# SOLUTION OF AN INVERSE PROBLEM BY CORRECTION OF TABULAR FUNCTION FOR MODELS OF NONLINEAR DYNAMIC SYSTEMS*

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Abstract: In this paper, we present a solution to the problem of correction of parameterized tabular nominal functions for the motion equations in a model of a nonlinear dynamical system using observations in discrete time. The correction vector is determined by the mean of the multi-polynomial approximation algorithm (MPA-algorithm) using observations of the noise functions of the components of the state vectors. The method of correction of tabular functions is demonstrated by correcting 204 parameters in an example involving a mathematical model of the motion of an F-16 aircraft.

## 1 INTRODUCTION

In this paper, an algorithm is presented for the a numerical process of correction of a nominal mathematical model of a nonlinear dynamical system based on experimental data. The problem of estimation of the vectors of mathematical model parameters (the traditional problem of identification of the constant unknown parameters) has been considered in many prior works (see, for example, Klein and Morelli, 2006, Cappe et al., 2005, Gordon et al., 1993, Doucet et al., 2000, Doucet et al., 2001, Ristic et al., 2004, Gosh et al., 2008, Namdeo et Manohar, 2007, Cotter et al., 2009, Boguslavskiy, 1996, Boguslavskiy, 2006, Boguslavskiy, 2008 and Boguslavskiy, 2009).

The statement of the problem we consider here differs from the traditional problem in that the nominal model –the model before the correction– contains several nominal (previously obtained) tabular functions of the components of the state vectors. The problem of identifying the correction vectors is as follows: it is necessary to construct an algorithm to correct the tabular functions by processing the observation data . To do this it is necessary to identify parameters that are not constant and depend on a flowing state vector of a dynamic system. These parameters are hidden (sleeping); they do not influence the evolution of system if the current state vector has not visited the corresponding areas of the phase space.

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This singularity distinguishes our problem from traditional problems in which the evolution of the state vector does not influence the constant unknown parameters. The task differs from the task set in (Cotter et al., 2009), where the Bayesian approach is used to estimate a function of time that belongs to the mathematical model of a dynamical system.

An example of a model with a tabular function is the mathematical model of an aircraft with aerodynamic characteristics, i.e., the dimensionless coefficients of aerodynamic forces and moments (Klein and Morelli, 2006), given from tables of functions of components of state vectors (e.g., angles of attack and sliding ) and components of the vector of control (e.g., angles of deviation of steering surfaces).

Modern computational methods and wind tunnel testing can provide, in many instances, comprehensive data about the nominal aerodynamic characteristics of the aircraft; these comprise the parameters of the mathematical model.

” However, there are still several motivations for identifying aircraft models from flight data, including:

1. Verifying and interpreting theoretical predictions and wind-tunnel test results (flight results can also be used to help improve ground-based predictive methods);

2. Obtaining more accurate and comprehensive mathematical models of aircraft dynamics for use in designing stability augmentation and flight control systems;



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3. Developing flight simulators, which require an accurate representation of the aircraft in all flight regimes (many aircraft motions and flight conditions simply cannot be duplicated in the wind tunnel or computed analytically with sufficient accuracy or computational efficiency);

4. Expanding the flight envelope for new aircraft, which can include quantifying stability and predicting or controlling the impact of aircraft modifications, configuration changes, or special flight conditions;

5. Verifying aircraft specification compliance" (Klein and Morelli, 2006).

The nominal parameters for the problem of identifying actual aerodynamic characteristics are values that correspond to knots of one-dimensional or two-dimensional tables.

The correction vector of nominal (rated) parameters, defined by an algorithm handling the streams of digital information from the aircraft transmitters, has a very high dimension that is on the order of several tens or hundreds.

It should be noted that at NASA, projects based on the theory and practice of identification of aircraft by means of test flights are widely applied. An application of (Klein and Morelli, 2006) in the internet software package SIDPAC is published in the MATLAB M-files language (systems identification programs for aircraft), representing an implementation of the numerous algorithms recommended by NASA for identification problems.

