EVALUATING MAXIMUM TRANSMISSION UNRELIABILITY IN PERSISTENT CSMA PROTOCOL

Dariusz Kościelnik and Marek Miśkowicz
Department of Electronics, AGH University of Science and Technology, Al. Mickiewicza 30, Cracow, Poland

Keywords: Carrier Sense Multiple Access, Performance Analysis, Analytical Modelling.

Abstract: The paper addresses the problem of evaluating the unreliability of transmission, undertaken by a given station, according to the persistent CSMA scheme. The unreliability of transmission is considered on the media access control level so it is defined by the probability that a given node participates in a collision. The presented results show that the maximum transmission unreliability is upper bounded by the persistence level (p), which is the main parameter of the protocol. The presented analysis is compared to the corresponding results for the non-persistent CSMA. As shown, both results are complementary because the maximum transmission unreliability in the non-persistent CSMA scheme is also bounded by the probability of choosing a single slot in the contention window.

1 INTRODUCTION

Although the carrier sense multiple access (CSMA) protocols have been introduced in the early 70s, due to their inherent flexibility and simplicity, they are in more advanced versions still widely used in contemporary networking, especially for wireless communication (e.g., Tay, Jamieson, Balakrishnan, 2004). In particular, the predictive CSMA protocol is employed in Local Operating Networks (LonWorks) commercial platform for sensor and control networking (Miśkowicz, Golański, 2006). On the other hand, the non-persistent CSMA scheme with a geometric distribution has been recently proposed for sensor networking (Tay, Jamieson, Balakrishnan, 2004; Miśkowicz, 2009a; Egea-López *et al.*, 2007).

The performance of the CSMA protocols have been investigated intensively for decades. The main criterion of performance analyses is evaluating the throughput-delay characteristics (Kleinrock, Tobagi, 1975; Lam, 1980), or the protocol energy efficiency in the context of wireless networking (Bruno, Conti, Gregori, E., 2002; Calì, Conti, Gregori, 2000).

Most of the performance analyses are based on classical approaches where the network load is assumed to contain an infinity number of stations that collectively produce (including both new and retransmitted packets) the traffic with Poisson arrivals (Kleinrock, Tobagi, 1975; Lam, 1980).

The Poisson model is an approximation of a large but finite population in which every station generates messages infrequently and each message can be successfully transmitted a long time before the station generates a next message.

The Poisson traffic model, based on the fundamental assumption of stochastically distributed independent stimuli, has been applied widely for performance evaluation of computer networks.

Since the assumption of lightly loaded network is invalid in many load scenaria in wireless local area networks (Bruno, Conti, Gregori, 2002; Calì, Conti, Gregori, 2000; Bianchi, 1998), and especially in networked sensor and control systems (Miśkowicz, 2009b), the other class of performance analyses deal with the load scenario of finite number of active stations in which every station may produce a significant portion of network traffic. In order to model the network operation under heavy load, it is assumed that all the stations are in the *asymptotic* (*saturation*) *conditions* where they have always a packet ready for transmission.

The present study deals with the performance analysis of the *persistent CSMA protocol* that belongs to one of generic CSMA schemes introduced in (Kleinrock, Tobagi, 1975). The paper contribution is the analytical proof that the unreliability of transmission undertaken by a given station, according to the persistent CSMA scheme, is

Kościelnik D. and Miskowicz M. (2010).
EVALUATING MAXIMUM TRANSMISSION UNRELIABILITY IN PERSISTENT CSMA PROTOCOL.
In *Proceedings of the 7th International Conference on Informatics in Control, Automation and Robotics*, pages 182-186
DOI: 10.5220/0002958301820186

upper bounded by the persistence level (p), which is the main parameter of the protocol.

The unreliability of transmission is considered on the media access control level so it is defined by the probability that a given node participates in a collision. The presented analysis is compared to the corresponding results for the non-persistent CSMA. As interesting, both results are complementary because the maximum transmission unreliability in the non-persistent CSMA scheme is also bounded by the probability of choosing a single slot in the contention window (Miśkowicz, Kościelnik, 2010).

The authors believe that the presented results contributes to better understanding of the persistent CSMA operation. To the best authors' knowledge, these results have not been yet published.

The performance analysis stated in the present paper belongs to the studies of persistent CSMA scheme for the network staying in the asymptotic conditions because the evaluation of the maximum transmission unreliability needs to feed a channel with heavy load.

