EFFICIENT IMPLEMENTATION OF CONSTRAINED ROBUST MODEL PREDICTIVE CONTROL USING A STATE SPACE MODEL

Amira Kheriji, Faouzi Bouani and Mekki Ksouri
Laboratory of Analysis and Control of Systems, National Engineering School of Tunis, B.P 37, 1002 Tunis, Tunisia

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Abstract: The goal of this paper is to evaluate the closed loop performances of a new approach in constrained state space Robust Model Predictive Control (RMPC) in the presence of parametric uncertainties. The control law is obtained by the resolution of a min-max optimization problem, initially non convex, under input and input deviation constraints, using worst case strategy. The technique used is the Generalized Geometric Programming (GGP) which is a global optimization method for non convex functions constrained in a specific domain. The key idea of the proposed approach is the convexification of the optimization problem allowing to compute the optimal control law using standard optimization technique. The proposed method is efficient since it guarantees set-point tracking different from the origin and non zero disturbances rejection. The efficiency of this approach is illustrated with two examples and compared with a recent state space RMPC algorithm.

1 INTRODUCTION

The MPC algorithms present a series of selling points over other methods amongst which stand out: its ability to handle non linear systems, multi input mutlti output systems as well as systems having input and/or state constraints. The model quality plays a vital role in MPC, but in reality there always exist model uncertainties, which may significantly degrade the system performances (Fukushima et al., 2007). Uncertainties can be represented in different forms reflecting in certain ways the knowledge of the physical mechanisms which cause the discrepancy between the model and the process (Camacho and Bordons, 2004). To describe the dynamic of the system, structured uncertainty was used by several Robust MPC (RMPC) works. A number of RMPC methods have been developed to cope with the presence of the uncertainties in the system model. A representative list of RMPC methods includes: (Campo and Morari, 1987), (Cordon and Boucher, 1994), (Kothare et al., 1996), (Rossiter and Kouvaritakis, 1998), (Huaizhong et al., 1998), (Lee and Kouvaritakis, 2000), (Ramirez et al., 2002), (Pannochia, 2004), (Fukushima et al., 2007), (Alamo et al., 2004), (Bouzouita et al., 2007), (Mayne et al., 2009), (Qian et al., 2010).

Most existing state-space RMPC algorithms are unable to control uncertain systems when the set-point is different from the origin or when it is changed such as LMI method introduced by (Kothare et al., 1996). Another limitation of this method consists on returning local optimum in some cases.

In the present work, we evaluate the closed loop performances of the proposed state space RMPC approach. This approach uses the state space output deviation method presented by (Watanabet et al., 1991) to compute the j step ahead output predictor with a finite prediction horizon since this method gives robust adaptive controlled results against the unknown plant parameters. Thus, the optimal control actions are determined by a min-max optimization problem. However, the criterion to be optimized is initially non convex relatively to the uncertain parameters and the control action. Hence, it can’t be solved by a standard optimization technique. To overcome this difficulty, the GGP method, which is a global optimization technique, is adopted to convexify the criterion by means of variable transformations.

The main features of the proposed algorithm are:

- guarantee non zero set-point tracking,
- move the system with time-varying model uncertainty form set-point to another without offset,
- satisfy process constraints,
- reject non zeros disturbances,

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- the on-line optimization algorithm is computed with a reasonable amount of time.

The efficiency of this algorithm is illustrated through two examples and compared with the method proposed by (Pannochia, 2004).

2 GENERALIZED PREDICTIVE CONTROL ALGORITHM

In this section, we will be based on the output deviation method introduced by (Watanabet et al., 1991) to compute the j step ahead output predictor value as well as the cost function. It is already proved that this method gives robust adaptive controlled result against the unknown plant parameters compared with the direct output method. The model considered at first for uncertain system is a linear discrete time single-input/single-output described by the following CARIMA model of the plant results performing an effective integral action:

$$A(q^{-1})\Delta y(k) = B(q^{-1})\Delta u(k) \tag{1}$$

where: - $\Delta y(k)$ and $\Delta u(k)$ are respectively the output and the input deviation system.

