VEHICLE ROUTING TO MINIMIZE MIXED-FLEET FUEL CONSUMPTION AND ENVIRONMENTAL IMPACT

O. Gusikhin[1], P. MacNeille[1] and A. Cohn[2]
[1]Ford Motor Company Research and Innovation Center, 2101 Village Road, Dearborn, MI 48121, U.S.A.
[2]University of Michigan, 2797 IOE Building, 1205 Beal Avenue, Ann Arbor, MI 48019, U.S.A.

Keywords: Fuel Economy, Heterogeneous Vehicle Routing Problem, Optimization.

Abstract: Efficient vehicle routing is critical to the operational profitability and customer satisfaction of vehicle fleet-related businesses, especially in light of increasing, and highly volatile, fuel prices. Growing pressures to reduce negative environmental impacts have suggested that a second metric (vehicle emissions) should also be considered in vehicle routing. Currently, the majority of existing tools use distance as a surrogate for cost. When considering a mixed fleet of multiple vehicle types, with individual vehicles within a fleet type also varying by age and vehicle health, this surrogate becomes significantly less accurate. Furthermore, using distance as a surrogate fails to capture the variations between city and highway driving, which are particularly striking for hybrid vehicles. We thus propose a new approach to the vehicle routing problem, specifically targeting applications with mixed fleets including clean-vehicle technologies, in recognition of the limitations of the existing approaches.

1 INTRODUCTION

Efficient vehicle routing is critical to the operational profitability and customer satisfaction of vehicle fleet-related businesses. The rising and highly-variable cost of fuel, as highlighted by the price spike in the summer of 2008, increases the importance of efficient vehicle routing. At the same time, growing environmental concerns suggest that cost is not the only metric of importance and that emissions should also be taken into account.

Currently, the majority of existing methods and software packages minimize travel distance as a surrogate for cost. Although there is a positive correlation between distance traveled and fuel consumed, it is not a perfect correlation. In particular, both fuel efficiency and emissions vary depending upon driving conditions (e.g. city vs. highway driving). This variability is even more pronounced in a heterogeneous fleet comprised of multiple vehicle types with a range of fuel-consumption and emissions characteristics.

In recent years, socio-economical pressures to reduce their fuel costs and carbon dioxide (CO_2) footprint have motivated many fleet operators to begin upgrading their fleets, focusing on clean-vehicle technologies and alternative fuels, including flex-fuel, hybrid vehicles, and plug-in hybrid electric vehicles (PHEV). As a result, many commercial fleets are currently composed of a heterogeneous set of vehicles, with noticeable variations in fuel economy and emissions across vehicle type. Furthermore, there are variations across vehicles even within a given vehicle type or common set of capabilities, because newer vehicles typically exhibit better fuel economy and better emission control technologies than older vehicles.

Consider the following examples of highly heterogeneous fleets: Florida Power and Lighting (www.fpl.com), the leader in green fleet initiatives, has a fleet of approximately 2400 vehicles, with half of the fleet powered by biodiesel, 300 hybrids and plug-in hybrids now in service, and plans to convert one-third of the vehicles to hybrid by the end of 2010.

A key issue in incorporating the fuel efficiencies and emissions of heterogeneous fleets within the vehicle routing problem is this: *The differences in fuel efficiencies and emissions across vehicle types are not exclusively proportional to the distance traveled, but are also highly dependant on the driving cycle.* For instance, a hybrid vehicle takes advantage of the regenerative braking that occurs in stop-and-go driving environments to charge a battery which can then be used to power the vehicle

Gusikhin O., MacNeille P. and Cohn A. (2010).
VEHICLE ROUTING TO MINIMIZE MIXED-FLEET FUEL CONSUMPTION AND ENVIRONMENTAL IMPACT.
In *Proceedings of the 7th International Conference on Informatics in Control, Automation and Robotics*, pages 285-291

when traveling at slow speeds. On average, fuel consumption is reduced about 20% by regenerative braking (Chan and Chau, 2001). Conversely, in highway driving, hybrids provide minimal, if any, improvement in fuel efficiencies over their conventional counterparts.

