2_e}(t),\\ \omega(t) &= \omega_r(t) + c_1 \sin z_{3_e}(t).\end{aligned} \tag{14}$$

Remark 1. *Due to the input time-delay, the system's control action is given by the delayed controller, i.e.,*

$$v(t-\tau) = v_r(t-\tau) + c_2 z_{1_e}(t-\tau) - c_3\omega_r(t-\tau) z_{2_e}(t-\tau),$$
$$\omega(t-\tau) = \omega_r(t-\tau) + c_1 \sin z_{3_e}(t-\tau).$$

The resulting control action provides an idea of how the system will behave. Intuitively, the robot will track the reference trajectory after a time τ. This will be examined in detail during the stability analysis.

Remark 2. *Although the stability analysis is unique for each control law, the delay compensation scheme is controller independent, so in this sense the generality of the scheme holds. Consider for example a controller which accounts for actuator saturation and collision avoidance such as in (Kostic et al., 2009).*

3.2 Stability Analysis

In order to describe the system's performance it becomes necessary to establish stability criteria. The system's control objectives are defined as follows,

- $q(t) \to q_r(t-\tau)$, the system converges to a delayed version of the reference trajectory;
- $z(t) \to q(t+\tau)$, the state estimator anticipates the system;
- $z(t) \to q_r(t)$, the state estimator converges to the reference trajectory.

Considering the control objectives and Remark 1, the following proposition is formulated.

Proposition 1. *Given the unicycle-type mobile robot (7) subject to a bilateral delay 2τ, the state estimator (9)-(13), and the control law (14), the robot will track a delayed version $q_r(t-\tau)$ of the reference trajectory.*

In accordance to Proposition 1 it follows that proving that the equilibrium point $(z_e(t), p_e(t)) = 0$ is stable satisfies the control objectives.

The error coordinates (11)-(12) are grouped and differentiated w.r.t time, and the tracking control law (14) is substituted in them. Given $\xi_1 = [z_{1_e}\ z_{2_e}\ p_{1_e}\ p_{2_e}]^T$ and $\xi_2 = [z_{3_e}\ p_{3_e}]^T$, the resulting closed-loop error dynamics are rearranged in the following cascaded form,

$$\dot{\xi}_1(t) = A_0(t,t-2\tau)\xi_1(t) + A_1\xi_1(t-2\tau) + g(t,t-2\tau,\xi_1(t),\xi_2(t),\xi_1(t-2\tau),\xi_2(t-2\tau)), \tag{15}$$

$$\dot{\xi}_2(t) = f_2(t,\xi_2(t),\xi_2(t-2\tau)), \tag{16}$$

where,

$$A_0(t,t-2\tau) = \begin{bmatrix} -c_2 & f_{12} & K_x & 0 \\ f_{21} & 0 & 0 & K_y \\ 0 & 0 & 0 & f_{34} \\ 0 & 0 & f_{43} & 0 \end{bmatrix},$$

$$A_1 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & K_x & 0 \\ 0 & 0 & 0 & K_y \end{bmatrix},$$

$$g(t,t-2\tau,\xi_1(t),\xi_2(t),\xi_1(t-2\tau),\xi_2(t-2\tau)) = \begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \\ 0 & g_{32} \\ 0 & g_{42} \end{bmatrix} \xi_2(t) + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ h_{31} & h_{32} \\ h_{41} & h_{42} \end{bmatrix} \xi_2(t-2\tau),$$

$$f_2(t,\xi_2(t),\xi_2(t-2\tau)) = \begin{bmatrix} -c_1 \sin z_{3_e}(t) + K_\theta \sin p_{3_e}(t) \\ K_\theta \sin p_{3_e}(t-2\tau) \end{bmatrix},$$