- Δ is the integral action which ensures offset-free steady-state response in the presence of variable set point.

- $A(q^{-1})$, $B(q^{-1})$ and $\Delta(q^{-1})$ are polynomials on q^{-1} with bounded coefficients:

$$A(q^{-1}) = 1 + a_1 q^{-1} + \ldots + a_{n_a} q^{-n_a} \tag{2}$$

$$a_i \in [\underline{a_i}, \overline{a_i}], 1 \le i \le n_a$$

$$B(q^{-1}) = b_0 q^{-1} + b_1 q^{-2} + \ldots + b_{n_b} q^{-(n_b+1)} \tag{3}$$

$$b_j \in [\underline{b_j}, \overline{b_j}], 0 \le j \le n_b$$

$$\Delta(q^{-1}) = 1 - q^{-1} \tag{4}$$

Then, equation 1 can be transformed using the observer canonical form into a state space model as follows:

$$\Delta x(k+1) = F\Delta x(k) + G\Delta u(k) \tag{5a}$$
$$\Delta y(k) = H\Delta x(k) \tag{5b}$$

where $\Delta x(k)$ is an n_a dimensional vector and F, G and H are represented by the following matrices:

$$F = \begin{bmatrix} -a_1 & 1 & 0 & \cdots & 0 \\ -a_2 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & & \vdots \\ -a_{n_a-1} & 0 & 0 & \cdots & 1 \\ -a_{n_a} & 0 & 0 & \cdots & 0 \end{bmatrix}, \quad G = \begin{bmatrix} b_0 \\ b_1 \\ \vdots \\ b_{n_a-2} \\ b_{n_a-1} \end{bmatrix} \tag{6a}$$

$$H = \overbrace{\begin{bmatrix} 1 & 0 & \cdots & 0 \end{bmatrix}}^{n_a} \tag{6b}$$

where $b_i = 0$ for $i > n_b$. Consequently, we can obtain using equation 5 the following state deviation at $k+j$ time:

$$\Delta x(k+j|k) = F^j \Delta x(k) + \sum_{i=1}^{j} F^{j-i} G \Delta u(k+i-1) \tag{7}$$

Then, it follows from equations 5 and 7, that the j-step ahead output predicted value is given by:

$$\begin{aligned} y(k+j|k) &= y(k) + \sum_{i=1}^{j} HF^i \Delta x(k) \\ &+ \sum_{i=1}^{j} \sum_{l=0}^{j-i} HF^l G \Delta u(k+i-1) \end{aligned} \tag{8}$$

Moreover, the cost function is defined by the following equation:

$$J = \sum_{i=1}^{H_p} (y(k+i|k) - w(k+i))^2 + \lambda \sum_{i=1}^{H_c} \Delta u(k+i-1)^2 \tag{9}$$

The output sequence on H_p prediction horizon can be written as follows:

$$Y = L_u \Delta U + f \tag{10}$$

where:

$$Y = [y(k+1|k), y(k+2|k), \ldots, y(k+H_p|k)]^T$$

$$\Delta U = [\Delta u(k), \Delta u(k+1), \ldots, \Delta u(k+H_c-1)]^T$$

The L_u with the (H_p, H_c) dimension and f which is an (H_p) dimensional vector are given by:

$$L_u = \begin{pmatrix} HG & 0 & \ldots & & 0 \\ HG+HFG & HG & 0 & \ldots & 0 \\ HG+HFG+HF^2G & HG+HFG & \ddots & & 0 \\ & & & & HG \\ \vdots & & \ddots & & \\ \sum_{j=1}^{H_c+1} HF^{j-1}G & & \ldots & & HG+HFG \end{pmatrix}$$

$$f = \begin{pmatrix} y(k) \\ y(k) \\ y(k) \\ \vdots \\ y(k) \end{pmatrix} + \begin{pmatrix} HF \\ HF + HF^2 \\ HF + HF^2 + HF^3 \\ \vdots \\ \sum_{i=1}^{H_p} HF^i \end{pmatrix} \Delta x(k)$$