The introduction of PHEVs, which use electrically-charged batteries for the initial portion of a trip and revert to gasoline-based power only when the battery has been depleted, presents even greater variability in terms of the correlations between distance and fuel utilization.

When we move beyond fuel costs to also consider the environmental impact of vehicle routing, the complexity grows further. Fuel consumption is effectively proportional to CO_2 emission, so fuel economy improvements are reflected in CO_2 reduction. Additionally, there may be requirements to minimize or eliminate conventional emissions such as volatile organic components (VOC) and nitrogen oxides (NO_x) emissions in highly populated areas, requiring the use of electric power or other clean alternative fuel options and thus limiting the feasible region of a vehicle routing problem.

In this paper, we consider ways to explicitly capture fuel consumption and emissions in a mixed-fleet vehicle routing program and analyze the opportunities for simultaneously reducing costs and negative environmental impacts. In Section 2, we review factors that influence fuel consumption and emissions and consider ways to estimate these metrics for a given route. In Section 3, we suggest a number of formulations for this new variation of the vehicle routing problem. We also outline a solution approach based on *composite variable modeling*. In Section 4, we provide a numerical example and analysis to highlight the benefits of our proposed approach and we then offer conclusions and suggested areas for future research in Section 5.

2 ESTIMATION OF FUEL ECONOMY & VEHICLE ENVIRONMENTAL IMPACT

Route planning is done by representing the road system as a graph in which intersections are nodes and road segments are arcs. To determine the best route from an origin node to a destination node, each arc is associated with a cost that represents distance, travel time, or fuel consumption. Then Dijkstra's algorithm (or an equivalent) is used to find the lowest cost path which is then inversed mapped to the road system for visualization and navigation of the preferred route.

Total fuel cost along a given route is the product of the total consumption of each type of fuel and the per-unit cost for that fuel. Total fuel consumption along an arc is largely dependant on the vehicle specific load (VSL) opposing vehicle motion multiplied by the distance traveled against the VSL to give the total energy required to travel the arc. This energy can be readily converted into fuel. For example, about 0.003 gallons of gasoline or 0.002 kWh of electricity are needed to push against a pound of force over a one mile stretch of road. These factors will vary, however; with the efficiency of the energy conversion which depends on many factors.

The VSL depends largely on external factors that include the drive cycle, aerodynamic drag, rolling resistance, parasitic drag, and gradient drag. These factors in turn are dependant on several external factors including weather conditions, road and traffic conditions and topography. Most of the external factors can be reasonably well estimated using well known engineering formulas; however, the VSL also depends on the pattern of acceleration/deceleration that takes place along the branch. Driving pattern effects depend largely on complex interactions between the driver, the road, the powertrain, and traffic conditions. Thus, these effects are difficult to predict.

One approach to predicting these complex factors is by classifying road, traffic, and driver and drivetrain combinations into load effects. The US Environment Protection Agency (EPA) provides miles per gallon estimates for highway and city for all vehicles sold in the US in the last 15 years currently based on two standard driving cycles. Although these estimates may not be adequate to accurately forecast the specific fuel consumption, they nevertheless can provide a reasonable basis for the comparative analysis between different vehicles.

A more detailed classification is described in Brundell-Freij and Ericsson (Brundell-Freij and Ericsson, 2005), where a classification system has been developed based on extensive data collected from instrumented vehicles. Four variables relating to the road type were found to be significant: 1) occurrence and density of junctions controlled by traffic lights, 2) speed limit, 3) function of the street, and 4) the type of neighborhood. A large effect was attributed to the power-weight ratio of the vehicle, which presumably is descriptive of the drivers that choose a vehicle with a specific power-weight ratio.

A challenge in using instrumented vehicles is in the ability to collect adequate data. For example, in the Brundell-Freij and Ericsson study much of the data was collected in Lund, Sweden. This location has limited topography, so road gradient was not captured, although it is well known that road gradient is an important factor in situations with topographic relief. For example, (Tavares et al., 2009) develops a topography-based routing algorithm for waste collection vehicles in a mountainous area and demonstrates that the proposed approach allows reduction of fuel consumption despite increasing in the distance traveled. Also road design and traffic control policies may vary considerably between political jurisdictions, as may the mix of vehicles. One way to overcome these difficulties is to use vehicle and traffic modeling to determine the significant factors for classification.