The most common method of identification is the known nonlinear method of least squares, where the sum of the squares of the discrepancies, i.e., the differences between the actual measurements and their rated analogues, obtained by numerical integration of the system's equations of motion is computed for some realization of a vector of unknown parameters. The outcome of a successful identification accepts the vector of parameters, supplying a global minimum to the mentioned sum of squares of the discrepancies.

It is necessary to note that this criterion is statistically justified only for linear problems of identification, problems in which the measurements are linear in the unknown vector of parameters.

Significant computing difficulties arise when implementing a nonlinear method of least squares to correct the nominal parameters of an aircraft according to its test flights. The difficulties arise due to the large dimension of the correction vector and due to the existence of numerous relative minima for the sum of the squares of the discrepancies as functions of the correction vector, and also because of the use of variants of Newton's method, which requires a sequence of local linearizations to define the stationary points of the function.

The authors of the monograph (Klein and Morelli, 2006) presented a detailed exposition and analysis of known algorithms for the identification of parameters of the dynamic systems in chapters of[1]:[4 - 8]. However, only a regression method can be used for a practical investigation. The regression method presented in (Klein and Morelli, 2006) solves this problem subject to the following restrictions:

1. All components of the state vector are measured.

2. The algorithm builds a vector of estimates for the vector of derivatives $\dot{dx}$ at the moments of measurement,

3. The vector functions on the right-hand side of the equations of motion linearly depend on the estimated vectors.

4. Prohibition of mathematical modeling without the use of a Monte-Carlo method to analyze the theoretical observability of the components of the identified parameter vector if the laws of control are set beforehand by test flights of the aircraft and information about random errors of its transmitters.

In the monograph (Cappe et al., 2005), the problem of estimating the parameters is considered within the limits of the common problem of smoothing; this consists of the problem of constructing approximate conditional expectations for elements in a non-observable sequence if these elements influence observable elements by means of a given statistical mechanism. Various approaches to solving the smoothing problem by means of expectation-maximization (EM) methods are stated and investigated. However, the maximization operation requires the definition of a point global (but non-local!) extremum, that is generally not guaranteed by numerical methods.

In the last ten years, a significant number of studies (Gordon et al., 1993; Doucet et al., 2000; Doucet et al., 2001; Ristic et al., 2004; Gosh et al., 2008; Namdeo et Manohar, 2007), were published that represented the basic solution of the problem as the definition of the conditional expectation of a vector of parameters for a mathematical model of a nonlinear dynamical system. By means of multiple applications of Bayes' formula to the state vectors and with numerical quadratures, it is easy to determine the recurrence equations for the probability density function (pdf) of state vectors of the dynamical system. The actual solution of these recurrence equations, however, is not feasible because of the unwieldy dimensions of the integrals involved (Namdeo et Manohar, 2007). Therefore, several alternative strategies have been developed. One set of such alternatives consists of developing suboptimal filtering, such as that based on linearization or transformations, and the other consists of methods that employ Monte Carlo simulation strategies (e.g., estimation with the sequential importance sampling particle filter ) to approximately evaluate the multidimensional integrals in a recursive manner.

This direction is perceptive, but it is bulky and not suitable for the solution of practical applied (instead of model!) problems of nonlinear identification.

The MPA algorithm (Boguslavskiy, 1996; Boguslavskiy, 2006; Boguslavskiy, 2008; Boguslavskiy, 2009) makes use of this paper for the correction of tabular functions. The MPA algorithm is a new recursive algorithm that asymptotically and accurately solves the nonlinear problem of construction of the conditional expectation vectors of the vector of parameters for mathematical models of nonlinear dynamical systems, including random errors and perturbations with given distributions. Therefore, the MPA algorithm approximately solves the problem of quasi-optimal mean-squares estimation.

The MPA algorithm has none of the disadvantages of the NACA algorithm noted above, and in principle it differs from all of the abovementioned algorithms; a theoretical proof of its accuracy is presented in (Boguslavskiy, 1996; Boguslavskiy, 2006; Boguslavskiy, 2008; Boguslavskiy, 2009).