Hence, the cost function of equation 9 is equivalent to:

$$J = (Y - W)^T (Y - W) + \lambda \Delta U^T \Delta U \qquad (11)$$

where Y is given by equation 10, λ is the weighting factor and W is the sequence of set-points on H_p prediction horizon:

$$W = [w(k+1), \ldots, w(k+H_p)]^T$$

3 PROBLEM STATEMENT

The strategy used to find the optimal control law is the minimization of the worst case objective function. The min-max problem is the following:

$$\min_{\Delta U(k) \in M} \max_{\substack{a_i \in [\underline{a_i}, \overline{a_i}] \\ b_j \in [\underline{b_j}, \overline{b_j}]}} J(\Delta U, a_i, b_j) \qquad (12)$$

where J is given by equation 11 and the set M represents the set of constraints on input and input deviation signals which can be described by: $M = \{\forall \Delta U : C\Delta U \leq D\}$ (Ramirez et al., 2002).

The maximization is over the bounds of A and B polynomial coefficients. This maximization would lead to a worst case value of J over all the values of a_i and b_j belonging respectively to $[\underline{a_i}, \overline{a_i}]$ and $[\underline{b_j}, \overline{b_j}]$ (Bouzouita et al., 2007). Therefore, it is deduced from equations 10 and 11, that the objective function J is non convex relatively to F, G and ΔU (see section 5 for more details). Hence, it is non convex relatively to the uncertain parameters a_i and b_j. Effective algorithm is proposed in the present paper to solve this maximization problem and obtain the global optimality within a good precision. The main idea of the GGP is to convexify the objective function and the constraints by applying different variable transformation techniques. Furthermore, this worst case value is minimized over present and future control moves $\triangle U = [\triangle u(k), ..., \triangle u(k + H_c - 1)]$. We present now the global optimization method (GGP) which allows us to solve the maximization problem of equation 12. This optimization problem can be converted to the given one:

$$\min_{\substack{a_i \in [\underline{a_i}, \overline{a_i}] \\ b_j \in [\underline{b_j}, \overline{b_j}]}} -J(\Delta U, a_i, b_j) \qquad (13)$$

Generalized geometric programming is an optimization technique for solving a class of non convex non linear programming problems (Tsai et al., 2007). The GGP problems occur frequently in engineering design, chemical process industry and management (Tsai, 2009), (Nand, 1995), (Chul and Dennis, 1996), (Maranas and Floudas, 1997) and (Porn et al., 2007). This class concerns the optimization problems with the objective function and constraints are in polynomial forms. Several specialized approaches have been proposed to locate the global optimum based mainly on variable transformations. Hence, the strategy of this technique is to replace all non convex signomials of the objective function with specific features into convex terms according to some specific transformation rules which will be formulated in next section.

4 CONVEXIFICATION STRATEGY OF THE GGP APPROACH

The mathematical formulation of a GGP problem is expressed as follows (Tsai, 2009):

$$\min_X Z(X) = \sum_{j=1}^{T_0} c_j z_j \qquad (14)$$

subject to:

$$\sum_{q=1}^{T_k} h_{k_q} z_{k_q} \leq l_k, k = 1, \ldots, K \qquad (15a)$$

$$z_p = x_1^{\alpha_{p_1}} x_2^{\alpha_{p_2}} \ldots x_n^{\alpha_{pn}}, p = 1, \ldots, T_0, \qquad (15b)$$

$$z_{k_q} = x_1^{\beta_{k_{q1}}} x_2^{\beta_{k_{q2}}} \ldots x_n^{\beta_{k_{qn}}}, k = 1, \ldots, K, q = 1, \ldots, T_k \qquad (15c)$$