2 ANALYTICAL MODEL OF PERSISTENT CSMA

2.1 Persistent CSMA Specification

The *persistent CSMA* scheme belongs to the slotted-CSMA protocol where the channel idle time is divided into fixed length intervals. All the stations in the network are synchronized and forced to start a transmission only at the beginning of a slot.

In the network that operates according to the persistent CSMA, when a station has a new message to transmit, it senses the channel. If the channel is detected to be idle, then it transmits a message with the probability p, while with probability 1-p, it delays the message transmission to the next time slot. The slot duration is determined by the network propagation delay.

By a comparison, in the non-persistent CSMA, when the station senses the channel to be idle, it draws a number of a slot from a set of slots included in the *contention window*. The probability distribution of a random slot selection is uniform.

In the persistent CSMA protocol, the number of empty slots preceding a (successful or unsuccessful) transmission of a data packet is theoretically unbounded because the probability of starting transmission is defined by the geometric distribution where a success occurs with the probability p, and a failure with the probability (1-p). The mean number of trials undertaken by a given station equals 1/p. On the other hand, in the non-persistent CSMA protocol, the maximum number of empty slots before (successful or unsuccessful) transmission of a data packet equals (W-1), and the mean number (W-1)/2 where W is a number of slots in the contention window.

2.2 Collision Probability in Single Transmission Attempt

The probability $p_{coll(1)}^{(k)}$ that a certain station is involved in collision in the kth transmission attempt is defined by the product of the following probabilities:

$$p_{coll(1)}^{(k)} = p_{coll(11)}^{(k)} p_{coll(12)}^{(k)} \quad (1)$$

where $p_{coll(11)}^{(k)}$ is the probability that all the contending stations had not started the transmission in the previous 1,…,k-1 transmission attempts, and $p_{coll(12)}^{(k)}$ is the probability that at least one from the s=1,2,3,… contending stations apart from a selected station starts the transmission in the kth transmission attempt. The former probability $p_{coll(11)}^{(k)}$ is given by the formula:

$$p_{coll(11)}^{(k)} = \prod_{x=1}^{k-1} (1-p)^{s+1} = (1-p)^{(s+1)(k-1)} \quad (2)$$

The latter probability $p_{coll(12)}^{(k)}$ is defined as follows:

$$p_{coll(12)}^{(k)} = p \sum_{x=1}^{s} C_s^x p^x (1-p)^{s-x} \quad (3)$$

where $C_s^x = s!/[(s-x)!\,x!]$ is the binomial coefficient, and s, s>1 is an integer.

Thus, the $p_{coll(1)}^{(k)}$ is given as:

$$p_{coll(1)}^{(k)} = p(1-p)^{(s+1)(k-1)+s} \left(\sum_{x=0}^{s} \left[C_s^x \left(\frac{p}{1-p} \right)^x \right] - 1 \right) \quad (4)$$

The formula (4) may be transformed as follows:

$$p_{coll(1)}^{(k)} = p(1-p)^{(s+1)(k-1)+s} \left(\left(\frac{1}{1-p} \right)^s - 1 \right) \quad (5)$$

because according to the Newton's generalized binomial theorem:

$$\sum_{x=0}^{s} C_x^s \left(\frac{p}{1-p}\right) = \left(\frac{p}{1-p}+1\right)^s \qquad (6)$$

As follows from (3), the probability $p_{coll(12)}^{(k)}$ does not depend on k but only on the number of contenders s. On the other hand, the probabilities $p_{coll(11)}^{(k)}$ (see (2)), and consequently $p_{coll(1)}^{(k)}$ also (see (5)), is a decreasing function of the number transmission attempt k.

In Fig. 1, the plot of the probability $p_{coll(1)}^{(k)}$ versus the number of transmission attempt k for selected numbers of the contending stations s={1,3,5} for p=1/16 according to (5) is shown.