with,

$$f_{12} = (1+c_3)\omega_r(t),$$
$$f_{21} = -\omega_r(t),$$
$$f_{34} = -f_{43} = \omega_r(t-2\tau),$$
$$g_{11} = c_1 z_{2_e}(t)\int_0^1 \cos(s z_{3_e}(t))ds + v_r(t)\int_0^1 \sin(s z_{3_e}(t))ds,$$
$$g_{21} = (v_r(t) - c_1 z_{1_e}(t))\int_0^1 \cos(s z_{3_e}(t))ds,$$
$$g_{12} = -K_\theta z_{2_e}(t)\int_0^1 \cos(s p_{3_e}(t))ds,$$
$$g_{22} = K_\theta z_{1_e}(t)\int_0^1 \cos(s p_{3_e}(t))ds,$$
$$g_{32} = (v_r(t-2\tau) + c_2 z_{1_e}(t-2\tau) - c_3\omega_r(t-2\tau) z_{2_e}(t-2\tau)) \cdot \int_0^1 \sin(s p_{3_e}(t))ds,$$
$$g_{42} = (v_r(t-2\tau) + c_2 z_{1_e}(t-2\tau) - c_3\omega_r(t-2\tau) z_{2_e}(t-2\tau)) \cdot \int_0^1 \cos(s p_{3_e}(t))ds,$$
$$h_{31} = c_1 p_{2_e}(t)\int_0^1 \cos(s z_{3_e}(t-2\tau))ds,$$
$$h_{41} = -c_1 p_{1_e}(t)\int_0^1 \cos(s z_{3_e}(t-2\tau))ds,$$
$$h_{32} = -K_\theta p_{2_e}(t)\int_0^1 \cos(s p_{3_e}(t-2\tau))ds,$$
$$h_{42} = K_\theta p_{1_e}(t)\int_0^1 \cos(s p_{3_e}(t-2\tau))ds.$$

The definition of a persistently exciting (PE) signal is required in order to establish the stability of the cascaded system (15)-(16).

Definition 1. *A continuous function $\omega : \mathbb{R}^+ \to \mathbb{R}$ is said to be persistently exciting (PE) if $\omega(t)$ is bounded, Lipschitz, and constants $\delta_c > 0$ and $\varepsilon > 0$ exist such that,*

$$\forall t \geq 0,\ \exists s:\ t-\delta_c \leq s \leq t \ \text{ such that } \ |||\omega(s)| \geq \varepsilon.$$

It is assumed that the desired rotational velocity ω_r, $i = 1,2$ is PE. Consequently, the time varying term $\omega_r(t-2\tau)$, in the matrix function $A_0(t,t-2\tau)$ may be renamed as $\bar{\omega}_r(t)$, resulting in a matrix $\bar{A}_0(t)$. Note that the matrix function, although time-varying, will always have entries in the same positions. From a practical viewpoint the matrix's entry values will change with time, but not its structure.

The following theorem establishes the local stability of the equilibrium point $(z_e(t), p_e(t)) = 0$, and hence the fulfillment of the control objectives.

Theorem 1. *Consider a unicycle-type mobile robot subject to a bilateral time-delay 2τ with a posture kinematic model given by (7). Suppose that the robot's reference trajectory is generated by an exosystem with dynamics (2). The estimator (9) together with the correcting term (13) are used to generate the system's control input (14), which is applied to the robot after a delay τ. Suppose that the reference rotational velocity $\omega_r(t)$ is PE, that the controller gains satisfy $c_1 > 0$, $c_2 > 0$, and $c_3 > -1$, and that the correcting term gains satisfy $K_x, K_y, K_\theta < 0$. If*

- *subsystem $\dot{\xi}_1(t) = \bar{A}_0(t)\xi_1(t) + A_1\xi_1(t-2\tau)$ in (15) is at least locally exponentially stable (LES);*
- *function g in (15) is bounded;*
- *system (16) is LES;*

then the equilibrium point $(z_e(t), p_e(t)) = 0$ of the closed-loop error dynamics (15)-(16) is LES.

Proof. For the sake of brevity only a sketch of the proof will be provided.

The theorem is derived from the results for cascaded systems presented in (Panteley and Loría, 1998), which have been successfully applied to the tracking control of a mobile robot in (Jakubiak et al., 2002) and to the mutual synchronization of two robots in (van den Broek et al., 2009).

3.2.1 Stability of $\xi_1(t) = 0$

Subsystem $\dot{\xi}_1(t)$ in (15) is a linear time-varying (LTV) system which can be separated as follows,

$$\begin{bmatrix} \dot{z}_{1_e}(t) \\ \dot{z}_{2_e}(t) \end{bmatrix} = \begin{bmatrix} -c_2 & (1+c_3)\omega_r(t) \\ -\omega_r(t) & 0 \end{bmatrix} \begin{bmatrix} z_{1_e}(t) \\ z_{2_e}(t) \end{bmatrix} + \begin{bmatrix} K_x & 0 \\ 0 & K_y \end{bmatrix} \begin{bmatrix} p_{1_e}(t) \\ p_{2_e}(t) \end{bmatrix}, \tag{17}$$

$$\begin{bmatrix} \dot{p}_{1_e}(t) \\ \dot{p}_{2_e}(t) \end{bmatrix} = \begin{bmatrix} 0 & \bar{\omega}_r(t) \\ -\bar{\omega}_r(t) & 0 \end{bmatrix} \begin{bmatrix} p_{1_e}(t) \\ p_{2_e}(t) \end{bmatrix} + \begin{bmatrix} K_x & 0 \\ 0 & K_y \end{bmatrix} \begin{bmatrix} p_{1_e}(t-2\tau) \\ p_{2_e}(t-2\tau) \end{bmatrix}. \tag{18}$$