## 2 A STATEMENT OF THE PROBLEM OF CORRECTION OF FUNCTIONS DETERMINED IN THE FORM OF TABLES

Let $x \in R^m$ be a moving state vector of the mathematical model of the dynamical system and the components of the vector $z$ be a subset of the components of $x$ : $z \in x, z \in R^r, r < m$. The vector $z$ is an argument of the tabulated functions. We suppose that the vectors $z$ belong to the interior boundary of a parallelepiped $\Omega \in R^r$ under all realizable conditions of the dynamical system – a realizable vector of functions of $t : u(t)$. Choose a Cartesian coordinate system for $R^r$ with axes parallel to the edges of the parallelepiped $\Omega$. The parallelepiped $\Omega$ is covered by nodes numbered $s^r$, where $s$ is given as an integer.

Here $s^r$ is the number of nodes on which the table determines the tabulated functions. We denote each node by $z(i_1,...,i_r), 1 \leq i_1,...i_r \leq s$, where the integers $i_1,...i_r$ are the indices of the coordinates of this node. The nodes $z(i_1,...,i_r)$ are vertices .of the small parallelepipeds belonging to $\Omega \in R^r$.

We suppose that a presentation ( a description) of the nominal mathematical model contains $K$ tabular functions $\vartheta_j(z(i_1,...,i_r)), j = 1,...,K$, which are given on the specified nodes. The values $\vartheta_j(z(i_1,...,i_r))$ are the nominal values of the tabular function, which, according to the preceding experiments or in theory, all represent the vertices of the small parallelepipeds.

Each tabular function $\vartheta_j(z(i_1,...,i_r))$ Is the skeleton of a continuous function $f_j(z), j = 1,...,K$ as follows : 1) on the vertices of the small parallelepiped the values of the continuous function coincide with the values of the tabular function; 2) on other points of the parallelepiped, the values of the continuous function are linear combinations of the values of the tabular function at the vertices, such that these values are multilinear functions of the values of the tabular function.

Let $L_1,...,L_r$ be the lengths of the edges of any of the small parallelepipeds. Then in the coordinates $tt^1(\alpha_1),...,tt^r(\alpha_r)$ of the $2^r$ vertices, we can write the formula $tt^k(\alpha) = tt_0^k + L_k\alpha_k, k = 1...,r$, where $\alpha_1...,\alpha_r$ are independently determined with the values 0, 1.The values $\alpha_1 = 0,...,\alpha_r = 0$ correspond to the minimal values of all coordinates for a given small parallelepiped.

Let $z^1,...,z^r$ be the components of the vector $xx$, $\varphi_0(z^k) = L_k^{-1}(tt^k(1) - z^k), \varphi_1(z^k) = L_k^{-1}(z^k - tt^k(0)), k = 1,...,r$ be linear functions of these components $\varphi_0(z^k) = 0$ if $z^k = tt_0^k, \varphi_0(z^k) = 1$ if $z^k = tt_0^k + L_k$, $\varphi_1(z^k) = 0$ if $z^k = tt_0^k + L_k, \varphi_1(z^k) = 1$ if $z^k = tt_0^k$.

Then the components of the multilinear function take the form

$$f_j(z^1,...,z^r) = \sum_{\alpha_1,\cdots,\alpha_r=0,1} \varphi_{\alpha_1}(z^1)\cdots\varphi_{\alpha_r}(z^r)$$

$$\vartheta(tt_0^1 + L_1\alpha_1,...,tt_0^r + L_r\alpha_r), \qquad (2.1)$$

where the sum is over all binary values of an aspect $\alpha_1 \cdots \alpha_r$ and over all $2^r$ of the items.

The vector $f_j(z^1,...,z^r)$ is a nominal representation of the continuous functions defined by the tabular functions $\vartheta_j(z(i_1,...,i_r))$.