$$X = (x_1, \ldots, x_n) \qquad (15d)$$

$$x_i > 0 \text{ for } 1 \leq i \leq n \qquad (15e)$$

$$\underline{x_i} \leq x_i \leq \overline{x_i} \qquad (15f)$$

Following the GGP formulation, the proposed method can be solved with only positive variables due to the logarithmic/exponential transformation used in the convexification strategy. Therefore, this transformation requires to replace x_i by e^{y_i}. Hence, x_i must be strictly positive. However, in several problems the polynomial variables can be negative. To overcame this limitation, a simple variable translation allows taking into account negative variables. Consequently, following the negative translation variable technique, the definition set of the polynomial variable of the objective function of equation 14 is $\Re_+^n$. Using equations 14 and 15 , the polynomial can be written as follows:

$$\min \sum_{j=1}^{T_0} c_j x_1^{\alpha_{p1}} x_2^{\alpha_{p2}} \ldots x_n^{\alpha_{pn}}, p = 1, \ldots, T_0 \quad (16)$$

In fact, the signomial function Z(X) is a sum of monomial terms $f_j(X)$ given by the following equation:

$$f_j(X) = c_j x_1^{\alpha_{p1}} x_2^{\alpha_{p2}} \ldots x_n^{\alpha_{pn}}, j = 1, \ldots, T_0 \quad (17)$$

Based on the given three propositions (Tsai et al., 2007), we can judge either each monomial term of the polynomial is convex or not.

Proposition 1. The function $f(X) = c\Pi_{i=1}^{n} x_i^{\alpha_i}$ is convex in $\Re_+^n$ if $c \geq 0$, $x_i \geq 0$ and $\alpha_{p_i} \leq 0$ (for all $i = 1, \ldots, n$).

Proposition 2. The function $f(X) = c\Pi_{i=1}^{n} x_i^{\alpha_i}$ is convex in $\Re_+^n$ if $c \leq 0$, $x_i \geq 0$, $\alpha_{p_i} \geq 0$ (for all $i = 1, \ldots, n$) and $(1 - \sum_{i=1}^{n} \alpha_i) \geq 0$.

Proposition 3. The function $f(X) = cexp(r_1 x_1 + r_2 x_2 + \ldots + r_n x_n)$ is convex in $\Re_+^n$ if $c \geq 0$ and $r_i \in \Re$. Hence, if one of the three above propositions is not satisfied for a signomial, by applying the following transformation rules we can convexify it:

Rule 1. If $c > 0$ and $\alpha_i > 0$, then $cx_1^{\alpha_{p1}} x_2^{\alpha_{p2}} \ldots x_n^{\alpha_{pn}} = cexp(r_1 y_1 + r_2 y_2 + \ldots + r_n y_n)$ where $y_i = log(x_i)$, $i = 1, \ldots, n$.

Rule 2. If $c < 0$, $\alpha_i > 0$ and $\sum_{i=1}^{n} \alpha_i > 1$, then $cx_1^{\alpha_{p1}} x_2^{\alpha_{p2}} \ldots x_n^{\alpha_{pn}} = cX_1^{\alpha_1/R} \ldots X_m^{\alpha_m/R}$ where $x_i = X_i^{1/R}$, $i = 1, \ldots, n$ and $R = \sum_{i=1}^{n} \alpha_i$.

5 SUMMARY OF THE STATE SPACE RMPC ALGORITHM

In this section, we provide a summary of the needed steps to find the optimal control law using the new proposed RMPC method in the state space model:

1. Fix the upper and lower bounds of a_i and b_j which are $\underline{a_i}$, $\overline{a_i}$ $(i = 1, \ldots, n_a)$, $\underline{b_j}$ and $\overline{b_j}$ $(j = 0, \ldots, n_b)$. Several works have been published addressing facets of finding model uncertainty bounds (Messaoud and Akoum, 2000), (Messaoud and Favier, 1994).
2. Find the optimum values of a_i and b_j by solving the minimization optimization problem of equation 13. This problem is initially non convex. By applying the transformation techniques (exponential and power transformations) of the GGP method, the transformed problem (objective function and constraints) becomes convex. The GGP technique is applied with a polynomial form.
3. Find ΔU, the solution of the minimization problem of equation 12 with the optimal values of a_i and b_j found in step 2.
4. Inject the control action in the plant to find the state and the output actions of the future sequences.
5. Go to step 2 and repeat with the optimal value of the control signal found in step 3.