Note that the output of system (18) perturbs system (17) in a similar way as in the cascaded structure (15)-(16). Nevertheless, the coupling term in (17) will vanish if (18) converges to zero. The reason for this is that the coupling term does not depend on $z_{1_e}(t)$ or $z_{2_e}(t)$, which is not the case of the coupling term in (15)-(16), which includes ξ_1 and ξ_2. Consequently, the stability of $\dot{\xi}_1(t) = \bar{A}_0(t)\xi_1(t) + A_1\xi_1(t-2\tau)$ can be established by proving the stability of (17) and (18) separately.

The time-delay in (18) is approximated by a Taylor series expansion and the stability of the subsystem is established using the resulting characteristic polynomials, which require that $K_x, K_y < 0$.

Knowing that the coupling term vanishes, the stability of the first term in (17) has already been studied in (van den Broek, 2008) and (van den Broek et al., 2009), and can be established by a well-known result in the adaptive control field for LTV systems. The requirement is that that $\omega_r(t)$ be PE and that the controller gains satisfy $c_2 > 0$ and $c_3 > -1$.

3.2.2 Assumption on g

The approach taken here is to express the indeterminate forms in g_{ij},

$$\int_0^1 \cos(sx)ds = \begin{cases} \frac{\sin(x)}{x} & \text{for} \quad x \neq 0 \\ 1 & \text{for} \quad x = 0 \end{cases},$$

$$\int_0^1 \sin(sx)ds = \begin{cases} \frac{1-\cos(x)}{x} & \text{for} \quad x \neq 0 \\ 0 & \text{for} \quad x = 0 \end{cases},$$

as,

$$\lim_{x\to 0} \tfrac{\sin(x)}{x} = 1, \quad \lim_{x\to 0} \tfrac{1-\cos(x)}{x} = 0. \tag{19}$$

by applying L'Hopital's rule. Using (19) and the properties of vector norms, and given that the Frobenius norm and the 2-norm are consistent, it is possible to obtain explicit expressions for the bounds on g_{ij}.

3.2.3 Local Exponential Stability of $\xi_2(t) = 0$

Consider subsystem $\dot{\xi}_2(t) = f_2(t, \xi_2(t), \xi_2(t-2\tau))$. Clearly, $\dot{p}_{3_e}(t) = K_\theta \sin p_{3_e}(t-2\tau)$ can be linearized and will be exponentially stable provided $K_\theta < 0$. On the other hand, $\dot{z}_{3_e}(t) = -c_1 \sin z_{3_e}(t) + K_\theta \sin p_{3_e}(t)$ can also be linearized. As with $\dot{\xi}_1(t)$, $\dot{z}_{3_e}(t)$ is perturbed through a coupling term that will vanish as $\dot{p}_{3_e}(t)$ converges to zero. Requiring that $c_1 > 0$ will ensure exponential stability of the remaining linearized dynamics. This completes the proof. □

4 EXPERIMENTAL RESULTS

Two equivalent multi-robot platforms exist at the Eindhoven University of Technology (TU/e) and at the Tokyo Metropolitan University (TMU). The bilateral time-delay compensation scheme is implemented in them, meaning that a mobile robot located at TU/e can be controlled from TMU and viceversa.

4.1 Experimental Platform Description

The experimental platforms' design objectives encompass cost, reliability and flexibility. The hardware and software choices together with the implementation of the setup at TU/e are discussed in greater detail in (van den Broek, 2008) (cf. Fig. 3). The setup has already been used to implement cooperation, coordination, collision avoidance and servo vision algorithms. The platform at TMU has similar characteristics, only differing from the one at TU/e in its size and vision calibration algorithm.