The corrections of the $K$ tabular functions $\vartheta_j(z(i_1,...,i_r)), j = 1,...,K$ replace the values $\vartheta_j(...)$ with values $\vartheta_j(...) + \theta_j(...)$, where the correction terms $\theta_j(...)$ are the results of processing the new data.

We shall designate by $f_j(z^1,...,z^r,\theta_j)$ the functions obtained from $f_j(z^1,...,z^r)$ by substituting) functions $\vartheta_j(...) + \theta_j(...)$ for $\vartheta_j(...)$ in (2.1)

The new data are the results of observations values $y_1,...,y_N$ where $y_k = H_k(x_k) + \xi_k$, $y_k, x_k, k = 1,...,N$ are the results of observations of the vectors $x$ at discrete instants, $H_k(...)$ are the given functions, and $\xi_k$ are random errors with a given distribution.

Using the MPA algorithm, the correction vector is quasi-optimal in the mean square estimation of the variations of the components of the nominal vectors $\vartheta_j(z(i_1,...,i_r)), j=1,...,K$. The experiment responds to these variations with new data about the dynamical system. The vector of the estimates has $K \times s^r$ components $\hat{\theta}_j(i_1,...,i_r), j=1,...,K, 1 \le i_1,...,i_r \le s$.

The MPA algorithm is the Bayesian estimator of the parameters (Boguslavskiy, 2006). A priori information, which is necessary for the training and adjustment of the MPA algorithm by means of the Monte Carlo method, includes the segment lengths of expected scattering of values $\theta_j(...)$ of the corrections of the nominal tabular functions, expressed in terms of components of these functions. Therefore, the sum $\vartheta_j(...)+\theta_j(...)$ replaces random values $\vartheta_j(...)(1+\rho_j\varepsilon(i_1,...,i_r))$, where $0<\rho<1$ and random the values $\varepsilon(...)$ are uniformly distributed on the segment $[-1,1]$.

If the nominal tabular function is smooth with respect to the components of the vector *xx*, i.e., it varies smoothly with changes in this component, then it is appropriate to require this smoothness to be preserved after the correction. The requirement is satisfied if the correction includes the increments. of the nominal values of the tabular function, This increments are the tabular functions obtained by varying a given component in the transfer from the given node to the subsequent node. These increments are approximately the derivatives of the tabular functions with respect to the given component.

If by a movement of the dynamical system the moving vector $z$ is found inside or on the boundary of any small parallelepiped, then the equations of the model use domains of the nominal tabular functions $\vartheta_j(z(i_1,...,i_r))$ and the corresponding continuous functions for which the points $z(i_1,...,i_r)$ belong to this small parallelepiped Therefore the correction of values $\vartheta_j(z(i_1,...,i_r))$ is impossible if by a given $u(t)$ the current state vector $x$ does not visit at any instant the small parallelepiped with the vectors $z(i_1,...,i_r)$. The significant influence of the area circumscribed by the control $u(t)$ on the correction result essentially distinguishes the problem of identification of the vector correction of tabular functions from the traditional problem of the identification of the parameters.

We illustrate the definitions on an example modeling the motion of an aircraft. The continuous functions corresponding to the tabular functions are piecewise linear approximations for the dimensionless coefficients of the aerodynamic forces and moments as the functions of the angles of attack.

We present a formalized statement of the problem of the correction of the tabular function, which is the problem of quasi-optimal identification of the variations of this function. The variations are estimated using data for the observed motion of the system. The equations of the motion model are written in the form

$$dx/dt = F(x, f_1(z^1,...,z^r,\theta_1),..., f_K(z^1,...,z^r,\theta_K),u,t). \quad (2.2)$$

The equation of the observation is written in the form

$$y_k = H_k(x_k) + \xi_k, \quad (2.3)$$

where $k=1,...,N$.