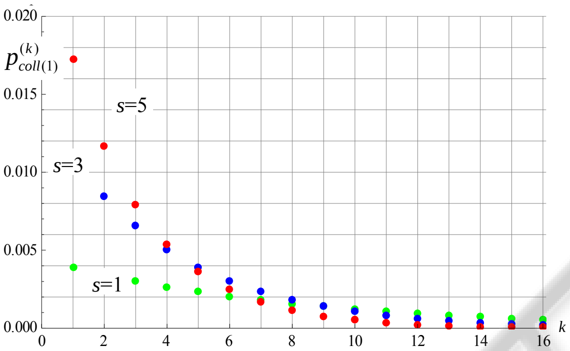

Figure 1: The probability $p_{coll(1)}^{(k)}$ that a certain station is involved in collision in the kth transmission attempt versus the number k for various population sizes of the contending stations s according to (5) for p=1/16 and s={1,3,5}.

In Fig. 2(a,b), the plots of the probability $p_{coll(1)}^{(k)}$ versus the number of the contending stations s in selected transmission attempts $k;k$={1,2,5,9} according to (5) are presented.

As seen in Fig. 2a, the probability $p_{coll(1)}^{(k)}$ that a given station participates in collision for k≥2 is a unimodal function of s that is maximized for a certain number of contending stations $s_0^{(k)}$. The $s_0^{(k)}$ is a decreasing function of k,k≥2. Instead, for k=1, the $p_{coll(1)}^{(k)}$ increases strictly with s and approaches a horizontal asymptote $p_{coll(1)}^{(k)} = p$ as may be analytically derived (Fig. 2b):

$$\lim_{s\to\infty} p_{coll(1)}^{(1)} = \lim_{s\to\infty} (1-p)^s p \cdot \left(\left(\frac{p}{1-p}+1\right)^s - 1\right) = p \qquad (7)$$

In particular, $p_{coll(1)}^{(k)}$ reaches 0.0625 for p=1/16=0.0625 for high s as seen in Fig.2b.

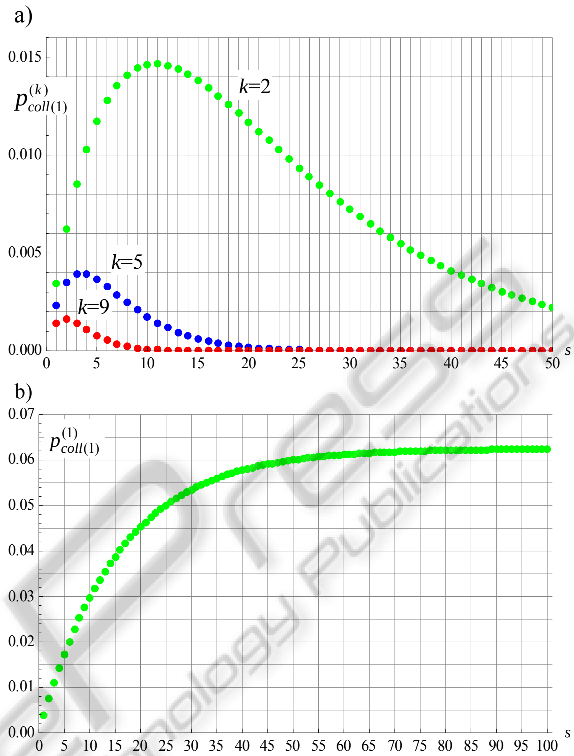

Figure 2: The probability $p_{coll(1)}^{(k)}$ versus the population of the contending stations s for p=1/16=0.0625 and selected transmission attempts k={2,5,9} (a), and k={1} (b).

3 MAXIMUM TRANSMISSION UNRELIABILITY IN PERSISTENT CSMA

As stated, the probability $p_{coll(1)}^{(k)}$ that a certain station is involved in collision in the kth transmission attempt is defined by the formula (5). The total probability $p_{coll(1)}^{(1\div k)}$ that a certain station participates in collision at most at the kth transmission attempt is defined as a sum:

$$p_{coll(1)}^{(1\div k)} = \sum_{x=1}^{k} p_{coll(1)}^{(x)} \qquad (8)$$

where $p_{coll(1)}^{(k)}$ is defined by (5).

By setting (5) to (8):

$$p_{coll(1)}^{(1\div k)} = \sum_{x=1}^{k} p(1-p)^{(s+1)(x-1)+s}\left(\left(\frac{1}{1-p}\right)^{s} - 1\right) \qquad (9)$$

The probability $p_{coll(1)}^{(1\div k)}$ in the formula (9) is defined as a geometric series with the first term equal to $p[1-(1-p)^s]$, and the ratio equal to $(1-p)^{s+1}$ so it might be computed as:

$$p_{coll(1)}^{(1\div k)} = \frac{p\cdot\left(1-(1-p)^s\right)\left(1-(1-p)^{(s+1)k}\right)}{1-(1-p)^{s+1}} \qquad (10)$$

The plot of the probability $p_{coll(1)}^{(1\div k)}$ versus k according to (10) is shown in Fig. 3 for p=1/16 and s={1,2,5,10}.

