NORMAL FLAT FORMS FOR A CLASS OF 0-FLAT AFFINE DYNAMICAL SYSTEMS AND ITS APPLICATION TO NONHOLONOMIC SYSTEMS

S. Bououden, D. Boutat
PRISME - ENSI de Bourges, 10 boulevard Lahitolle, 18020, Bourges cedex, France

F. Abdessemed
Faculty of Engineering, Electronic Departement, Batna University, 05000, Algeria

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Abstract: In this contribution normal flat forms are used to achieve stable tracking control for nonlinear flat systems. Our approach is based on a nonlinear transformations in order to derive two 0-flat normal forms for a class of non-linear systems, a dynamical control law is then proposed to achieve stable trajectory tracking. Finally, This method is generalized to analysis and control a class of a 0-flat affine nonlinear multi-input dynamical systems for which we can build flat outputs to give structural normal flat forms. The computer simulations are given in the paper to demonstrate the advantages of the method.

1 INTRODUCTION

The control of nonholonomic dynamic systems has received considerable attention during the last years and become a popular subject in the nonlinear control. One a reasons for this, in real world, the nonholonomic systems are frequently used to describe some pratical control systems such as mobile robot, car-like vehicule, and under-actuated satellites, can all be modeled as nonholonomic control systems or nonholonomic maneuvers. Hence, control problems involve them have attracted attention in the control community.

Different methods have been applied to solve motion control problems. (Kanayama et al., 1991) propose a stable tracking control method for nonholonomic vehicule using a Lyapunov function. (Lee et al., 1998) solved tracking control using backstepping and in (Lee and Tai, 2001) with saturation constraints. Furthermore, most reported designs rely on intelligent control approaches such as Fuzzy Logic Control (Pawlowski et al., 2001), (Tsai et al., 2004), Neural Networks (Song and Sheen, 2000), and (Chwa, 2004) used a sliding mode control to the tracking control problem. (Fierro and Lewis, 1995) propose a dynamical extention that makes possible the integration of kinematic and torque controller for a nonholonomic mobile robot. (Fukao et al., 2000), introduces an adaptive tracking controller for the dynamic model of mobile robot with unknown parameters using backstepping. However the field of control of such systems is still open to develop other control strategie.

Application of flatness to problems of engineering interest have grown steadily in recent years. Michel Fliess et al. (Fliess et al., 1992), (Fliess et al., 1995) introduced the concept of flat outputs, these outputs guarantee that the problem will be put in term of control algorithm for motion planning, trajectory generation and stabilization. A limitation of flatness is that there does not exist necessary and sufficient conditions to determine if a general system is differentially flat and there no algorithm to compute the flat outputs. Nevertheless, it is well-known that all controllable linear systems can be shown to be flat. Indeed, any system that can be transformed into a linear system by changes of coordinates, static feedback transformations, or dynamic feedback transformations is also flat (Jakubczyk and Respondek, 1980), (Hunt et al., 1983).

We present in this paper two normal flat forms, It deals with sufficient geometrical conditions which enable us to conclude if a given nonlinear controllable dynamical system can be transformed, by means of change of coordinates, to one of these normal forms.

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In the same way it gives an algorithm to compute the flat outputs. As an illustration to the proposed approach, a trajectory tracking of a nonholonomic uniaxial vehicule is simulated. As we will show with this example, for this particular class, our method presents a new direction to solve the flatness problem.

The paper is organized as follows. In section 2 we address notations, definitions and our problem statement, we describes the classes of 0-flat systems study. The necessary and sufficient geometrical conditions for affine dynamical systems are presented in Section 3. In sections 4 provides illustrative examples and simulations. Some conclusions are presented in section 5.

2 DEFINITION AND PROBLEM STATEMENT

Let us consider the following class of multivariable nonlinear systems described in state space form by equations of the following kind In the next section, we will recall the concept of 0-flat systems

$$\dot{x} = f(x) + \sum_{i=1}^{m} g_i(x)u_i \tag{1}$$

where $x \in \chi \subseteq \Re^n, u \in U \subseteq \Re^m$ and f is a smooth function on $\chi \times U$.