Modeling tools such as the Powertrain System Analysis Toolkit (PSAT) (PSAT, 2008) or The MathWorks Simulink/SimDriveLine (Rose-Hulman, 2005) can be used to estimate the energy or fuel along a route given vehicle design parameters, external load factors such as road gradient and the driver's torque demand along the route. Vehicle design parameters can be obtained from vehicle manufacturers and external factors from published maps. Driver's torque demand involves the psychophysics of driving and has been simulated using software such as VISSIM or MISSION (PTV AG, 2009), (Wiedemann et al., 1991), (Busawon et al., 2006), (Noland and Quddus, 2006).

Vehicles in the fleet may be instrumented to collect actual fuel economy data along the branches they travel. The data may be recovered from the vehicle at a download site and stored in a database. Periodically the database may be used to automatically refine the costs assigned to a class of road segments, and to reclassify segments as needed.

3 VEHICLE ROUTING TO MINIMIZE FUEL CONSUMPTION

The Vehicle Routing Problem to Minimize Mixed-Fleet Fuel Consumption and Environmental Impact (VRPMF) belongs to the class of heterogeneous fleet vehicle routing problems (HVRP). (Baldacci et al., 2008) provides a comprehensive classification and review of the main approaches proposed for VRP with a heterogeneous fleet. Specifically, the problem being considered in the paper represents a variant of HVRP with Vehicle-Dependent Routing Costs (HVRPD). They note that solution approaches to this difficult family of problems, both in the literature and in commercial applications, have predominantly been heuristic in nature. These are typically adaptations or extensions of solution techniques for traditional VRP and VRP with Time Windows.

In order to capture the complexities (and, in particular, the non-linearities) of VRPMF, we instead propose to leverage the use of *composite variable modeling* (CVM) to capture the complex real-world details associated with accurately modeling the fuel cost (and associated emissions) of a prescribed route. The idea behind CVM (Cohn, 2002), (Barlatt, et al., 2009) is to embed modeling complexity into the variable definition rather than capturing it explicitly in a model which may then become intractable. For example, in VRPMF, explicitly modeling the cost functions described in Section 2 within the framework of a traditional VRP would make an already difficult problem unsolvable. However, it is far easier to calculate the cost of a given route (for a given vehicle) off-line. We can then formulate a master problem in which each variable represents the assignment of a specific route to a specific vehicle. For a given route, we can compute the total fuel consumption for a given vehicle, and thus the total cost is just the sum of the chosen assignments. Similarly, a route pre-specifies all the customer demands that it meets, and thus we only need two sets of constraints. The first ensures that each vehicle is assigned to at most route and the second ensures that each customer demand is met exactly once.

The challenge, then, is to address the exponentially large number of potential variables. Clearly not all of the exponentially-large set of feasible routes (and their corresponding costs) can be generated. Even if they could, it would not be possible to solve the resulting exponentially-large set partitioning problem. Instead, *column generation* techniques (Desaulniers et al., 2005) (originally developed as part of Dantzig-Wolfe Decomposition) can be employed. The idea behind column generation for solving a linear program with an exponential number of variables is to identify candidate pivot variables for the simplex method not by pricing each variable’s reduced cost directly, but rather by solving a secondary optimization problem (often called a *sub problem*) which seeks the feasible variable with the most negative reduced cost. If this yields a negative reduced cost variable, then the simplex pivot occurs and the algorithm proceeds. If the most negative reduced cost variable is strictly non-negative, then a certificate of optimality is achieved and the algorithm terminates with a provably-optimal solution.

In Dantzig-Wolfe Decomposition, the inherent structure of a problem leads to a sub-problem that is pre-defined. Furthermore, it is itself a linear program and thus straightforward to solve. In CVM, because the variable definition is chosen specifically to overcome challenges of a traditional formulation, the sub problem reflects these challenges.