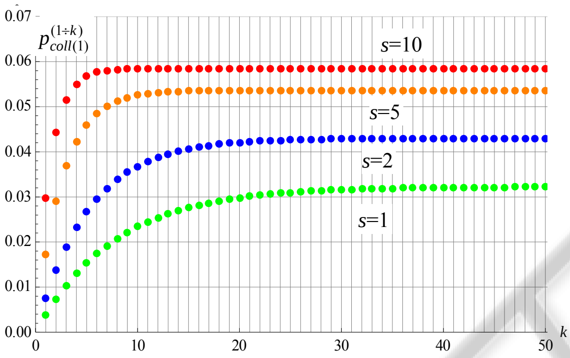

Figure 3: The probability $p_{coll(1)}^{(1\div k)}$ according to (10) for p=1/16 and s={1,2,5,10}.

As seen in Fig. 3, each curve approaches a horizontal asymptote with growing number of transmission attempt k. These asymptotes corresponding to the limits:

$$p_{coll(1)} = \lim_{k\to\infty} p_{coll(1)}^{(1\div k)} \qquad (11)$$

for various s and denoted by $p_{coll(1)}$ defines the probability of collision in any attempt in a given transmission cycle.

By setting (10) to (11):

$$p_{coll(1)} = \frac{p\cdot\left(1-(1-p)^s\right)}{1-(1-p)^{s+1}} \qquad (12)$$

As follows from (12), the $p_{coll(1)}$ depends both on the p value and the number of contending stations s. The plots of $p_{coll(1)}$ versus the persistence level p for various numbers of contending stations s is presented in Fig. 4.

As seen in Fig. 4, the probability $p_{coll(1)}$ grows with increasing p but it is at the same time smaller than p for any number of contending stations s.
This conclusion may be also derived analytically on the basis of (12) as follows:

$$p_{coll(1)} = \frac{p\cdot\left(1-(1-p)^s\right)}{1-(1-p)^{s+1}} < p \qquad (13)$$

because $\frac{1-(1-p)^s}{1-(1-p)^{s+1}} < 1$ for s≥1, and furthermore:

$$\lim_{s\to\infty} p_{coll(1)} = \lim_{s\to\infty} \frac{p\cdot\left(1-(1-p)^s\right)}{1-(1-p)^{s+1}} = p \qquad (14)$$

Thus, the probability $p_{coll(1)}$ of collision in any attempt in a given transmission cycle is upper bounded by the persistence level p regardless of the number of contending stations s.

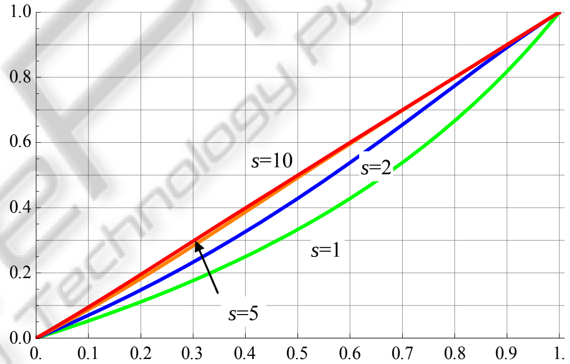

Figure 4: Plots of $p_{coll(1)}$ vs. the persistence level p for s={1,2,5,10}.

In Fig. 5, the plots of the $p_{coll(1)}$ versus s for p={1/4, 1/16, 1/64} are presented. Finally, in Fig. 6, the $p_{coll(1)}$ versus s and p in 3-D plot are illustrated.

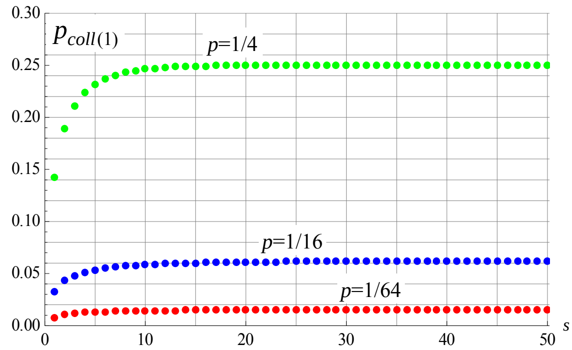

Figure 5: Plots of the $p_{coll(1)}$ versus s for p={1/4, 1/16, 1/64}.