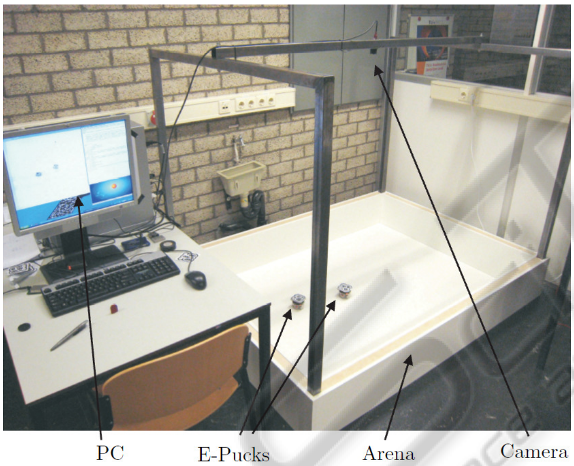

Figure 3: Experimental setup at TU/e.

Mobile Robot. The unicycle selected is the e-puck, (Mondada et al., 2009), whose wheels are driven by stepper motors that receive velocity control commands over a BlueTooth connection.

Vision. Each robot is fitted with a fiducial marker of 7 by 7 cm, collected by an industrial FireWire camera, interpreted in the program reacTIVision (reacTIVision, 2009), and calibrated by means of a global transformation (TU/e) or a grid (TMU).

Driving Area. The driving area is of 175×128 cm for TU/e and 100×50 cm for TMU, and is determined based on the required accuracy, the camera lens, and the height at which the camera is positioned.

Software. The e-puck robots and reacTIVision's data stream can be managed in C, Python, or Matlab script. In this work, the controller implementation and signal processing is carried out in Python, (Python, 2009).

Bandwidth and Sampling Rate. Using vision as the localization technique diminishes the system's bandwidth and results in a sampling rate of 25Hz.

4.2 Data Exchange over the Internet

Due to its widespread availability and low cost, the Internet is chosen as the communication channel to exchange data between TU/e and TMU.

Data Exchange. A Virtual Private Network (VPN) is established between TU/e and TMU in order to implement a reliable and secure data exchange.

Socket Configuration. Data is exchanged between TU/e and TMU as soon as it becomes available using non-blocking Transmission Control Protocol (TCP) sockets running the Internet Protocol (IP). The system's low bandwidth allows the use of the TCP, which guarantees reliable and orderly data delivery.

Data Payload. The variables exchanged amount to the current time instant and control signals from the control side to the system, and to the position and orientation values from the system side to the controller.

4.3 Implementation Issues

One of the main implementation issues of the proposed time-delay compensation strategy is the accurate modeling and characterization of the time-delay induced by the communication channel. The use of predictor-like schemes is often discouraged because of their sensitivity to delay model mismatches, (Hokayem and Spong, 2006), specially when considering nonlinear systems and a communication channel such as the Internet. To this end, three methods that ease the implementation of the proposed compensation strategy are suggested. Their objective it to bring $\tilde{\tau}$ as close as possible to τ in practice.

Delay Measurement. The round trip delay between TU/e and TMU (and viceversa) has been measured during different times of the day, for a variable amount of time, and for a total time of around 60min. The mean delay value is approximately 265ms for

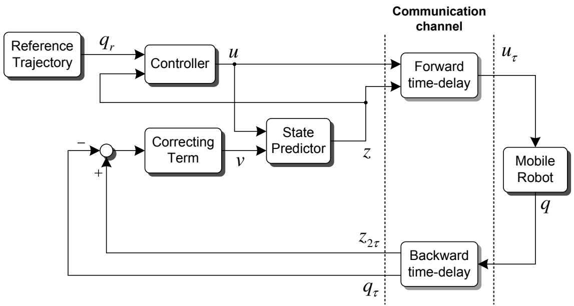

Figure 4: Time-delay compensation scheme block diagram representation without time-delay models.

both cases (267.4917ms TU/e-TMU, 269.5307ms TMU-TU/e). Occurrences of delays greater than 300ms where of 0.27% for TU/e-TMU and 0.34% for TMU-TU/e. Thus, the round trip delay can be modeled with enough accuracy even if the Internet is considered as the communication channel.

Time-stamping. Outgoing and incoming data on the controller side can be time-stamped in order to estimate the round trip delay for each pair of control signals and sensor data, setting the estimator's delay model accordingly.

Signal Bouncing. The estimator's output may be sent together with the control signals to the mobile robot, and then sent back to the controller without being modified. By using the communication channel itself to delay the estimator's output, modeling the time-delay is no longer necessary (cf. Figure 4).