Definition 1. The dynamical system (1) is flat if there exist m functions $y = (y_1, \dots, y_m)$ called the flat outputs such that:

1- $y = F(x, u, \dot{u}, \dots, u^{(r_1)})$ is a function of the state x, the input u, and its derivatives $u^{(i)}$

2- $x = \varphi(z, \dot{z}, \dots, z^{(r_2)})$ is a function of the flat outputs and their derivatives.

3- $u = \gamma(z, \dot{z}, \dots, z^{(r_2+1)})$ is a function of the flat outputs and their derivatives.

In this paper, we will deal with multi-input affine dynamical systems, without loss of generality, we will assume within this work that:

Assumption 1. The vector field $G = [g_1, \dots, g_m]$ is of rank m.
However, we don't assume that $\Delta = span(G)$ is involutive as it is the case in many researches which try to compute the inverse dynamics. We will characterize a class of dynamical systems for which the flat outputs are only functions of states x. This dynamical systems is called 0-flat: (Pereira, 2000)

$$y_i = F_i(x);\ where\ 1 \leq i \leq m \tag{2}$$

2.1 A Class of Structurally 0-Flat Dynamical System

It is well-known that the codimension one $(m = n - 1)$ controllable dynamical systems are 0-flat (Charlet and Lévine, 1989). Our result concerns a normal flat forms of affine dynamical systems with n states and $m = n - 2$ inputs (codimension 2). Our objective is to introduce our formulation.

2.2 Main Result

In the sequel we introduce the following notation $z_{ji} = z_{j,i} = y_j$ with $j = 1 : m$ relative to the flat output y_j, which means $i - 1$ derivation of the output y_j.

2.2.1 First 0-Flat Normal Form

Let us consider the following proposition written in terms of the z_{j1}, where $j = 1 : m$ state variables. Let us consider the following proposition

Proposition 1. The following dynamical system is 0-flat

$$\begin{aligned}
\dot{z}_{11} &= z_{12} + \sum_{i=3}^{n-2} \beta_{11}^i(z)u_i \\
\dot{z}_{12} &= \alpha_{12}(z) + u_1 + \beta_{12}^2(z)u_2 + \dots + \beta_{12}^m(z)u_m \\
\dot{z}_{21} &= z_{22} + \sum_{i=3}^{n-2} \beta_{21}^i(z)u_i \\
\dot{z}_{22} &= \alpha_{22}(z) + \beta_{22}^1(z)u_1 + u_2 + \dots + \beta_{22}^m(z)u_m \\
\dot{z}_{j1} &= \alpha_{j1} + u_j + \sum_{i=3, i\neq j}^{n-2} \beta_{j1}^i(z)u_i
\end{aligned} \tag{3}$$

with z_{11}, z_{21} as flat outputs, and $m = n - 2$.

Proof 1. Now we will show that the equations (3) represent a locally dynamical 0-flat system. Consider the following m equations :

$$E_1 = \dot{z}_{1,1} - z_{1,2} - \sum_{i=3}^{m} \beta_{11}^i(z)u_i = 0 \tag{4}$$

$$E_2 = \dot{z}_{2,1} - z_{2,2} - \sum_{i=3}^{m} \beta_{21}^i(z)u_i = 0 \tag{5}$$

$$E_j = \dot{z}_{j,1} - \alpha_{j,1} - u_j - \sum_{i=3, i\neq j}^{m} \beta_{j1}^i(z)u_i = 0 \tag{6}$$

where $3 \leq j \leq m$

Let $v = (z_{1,2}, z_{2,2}, u_3, \dots, u_m)$ be the vector of unknown system variables and let us compute the following partial derivative

$$\frac{\partial(E_1, \dots, E_m)}{\partial v} = -I_m + o(v) \tag{7}$$

Where I_m is the identity matrix and $O(v)$ represents the order one of the v variables. From the equations (4, 5, 6) and the fact that $\frac{\partial(E_1,\ldots,E_m)}{\partial v}$ is locally invertible then the implicite function theorem, allows us to conclude that there exists $\varphi_k()$ and $\gamma_k()$ functions such that

$$z_{k,2} = \varphi_k(y_j, \dot{y}_j, j = 1:m, k = 1:2) \quad (8)$$

$$u_k = \gamma_k(y_j, \dot{y}_j, j = 1:m, k = 3:m) \quad (9)$$

By replacing (8) and (9) in the second and the fourth dynamic equation of (3) we can get the inputs u_1 and u_2 as functions of $(y_j, \dot{y}_j)$ for $j = 1:m$ and their second derivatives $y_1^{(2)}$, $y_2^{(2)}$. In the next subsection we will give a slightly different 0-flat normal form which is related to more drastic conditions.