Perhaps the most closely related work to our proposed approach is that of Taillard (Taillard, 2005), who used a heuristic based on column generation techniques to solve HVRPD. Specifically, a large set of candidate routes were generated by solving separate homogeneous VRP problems for each fleet type. The final routes were then selected using a set partitioning formulation to ensure that all demands were met.

A key difference between our proposed approach and typical network-based routing problems is that the "cost" of a route cannot be computed simply by adding the individual arc costs (if so, the sub-problem would be a simple minimum cost flow problem). At first glance, it seems possible to formulate the sub problem as a network flow problem, where each node represents a customer or depot, each arc represents the driving from one node to another, and the cost associated with each arc can fully capture (based on off-line calculations) the cost of this driving. This is not quite true, however: The cost on a given arc is not independent of the other arcs that are also chosen for an individual vehicle's route. This is because the fuel consumption on an arc depends on the starting conditions of the vehicle at the first node. If the battery is fully charged, it may be possible to complete most of the driving without relying on gasoline, and the resulting cost will be lower, whereas if the battery is depleted, the cost of the arc will be much higher.

Therefore, a more sophisticated approach to solving the sub-problem must be employed. For example, we could take a *multi-label shortest path* approach (Desrochers and Soumis, 1988), which is similar to Dijkstra's shortest path algorithm, but with an added layer of complexity. Specifically, multiple metrics (not just cost) must be checked to determine whether a partial path can be pruned from consideration. One partial path dominates another only if it is less costly *and* covers the same amount of demand or more *and* has the same amount of remaining battery charge or more. Efficiently solving this sub-problem is the key to successfully solving the master problem.

We conclude this section by noting that this approach has the added advantage of allowing the user to trade off between time and solution quality. Specifically, the solution quality continues to improve as each new candidate route is added to the master problem for consideration, but high-quality feasible solutions can nonetheless often be found early in the process. Furthermore, this approach naturally lends itself to a parallel implementation. At the highest level, a separate sub-problem can be solved, in parallel, for each vehicle. Furthermore, these individual sub-problems themselves can leverage a parallel architecture for efficient search.

4 ILLUSTRATIVE EXAMPLE

This section provides a simplified illustrative example of VRPMF. We consider a fleet of two vehicles: the first is a 2009 Ford Taurus front-wheel drive gasoline engine vehicle and the second is 2010 Ford Escape 4-wheel drive hybrid vehicle. Figure 1 shows estimates of the miles per gallon (MPG) values and environmental scores as provided by the US Environmental Protection Agency (EPA) at www.fueleconomy.gov. Note that the Taurus gets 18 MPG in city driving and 28 MPG in highway driving, while the Escape hybrid gets 27 MPG on the highway and 30 MPG in the city. The environmental impact of each vehicle can be evaluated by its carbon footprint and air pollution score. The carbon footprint measures greenhouse gas emissions (primarily CO_2) that in turn impact climate change. CO_2 emissions are closely linked to fuel consumption, since CO_2 is the ultimate end product of burning gasoline. The Air Pollution score represents the amount of health-damaging and smog-forming airborne pollutants (such as carbon monoxide, CO, and oxides of nitrogen, NOx) that the vehicle emits on a scale from 0 (worst) to 10 (best). Note that there is little correlation between fuel consumption and these emissions; emissions primarily depend on the emission control technology. Taurus has an Air Pollution score of 6 and Escape Hybrid has a score of 8.

Suppose that we have eight customers distributed within a given geographical area as shown in Figure 2. The customers are labeled 1 to 8, while 0 is the depot. The driving distance between the depot and each of the customer sites is presented in Table 1. For this example, we assume that the distance data is symmetrical and shown as $X+Y$, where X is the

Table 1: Distance between depot and between customer's sites.