4.4 Experiments

In the first experiment a mobile robot at TMU is controlled from TU/e. The reference trajectory is a lemniscate with center at $[0.5\text{m}, 0.25\text{m}]$, a length and width of 0.2m, and a velocity multiplier of 0.2m/s. The scenario repeats in the second experiment, where a sinusoid with origin at $[0.1\text{m}, 0.25\text{m}]$, an amplitude of 0.15m, an angular frequency of 0.3rad/s, and a velocity multiplier of 0.01m/s constitutes the reference.

The system's initial condition is $q(0) = [0.3235\text{m}\ 0.1882\text{m}\ 0.2851\text{rad}]^T$ for the first experiment and $q(0) = [0.0225\text{m}\ 0.1821\text{m}\ 0.3916\text{rad}]^T$ for the second one. In both cases the estimator's initial condition is set to $z(0) = [0\ 0\ 0]^T$, the controller gains to $c_1 = 1.0$, $c_2 = c_3 = 2.0$ and the correcting term gains to $K_x = K_y = K_\theta = -0.6$. The sampling rate is 25Hz and the experiments' duration is 60sec and 120sec respectively. The round trip delay is modeled as 265ms based on measurements, although the estimator's output is in fact delayed 280ms since

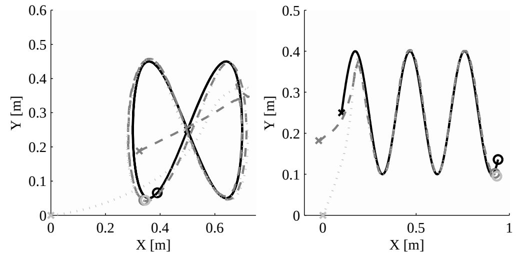

Figure 5: Reference, robot and predictor behavior in the X-Y plane for two different trajectories.

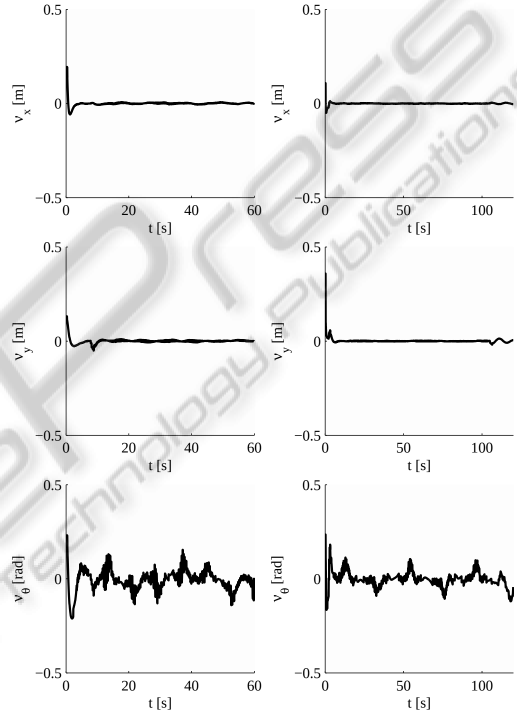

Figure 6: Practical convergence of the correcting terms.

only delay models which are multiples of 0.04 are allowed due to the setup's sampling time, meaning data is displaced 7 locations within the storage buffer.

The experimental results are shown in Figure 5 and 6 for both experiments. The first plots show the reference (black), robot (gray) and predictor (light gray) trajectories in the X-Y plane, with their initial and final position marked with a cross and a circle respectively. The plots in Figure 6 show the evolution of the correcting terms $\nu_x(t)$, $\nu_y(t)$, and $\nu_\theta(t)$ and how they practically converge to zero even in the presence of a delay model mismatch and considering a time-varying communication channel. The behavior of the proposed delay compensation strategy is consistent with the stability analysis presented and the tracking performance of the robot can be ensured even under a bilateral time-delay.

5 DISCUSSION

A compensation strategy to account for the negative effects of a bilateral time-delay affecting a unicycle-type mobile robot has been proposed. Several techniques to accurately reproduce the time-delay in the estimator have been presented, since this is required and often a cause of concern in predictor-like control strategies. Experiments show that the delay compensation strategy is robust to small delay model mismatches and delay variations using the Internet as a communication channel.

Future work includes a robustness analysis of the delay compensation scheme and a comparison with other predictor-like strategies. Additionally, techniques to synchronize as precisely as possible the controller and system sides are being studied in order to obtain a more accurate measurement of the system's performance. Integrating the concept of remote control of mobile robots with notions related to the long distance synchronization of robotic networks and extending the concepts of the estimator to a more general setting (other mechanical systems) also remain topics to be addressed in the future.

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