2.2.2 Second 0-Flat Normal Form

The second canonical system gives the missing variables from the successive derivation of the same flat output written in terms of the variables $z_{j1} = y_j, 1 \leq j \leq (n-2)$:

Proposition 2. The following dynamical system is 0-flat

$$\begin{aligned}
\dot{z}_{11} &= z_{12} + \sum_{i=2}^{n-2} \beta_{11}^i(z) u_i \\
\dot{z}_{12} &= z_{13} + \sum_{i=2}^{n-2} \beta_{12}^i(z) u_i \\
\dot{z}_{13} &= \alpha_{13}(z) + u_1 + \sum_{i=2}^{n-2} \beta_{13}^i(z) u_i \\
\dot{z}_{j1} &= \alpha_{j1(z)} + u_j + \sum_{i=2}^{n-2} \beta_{j1}^i(z) u_i
\end{aligned} \quad (10)$$

Where $2 \leq j \leq m$, and $m = n-2$.

Proof 2. The main difference from (3) concerns the fact that we assume the variable z_{13} not present in the dynamics $\dot{z}_{j1}$, for $1 \leq j \leq (n-2)$. Then we can conclude that

Condition 1. z_{13} must not be present in $\beta_{j1}^i(z)$ for $j = 1:m$, and $i = 2:m$

Condition 2. z_{13} must not be present in α_{j1} for $j = 2:m$.
Under these conditions we can use the same procedure as the canonical form (3) to solve the dynamical system (10). So let us consider the m equations:

$$E_1 = \dot{z}_{11} - z_{12} - \sum_{i=2}^{n-2} \beta_{11}^i(z) u_i = 0 \quad (11)$$

$$E_j = \dot{z}_{j1} - \alpha_{j1(z)} - u_j - \sum_{i=2}^{n-2} \beta_{j1}^i(z) u_i = 0 \quad (12)$$

We can put $v = (z_{12}, u_2, \ldots, u_m)$ the vector of unknown system variables. Let us compute the following partial derivative

$$\frac{\partial(E_1, E_2 \ldots, E_m)}{\partial v} = -I_m + O^1(v) \quad (13)$$

Where I_m is the identity matrix and $O^1(v)$ represents the order one of the v variables. From the equations (11, 12) and the fact that $\frac{\partial(E_1,E_2,\ldots,E_m)}{\partial v}$ is locally inversible then the implicite function theorem, allows us to conclude that there exists $\varphi_1()$ and $\gamma_k()$ functions such that

$$z_{1,2} = \varphi_1(y_j, \dot{y}_j, j = 1:m) \quad (14)$$

$$u_k = \gamma_k(y_j, \dot{y}_j, j = 1:m, k = 2:m) \quad (15)$$

By replacing (14) and (15) in the second and the third dynamic equation of (10) we can get the variable z_{13} as a function of $\dot{y}_1$, $y_1^{(2)}$ and y_i for $i = 1:m$. Also we get the input variable u_1 as a function of $\dot{y}_1$, $y_1^{(2)}$, $y_1^{(3)}$ and y_i for $i = 1:m$.

3 TRANSFORMATIONS

Now, we give some conditions for a class of nonlinear systems, for which we can transform a nonlinear dynamical system (1) in a new 0-flat normal forms. So we distinguished two cases:

Case 1. The controllability distribution has the following vector field:

$\Delta_1 = span\{g_1, g_2, \ldots, g_{n-2}, ad_f g_1, ad_f g_2\}$, with $dim(\Delta_2) = n$.