Further, we designate by $Y_N$ the vector whose components are $y_1,...,y_N$. In verifying the quality of the correction after realization of the estimations $\hat{\theta}_j(i_1,...,i_r), j=1...,K, 1\le i_1,...,i_r \le s$, we can use the sum of quadrates of differences, which is

$$\sum_{k=1,...,N} (y_k - \hat{y_k})^2,$$

where the values $\hat{y_k}$ are computed from (2.2), (2.3) after substituting the values $\hat{\theta}_j(i_1,...,i_r), i=1,...,K, 1 \le i_1,...,i_r \le s$ for the values $\theta_j(i_1,...,i_r), i=1,...,K, 1 \le i_1,...,i_r \le s$

# 3 IDENTIFICATION OF SOME PARAMETERS OF F-16 AIRCRAFT

*1. The Driving Equations*

It follows from monograph (Klein and Morelli, 2006) that the driving equations of an F-16 plane stabilize with respect to the magnitude of the airspeed velocity $V$ and roll angle rotation $\phi$ ($\dot{V}=\dot{\phi}=0$) and can be approximated in the following form:

$$\dot{\alpha} = q + (gcos\upsilon cos\phi + \bar{q}SC_Z/M)/V,$$

$$\dot{\beta} = -r + (gcos\upsilon sin\phi + \bar{q}SC_Y/M)/V,$$

$$p = 0,$$

$$\dot{q} = 160c_7r + c_6r^2 + \bar{q}S\bar{c}c_7C_m,$$

$$\dot{r} = -(c_2r + 160c_9)q + \bar{q}Sbc_9C_n,$$

where the pitch angle rotation $\upsilon$ and the yaw angle rotation $\phi \simeq$ are constants (within a small maneuvering time), the angle of attack is $\alpha$, the sideslip angle is $\beta$, $p,q,r$ are the body-axis components of the aircraft's angular velocity, $\bar{q}$ is the dynamic pressure, $S$ is the wing reference area, $b$ is the wing span, $\bar{c}$ is the mean aerodynamic chord of the wing, $M, c_2, c_6, c_7, c_9$ are constants (see (Klein and Morelli, 2006), $C_Y, C_Z, C_m, C_n$ are non-dimensional coefficients that are directly proportional to the aerodynamic forces and moments

$$C_Y = -0.02\beta + 0.086(\delta_r/30) + (b/2V)C_{Y_r}(\alpha)r,$$

$$C_Z = C_{Z_0}(\alpha)(1-(\beta\pi/180)^2) - 0.19(\delta_s/25) + (\bar{c}/2V)C_{Z_q}(\alpha)q,$$

$$C_m = C_{m_0}(\alpha,\delta_s) + (\bar{c}/2V)C_{m_q}(\alpha)q + 0.1C_Z,$$

$$C_n = C_{n_0}(\alpha,\beta) + C_{n_{\delta_r}}(\alpha,\beta)(\delta_r/30) + (b/2V)C_{n_r}(\alpha)r - 0.1(\bar{c}/b)C_Y,$$

$\delta_s$ is the stabilizer deflection,$\delta_r$ is the rudder deflection, and $\alpha,\beta,\delta_s,\delta_r$ are in degrees.

Further, the functions $C_{Y_r}(\alpha), C_{Z_0}(\alpha), C_{Z_q}(\alpha), C_{m_0}(\alpha,\delta_s), C_{m_q}(\alpha), C_{n_0}(\alpha,\beta), C_{n_{\delta_r}}(\alpha,\beta), C_{n_r}(\alpha)$ are nominal functions. They are defined from the vertices of the parallelepiped $\Omega \in R^4$ by $s = 8$. The functions accept ratings that are evaluated using experiments in a wind tunnel at a finite number of reference nodes, covering the domains $\Omega$. The vectors of the nominal experimental data are $\vartheta_1(\ldots),\ldots,\vartheta_8(\ldots))$ in the equations (2.1). For the functions $C_{Z_0}(\alpha), C_{Y_r}(\alpha), C_{Z_q}(\alpha), C_{n_r}(\alpha), C_{m_q}(\alpha)$ the vectors $\vartheta_1(\ldots),\ldots,\vartheta_5(\ldots)$ have dimensions $12\times 1$; accordingly, for the functions $C_{m_0}(\alpha,\delta_s), C_{n_0}(\alpha,\beta)$ the vectors $\vartheta_6(\ldots),\vartheta_7(\ldots)$ have dimensions $12\times 5$ and $12\times 7$. Furthermore, we do not correct the function $C_{n_{\delta_r}}(\alpha,\beta)$. Therefore, the number of nominal parameters defining these functions equals $12\times(5+5+7) = 204$.