To explain more step 2, we consider a simple example where the state matrix is $F = -a_1$, the input matrix is $G = b_0$ and the output matrix is $H = 1$. The controller parameters are: $H_p = 1$, $H_c = 1$ and $\lambda = 1$. Then using equations 8 and 9, the criterion J is written as following:

$$J = (y(k) - a_1 \Delta x(k) + b_0 \Delta u(k) - w(k+1))^2 + \Delta u(k)^2 \quad (18)$$

Consequently, after expanding equation 18, we observe that the J criterion is non convex relatively to x_1, x_2 and x_3 (according to proposition 1 and proposition 2).

6 SIMULATION EXAMPLES

In this section, the new RMPC method using state space description and based on GGP will be illustrated through two examples.

6.1 Example 1

The first example is a simple system described by the discrete state model given by equation 5, where the state matrices are:

$$F = -a_1 \quad , G = 0.11 \text{ and } H = 1$$

The uncertain variable bounds are:

$$-1.6 \leq a_1 \leq -1.2$$

This system is unstable for all values of a_1. The initial state points is fixed at $x(0) = x(1) = 0$. We consider the following control parameters: $H_p = 3$, $H_c = 1$ and $\lambda = 0.02$. The set-point is changed between 1 and -1. Moreover, constraints on control and control moves signals process have been taken into account. Their values are: $-3.5 \leq u(k) \leq 2.3$ and $-1.5 \leq \Delta u(k) \leq 1.5$. For model 1, $a_1 = -1$ and for model 2, $a_1 = -1.2$.

Fig. 1 shows the closed loop response of the system for the two models using the proposed state space RMPC approach based on the GGP technique. A load disturbance is added to the model output. This disturbance takes 0.2 for $15 \leq k \leq 25$ and $45 \leq k \leq 55$

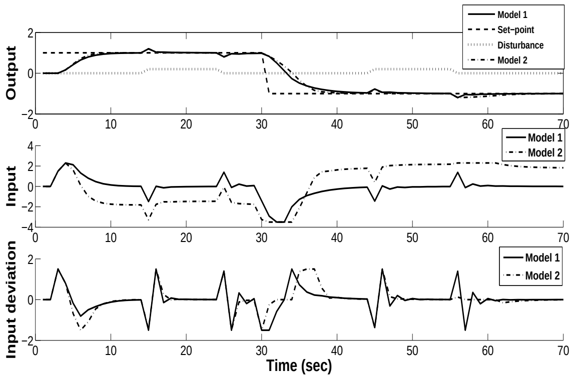

Figure 1: Closed-loop simulation results for model 1 and model 2.

and 0 else. The simulation results show good performances of the proposed approach. This approach successfully controls the above system. It achieves variable set point tracking and non zero disturbance rejection with respect to input and input deviation constraints. Moreover, the on-line optimization algorithm takes about $0.17s$ per sample time. Consequently, the proposed technique is accomplished in a reasonable amount of time.