Case 2. The controllability distribution has the following:

$\Delta_2 = span\{g_1, g_2, \ldots, g_{n-2}, ad_f g_1, ad_f^2 g_1\}$, with $dim(\Delta_2) = n$.

3.1 Case 1

Proposition 1. If the distribution $\overline{\Delta_1} = span\{g_1, g_2\} \subseteq \Delta_1$ is involutive, then the dynamical system (1) is 0-flat.

Proof 1. As $dim(\Delta_1) = n$ and $\overline{\Delta_1}$ is a 2-dimensional involutive distribution, there exist $n-2 = m$ independent functions of states x, $(y_i = z_{i,1}), 1 \leq i \leq m$ such that:

- $\overline{\Delta_2} = \bigcap_{i=1}^m \ker dy_i$ where $\ker dy_i$ means the kernel of the differential of the function y_i.

- $L_{[g_k,f]}z_{k,1} = 1$ for $k = 1:2$
- $L_{g_k}z_{k,1} = 1$ for $3 \leq k \leq m$

Now let us consider for $k = 1:2$ the following new variables: $z_{k,2} = L_f z_{k,1}$ where L_f is the Lie derivative in the direction of f. Therefore the set of the n variables: $(z_{i,1}), 1 \leq i \leq m$, $z_{1,2}$ and $z_{2,2}$ form a new coordinate system. For this the derivative of these variables give structural 0-flat normal form (3).

3.2 Case 2

Now let us describe the above conditions **(1), (2)** in (Proof 2) geometrically. For this one remarks that in (10) we have $g_1 = \frac{\partial}{\partial z_{1,3}}$. Therefore the independence of the others input directions $(g_i)_{2\leq i\leq m}$ from the variables $z_{1,3}$ can be described by the fact that:

$$[g_1, g_k,] \in span\{g_1, ad_f g_1\}.$$

Indeed $ad_f g_1 = \frac{\partial}{\partial z_{1,3}} + h(z)\frac{\partial}{\partial z_{1,2}}$. For second condition, $\alpha_{j,1} = L_f z_{j,1}$ for all $j \geq 2$. Then, if we want this function independent of the variable $z_{1,3}$, then we must have $L_{g_1}L_f z_{j,1} = 0$. As u_1 is not present in the last equations of (10) then $L_{g_1}z_{j,1} = 0$. Therefore, $L_{g_1}L_f z_{j,1} = 0$ is equivalent to $L_{ad_f g_1}z_{j,1} = 0$. Thus we can conclude that the distribution $\overline{\Delta_2} = \{g_1, ad_{fg_1}\}$ is involutive.

Proposition 3. If the distribution $\overline{\Delta_2} = \{g_1, ad_{fg_1}\} \subseteq \Delta_2$ is involutive and $[g_1, g_2] \in span\{g_1, ad_{fg_1}\}$ then the dynamical system (1) is 0-flat.

4 ILLUSTRATIVE EXAMPLE

In order to verify the performance of proposed methodology, as an illustration, we used a nonholonomic system. A nonlinear transformation is made in order to derive a 0-flat normal forms, the results obtained with our proposed control based on 0-flat normal forms of codimension 2 are used to control the nonlinear system in the aim to show its usefulness.

4.1 Application to a Nonholonomic Uniaxal Vehicule

The example we study is the kinematic model of a mobile car (see Figure 1), this system can be represented by the following set of equations:

$$\begin{cases} \dot{x}_1 = v cos\theta \\ \dot{x}_2 = v cos\theta \\ \dot{\theta} = \omega \end{cases} \quad (16)$$

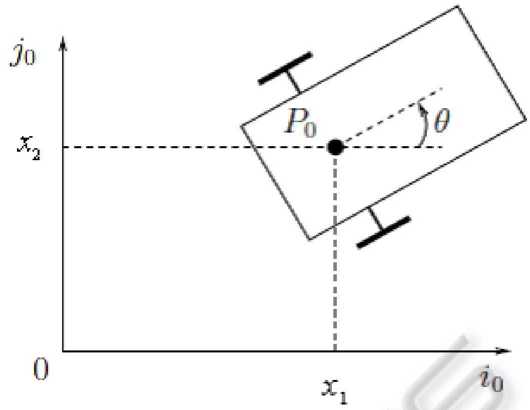

Figure 1: Coordinates system of car.