The software package *SIDPAC* contains a file *F16 AERO SETUP Generates aerodynamic data tables* with ten one- and two-dimensional tables of nominal experimental data.

We emphasize that the nominal functions mentioned above are nonlinear functions of the arguments.

*2. Parametric Model of Aerodynamic Parameters of the Subjects of Identification*

We suppose that for all functions except $C_{\delta_r}(\alpha,\beta)$, the nominal experimental data differ from the true data by some random error vectors, which are designated $\theta_1,\ldots,\theta_7$.

We consider the most complicated problem for the MPA algorithm, where at each of the points of the table, the actual parameter differs from the nominal parameter by a random magnitude subject to the a priori limits $\theta_i$.

After collecting the measurements of the parameters of the perturbed driving of the aircraft, the MPA identification algorithm should estimate the 204 components of the vector of random errors, generating the vector of differences between the actual and nominal parameters.

Let $A_i$ and $B_i$ $(i = 1,\ldots,204)$ be the $i$-th components of the nominal and actual (perturbed ) vectors of the aerodynamic parameters corresponding to the 204 actual parameters subject to identification.

We suppose a fair parametrical model:

$$B_i = A_i + \Delta_i,$$

The vector $\Delta$ is the vector of perturbations of nominal parameters, i.e., the vector of errors of the aerodynamic parameters, and its component estimates are subject to our identification. For the structure of these components, we give the formula

$\Delta_i = A_i\rho_i\varepsilon_i, 0 < \rho_i < 1, -1 \le \varepsilon_i \le 1.$

The positive number $\rho_i$ defines the maximum magnitude that the ratio of the random variable of perturbations $\Delta_i$ and nominal parameter $A_i$ is allowed to attain under the conditions of our identification algorithm. Each $\varepsilon_i$ is a random number that is uniformly and independently distributed.

*3. Transients of Characteristics of the Nominal and Perturbed Movements*

We suppose as above that the transients in the reduced driving equation of the aircraft F-16 are $\alpha,\beta,q,r$ over 20 sec., if at $t = 0$ $\alpha = \beta = 0.3rad., q = r = 10deg/sec$ and magnitudes $\delta_s,\delta_r$ are constant and equal to $10deg$.

We shall discuss the precision of the estimate under the following assumptions: during the 20 sec. period, the current magnitudes $\alpha,\beta,q,r$ are measured at intervals of 0.05 sec ($N = 1600$). We suppose that the random errors of measurement are discrete white noise, which is limited by the product of the true measured magnitudes on the magnitude of the set $\varepsilon$. We shall suppose that the MPA algorithm supplies the magnitudes $\widehat{\Delta}_i, i = 1,\ldots,204$, which are the estimates of the magnitudes $\Delta_i, i = 1,\ldots,204$. To characterize the relative precision of the identification of the random parameters $\Delta_i$ we define ratios $\varepsilon_i = (\widehat{\Delta}_i - \Delta_i)/\Delta_i$

We must emphasize that the state vector of the aircraft corresponding to the modeled transient does not visit all the reference points in which the nominal experimental data are set. Therefore, for some values of $i$, the magnitude $\varepsilon_i$ has an order of 1 or more. The corresponding values $\vartheta_i$ are not observable for the modeled transient, and also cannot be corrected by means of the MPA algorithm.

Table 1 presents a histogram of $\varepsilon_i$.