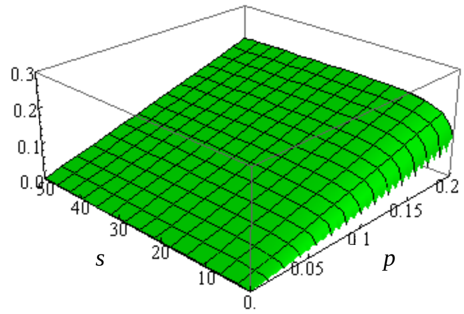

Figure 6: The 3-D plot of $p_{coll(1)}$ versus *s* and *p*.

By a comparison, the probability that a given station participates in collision for non-persistent CSMA with a number of *W* contending slots equals 1/*W* and is independent of the number of contenders (Koscielnik, Miskowicz, 2010).

4 CONCLUSIONS

We compare the maximum transmission unreliability in the non-persistent CSMA and persistent CSMA for the same average number of contention slots in both schemes. In the persistent CSMA, the latter equals simply 1/*p*. In the non-persistent CSMA, the contention window is constant in each transmission cycle and equals *W* slots.

As proved in (Koscielnik, Miskowicz, 2010), the maximum probability of participating in a collision in the non-persistent CSMA scheme is upper bounded by 1/*W*, that is, by the probability of a selection of a single slot in the transmission attempt. On the other hand, as follows from the present paper, maximum probability of participating in a collision by a given station, according to the persistent CSMA scheme, is upper bounded by the persistence level (*p*), which a main parameter of the protocol. Thus, the complementary results defined by (14) are valid for the persistent CSMA scheme.

REFERENCES

Tay, Y. C., Jamieson, K., Balakrishnan, H., 2004. "Collision-minimizing CSMA and its applications to wireless sensor networks", *IEEE Journal on Selected Areas in Communication*, vol.22, pp.1048-1057.

Miśkowicz, M., Golański R., 2006. LON technology in wireless sensor networking applications, *Sensors*, vol. 6, no. 1, pp. 30-48.

Miśkowicz, M, 2009. Average channel utilization of CSMA with geometric distribution under varying workload, *IEEE Transactions on Industrial Informatics*, vol. 6, no. 2, pp.123-131, 2009.

Egea-López, E., Vales-Alonso, J. Martínez-Sala, A. S., Bueno-Delgado, M. V., M. V. García-Haro, J., 2007. Performance evaluation of non-persistent. CSMA as anti-collision protocol for active RFID tags", *Lecture Notes in Computer Science LNCS* 4517, pp. 279-289.

Kleinrock, L., Tobagi, F.A, 1975. Carrier sense multiple-access modes and their throughput-delay characteristics, *IEEE Transactions on Communication*, vol. COM-23, no. 12, pp. 1400-1416.

Lam, S.S., 1980. A carrier sense multiple access protocol for local networks, *Computer Networks*, vol. 4, no. 1, pp. 21-32.

Bruno, R., Conti, M., Gregori, E., 2002. Optimization of efficiency and energy consumption in p-persistent CSMA-based wireless LANs, *IEEE Transactions on Mobile Computing*, vol. 1, no. 1, pp. 10-31.

Calì, F., Cont, M., Gregori, E., 2000. Dynamic tuning of the IEEE 802.11 protocol to achieve a theoretical throughput limit, *IEEE/ACM Transactions on Networking*, vol. 8, no. 6, pp. 785-799.

Miśkowicz, M., 2009. Access delay in LonTalk MAC protocol, *Computer Standards & Interfaces*, vol. 31, no. 3, pp. 548-556.

Bianchi, G., 1998. IEEE 802.11—Saturation throughput analysis, *IEEE Communications Letters*, vol. 2, no. 12, pp. 318-320.

Miśkowicz, M., Kościelnik, D., 2010. On the upper bound of transmission unreliability in memoryless backoff contention, *Proceedings of IEEE International Workshop on Factory Communication Systems WFCS 2010*, Nancy, France, May 2010.