where
(x_1, x_2) represents the cartesian position of the centre of mass of the car,
θ its inclination with respect to the horizontal axis,
(v, ω) its forward and angular velocities respectively.
the model (16) is not static feedback linearizable. However, the problem can be solved by introducing the new state $x_4 = \sqrt{\dot{x_1}^2 + \dot{x_2}^2}$.
Resulting in the extended static feedback linearizable system described by:

$$\begin{cases} \dot{x}_1 = x_4 cos(x_3) \\ \dot{x}_2 = x_4 sin(x_3) \\ \dot{x}_3 = u_1 \\ \dot{x}_4 = u_2 \end{cases} \quad (17)$$

The outputs are the states variables: $x^T = (x_1, x_2, x_3, x_4)$, the above equations can be written in the following form:

$$\dot{x} = f(x) + g_1(x)u_1 + g_2(x)u_2; \quad (18)$$

where $f(x), g_1(x), g_2(x)$ are the smooth vector fields. We will transform the nonlinear dynamical system (17) in a structural 0-flat normal form, the distribution
$\overline{\Delta_1} = \{g_1, g_2\}$ has dimension 2.
The bracket $[g_1, g_2] = 0$ which means that $\overline{\Delta_1}$ is involutive. Then there exists two functions $y_1(x)$ and $y_2(x)$ such that:

$$dy_1(x).\overline{\Delta}_1 = 0 \quad (19)$$

$$dy_2(x).\overline{\Delta}_1 = 0 \quad (20)$$

From (19) and (20) we can conclude that $\frac{\partial y_i}{\partial x_3} = 0$ and $\frac{\partial y_i}{\partial x_4} = 0 \ \forall x_1, x_2$. Then it is enough to set $y_1(x)$ a function of x_1 and $y_2(x)$ any function in terms of x_2. Let us consider $y_1 = x_1$ and $y_2 = x_2$.
Now let us consider for $k = 1:2$ the following variables: $z_{k,1} = y_k$ and the new coordinate variables:
$z_{1,2} = L_f z_{1,1}$ and $z_{2,2} = L_f z_{2,1}$.

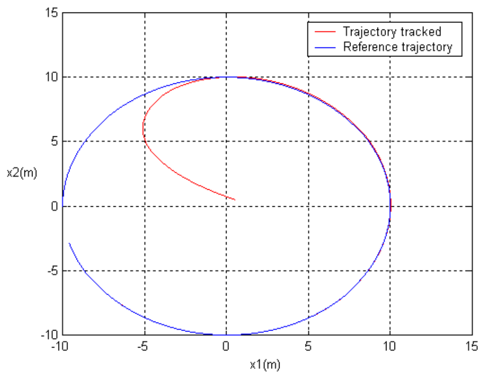

Figure 2: Trajectory tracked by uniaxal vehicule.

For this the derivative of these variables give structural 0-flat normal form (3).

$$\begin{aligned} \dot{z}_{11} &= z_{12} + \sum_{i=3}^{n-2} \beta_{11}^{i}(z)u_i \\ \dot{z}_{12} &= \alpha_{12}(z) + u_1 + \beta_{12}^{2}(z)u_2 + \ldots + \beta_{12}^{m}(z)u_m \\ \dot{z}_{21} &= z_{22} + \sum_{i=3}^{n-2} \beta_{21}^{i}(z)u_i \qquad (21) \\ \dot{z}_{22} &= \alpha_{22}(z) + \beta_{22}^{1}(z)u_1 + u_2 + \ldots + \beta_{22}^{m}(z)u_m \end{aligned}$$

The system is codimension 2, the canonical form can be expressed as follows:

$$\begin{cases} \dot{z}_{11} = z_{12} \\ \dot{z}_{12} = \alpha_{12}(z) + \beta_{11}^{1}(z)u_1 + \beta_{12}^{2}(z)u_2 \\ \dot{z}_{21} = z_{22} \\ \dot{z}_{22} = \alpha_{22}(z) + \beta_{22}^{1}(z)u_1 + \beta_{22}^{2}(z)u_2 \end{cases} \qquad (22)$$

where $\alpha_{12} = 0, \alpha_{22} = 0$
$\beta_{12}^{1}(z) = x_4 sin(x_3); \beta_{12}^{2}(z) = x_4 cos(x_3)$
$\beta_{22}^{1}(z) = cos(x_3); \beta_{22}^{2}(z) = sin(x_3)$