Table 1.

| | | |
|---|---|---|
| $2 \ge \lvert\varepsilon_i\rvert \ge 1$ | $1 \ge \lvert\varepsilon_i\rvert \ge 0.5$ | $0.5 \ge \lvert\varepsilon_i\rvert \ge 0.25$ |
| 37 | 44 | 37 |
| $0.25 \ge \lvert\varepsilon_i\rvert \ge 0.1$ | $0.1 \ge \lvert\varepsilon_i\rvert \ge 0.05$ | $0.05 \ge \lvert\varepsilon_i\rvert$ |
| 29 | 17 | 17 |

The practical purpose of identification is to correct the nominal experiment data, and the outcome is to replace the nominal aerodynamic parameters with new parameters. If the errors of identification are small, the driving characteristics of the aircraft, obtained by numerical integration after correction, should be close to the perturbed driving characteristics of the aircraft, as discovered early in real flight or by means of modeling. In a real flight situation, the errors of the nominal experimental data can only be estimated only over time and by observing the perturbed driving characteristics.

In Table 2 , we present the ratios of the difference between the characteristics of the corrected and perturbed movements and the difference between the characteristics of the nominal and perturbed movements as functions of discrete time with increments of 1 sec.

In Table 2, for example, the expression $\delta^{c,p}_{n,p}\alpha$ designates the difference ratio $(\alpha(corr) - \alpha(perturb))/(\alpha(nomin) - \alpha(perturb))$.

The labels $\delta^{c,p}_{n,p}\beta$, $\delta^{c,p}_{n,p}q$, $\delta^{c,p}_{n,p}r$ are defined similarly.

These ratios show how quickly the MPA algorithm reduces the difference between the corrected and perturbed movements compared to the difference between the nominal and perturbed movements.

Table 2.

| sec | $\delta^{c,p}_{n,p}\alpha$ | $\delta^{c,p}_{n,p}\beta$ | $\delta^{c,p}_{n,p}q$ | $\delta^{c,p}_{n,p}r$ |
|---|---|---|---|---|
| 0 | -0.086 | -0.050 | 0.215 | -0.040 |
| 1 | -0.094 | -0.064 | 0.048 | -0.112 |
| 2 | -0.112 | -0.067 | 0.012 | -0.329 |
| 3 | 0.110 | -0.054 | -0.011 | -0.024 |
| 4 | -0.082 | -0.050 | -0.022 | -0.130 |
| 5 | -0.019 | -0.046 | -0.040 | -0.083 |
| 6 | -0.022 | -0.042 | -0.168 | -0.114 |
| 7 | -0.022 | -0.039 | -0.003 | -0.126 |
| 8 | -0.022 | -0.036 | -0.0105 | -0.141 |
| 9 | -0.022 | -0.034 | -0.029 | -0.154 |
| 10 | -0.022 | -0.032 | 0.001 | -0.079 |
| 11 | -0.022 | -0.031 | -0.009 | 0.001 |
| 12 | -0.022 | -0.030 | -0.007 | 0.108 |
| 13 | -0.022 | -0.028 | -0.008 | 0.233 |
| 14 | -0.022 | -0.026 | -0.008 | 0.297 |
| 15 | -0.022 | -0.025 | -0.009 | 0.345 |
| 16 | -0.022 | -0.024 | -0.009 | 0.0382 |
| 17 | -0.022 | -0.022 | -0.009 | 0.434 |
| 18 | -0.022 | -0.021 | -0.009 | 0.486 |
| 19 | -0.022 | -0.020 | -0.009 | 0.599 |
| 20 | -0.022 | -0.019 | -0.010 | 0.722 |

It follows from Table 2 that the corrected characteristics become close to, and often coincide with, the perturbed driving characteristics (to within 2 digits after the decimal point), in the absence of observational errors.

## 4 CONCLUSIONS

The data presented in this work show that a multi-polynomial approximation algorithm can form a computational basis for creating an effective solution for inverse problems, thus identifying the parameters of a nonlinear dynamical system, including the system of aerodynamic parameters of an aircraft.

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