6.2 Example 2: Comparison with Pannochia Method

In this example, we consider a jacketed continuous stirred tank reactor (CSTR) presented by Henson and Seborg (Henson and Seborg, 1997). After linearization around the middle-conversion open-loop unstable steady-state and discretization with a sampling time of $5s$ (Pannochia, 2004), we obtain the following state space model matrices:

$$F = \begin{bmatrix} -a_1 & 1 \\ -a_2 & 0 \end{bmatrix}, \; G = \begin{bmatrix} b_0 \\ b_1 \end{bmatrix} \; H = \begin{bmatrix} 1 & 0 \end{bmatrix}$$

where the uncertainty variables are bounded as follows: $-2.3006 \leq a_1 \leq -2.1617$, $1.1555 \leq a_2 \leq 1.2863$, $0.2022 \leq b_0 \leq 0.2153$ $-0.1804 \leq b_1 \leq -0.1718$. Model 1 is described by the following state matrices:

$$F_1 = \begin{bmatrix} 2.1617 & 1 \\ -1.1555 & 0 \end{bmatrix}, \; G_1 = \begin{bmatrix} 0.2022 \\ -0.1718 \end{bmatrix} \; H_1 = \begin{bmatrix} 1 & 0 \end{bmatrix}$$

However, for model 2 we consider the following state matrices:

$$F_2 = \begin{bmatrix} 2.3006 & 1 \\ -1.2863 & 0 \end{bmatrix}, \; G_2 = \begin{bmatrix} 0.2153 \\ -0.1804 \end{bmatrix} \; H_2 = \begin{bmatrix} 1 & 0 \end{bmatrix}$$

Fig.2 compares the closed loop performances of the proposed optimization algorithm using GGP technique and the RMPC method presented by (Pannochia, 2004) using the above system for the two models. The control parameters are the following: $H_c = 2$, $\lambda = 0.002$. In the proposed approach $H_p = 3$. The initial state is x=$\begin{bmatrix} 0 \\ 0 \end{bmatrix}$. Inputs constraints are fixed as follows: $-10 \leq u(k) \leq 10$. In the proposed approach, we suppose that the future set-points are unknown. Moreover, the two models are considered:

for $1 \leq i < 40$ the true system is model 1

for $i \geq 40$ the true system is model 2

Fig.2 shows a slightly difference between the two outputs. The response time of the Pannochia RMPC method is about $t_r = 4.87s$, however the one of the proposed RMPC method is $t_r = 6.1s$. Concerning the control signal, the proposed RMPC method shows less oscillations at the set point variations than the Pannochia RMPC method since it presents less of peaks.

(Pannochia, 2004) uses two algorithms: an off-line algorithm and an on-line one. The off-line algorithm computes a nominal system and a feedback gain design which guarantees the closed loop system stability. This algorithm solves a non convex min-max optimization problem. Hence, both the minimization and the maximization problem give local solutions. In fact, this limitation can affect the closed loop performance responses. However, The proposed approach uses only one on-line algorithm based on the GGP method which is a global optimization technique for non convex polynomial functions.

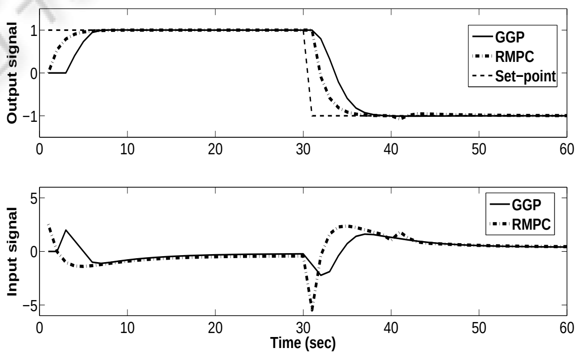

Figure 2: Closed-loop simulation results for models 1 and 2.

7 CONCLUSIONS

An examination of the closed loop performances of a new approach in constrained state space Robust Model Predictive Control (RMPC) in the presence of parametric uncertainties is presented. Based on simulation example results, we have shown that the proposed method is able to guarantee variable set-point tracking respecting to the input and input deviation constraints and to reject non zero disturbance. Moreover, our method features good performances in the on-line algorithm time computation and a simplicity of implementation. These features make this method particulary attractive for industrial applications. A comparison with a recent state space RMPC method is also given.

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