The main objective of the flatness based controller is to obtain the asymptotic tracking of a desired trajectory, let the system output be $y_1 = z_{11}$, $y_2 = z_{21}$, from (22) we can obtain the expressions of u_1, u_2 in terms of $(y_1, \dot{y}_1, y_2, \dot{y}_2)$, and the second derivatives of two first variables $\ddot{y}_1$, $\ddot{y}_2$. Let $\ddot{y}_1 = v_1, \ddot{y}_2 = v_2$ two new inputs control such that:

$$\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} \ddot{y}_{d_1} + k_d(\dot{y}_{d_1} - \dot{y}_1) + k_p(y_{d_1} - y_1) \\ \ddot{y}_{d_2} + k_d(\dot{y}_{d_2} - \dot{y}_2) + k_p(y_{d_2} - y_2) \end{bmatrix}$$

where $k_d, k_p > 0$ are control gains chose carefully to ensure exponential stability, and $\ddot{y}_d, \dot{y}_d, y_d$ are prescribed reference trajectories.
The control law:

$$\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} \beta_{11}^{1}(z) & \beta_{12}^{2}(z) \\ \beta_{22}^{1}(z) & \beta_{22}^{2}(z) \end{bmatrix}^{-1} \begin{bmatrix} v_1 - \alpha_{12}(z) \\ v_2 - \alpha_{22}(z) \end{bmatrix}$$

The controller gains are chosen to be:

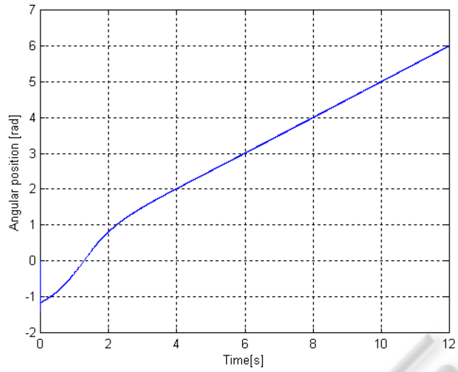

Figure 3: Unicycle orientation x_3.

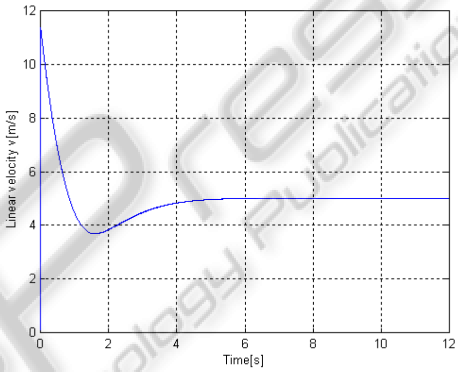

Figure 4: Linear velocity of unicycle vehicule.

$k_p = diag[1000, 1000]$, $k_d = diag[1000, 1000]$.

We consider the following continuously derivable desired trajectories:

$y_{d1} = rsin\frac{v_0}{r}t$, $y_{d2} = -rcos\frac{v_0}{r}t$ is assigned, which described a circular movement with radius $r = 10$ at the constant speed $v_0 = 5$.

The closed loop system was simulated using the initial conditions $x^T(0) = (0.5; 0.5; 0; 0.1)$ for system (17).

The trajectory tracked (see Fig. 2) is very close to the desired one, achieving by thus the control objective. Finally, angular position and linear velocity are depicted in the Fig. 3 and Fig. 4. It can be seen that our control scheme achieves satisfactory performances.

5 CONCLUSIONS

We described the development of a tracking controller based on normal flat forms. This method is generalized to analysis and control a class of a 0-flat affine nonlinear multi-input dynamical systems for which we can build flat outputs to give structural normal flat forms. Simulation results show an acceptable performance under the studied cases.

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