	0	1	2	3	4	5	6	7	8
0		14.5+ .9	19+ 1	23.9+ 2.4	0+ 11.6	0+ 9.8	0+ 13.6	12.5+ 10	15.5+ 1.1
1	14.5+ .9		4.2+ 1.5	10.2+ 1.8	0+ 6.3	0+ 10.3	0+ 12.1	12.2+ 2.6	0+ 17.9
2	19+ 1	4.2+ 1.5		5.7+ 2.2	2.5+ 9.1	4.2+ 11.6	11.4+ 4.7	7.7+ 2.9	20.2+ 2
3	23.9+ 2.4	10.2+ 1.8	5.7+ 2.2		0+ 6.7	0+ 10.6	0+ 6.5	0+ 3.8	14.3+ 1.5
4	0+ 11.6	0+ 6.3	2.5+ 9.1	0+ 6.7		0+ 4	14+ 6	17+ 7.5	0+ 11.6
5	0+ 9.8	0+ 10.3	4.2+ 11.6	0+ 10.6	0+ 4		0+ 3.8	0+ 8.6	0+ 7.7
6	0+ 13.6	0+ 12.1	11.4+ 4.7	0+ 6.5	14+ 6	0+ 3.8		0+ 4.7	7.4+ 4.1
7	12.5+ 10	12.2+ 2.6	7.7+ 2.9	0+ 3.8	17+ 7.5	0+ 8.6	0+ 4.7		12.5+ 2.5
8	15.5+ 1.1	0+ 17.9	20.2+ 2	14.3+ 1.5	0+ 11.6	0+ 7.7	7.4+ 4.1	12.5+ 2.5	

	
2009 Ford Taurus FWD	2010 Ford Escape Hybrid 4WD
EPA MPG City	
18	30
EPA MPG Hwy	
28	27
EPA Air Pollution Score	
6	8

Figure 1: Fleet Composition.

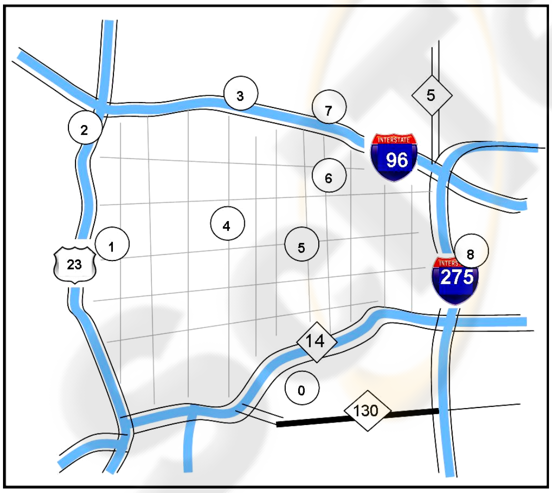

Figure 2: Vehicle Routing Problem.

number of highway miles and Y is the number of city miles.

Further, we assume that each service call requiresapproximately 90 minutes and that each service agent has to finish his/her route, starting and ending at the depot, within 8 hours.

We ignore breaks, and assume that there is no capacity limit on the routes other than the time limit. We begin by solving the traditional problem of minimizing the total fleet vehicle miles traveled (VMT). (In this small problem, this can be done by explicit enumeration). The total optimum VMT is 97.2. One vehicle visits customers 1, 2, 3, and 4 with a total travel distance of 47.3 miles, comprised of 22.9 city miles and 24.4 highway miles. The other vehicle visits customers 5, 6, 7, and 8 with a total travel distance of 49.9 miles, comprised of 21.9 city miles and 28 highway miles. The estimated travel time for the first route is 75 minutes and for the second route is 78 minutes. Note that in this variation of the problem, we do not differentiate between vehicles, as we are simply minimizing distance traveled.

For illustrative purposes, to estimate fuel consumption of each vehicle along the given routes we assume the estimated highway and city MPGs as defined in Figure 1. (Of course, real-world calculations are more complex as we discussed in section 2). As a result, the fuel consumption of the Taurus for the first route is 2.14 gallons and for the second route is 2.22 gallons. For the Escape Hybrid, the first route consumes 1.67 gallons and the second route consumes 1.77 gallons.

The optimal solution is to assign the Taurus to the first route, resulting in the consumption of 2.14 gallons, and to assign the Escape to the second route, with an estimated consumption of 1.67 gallons of gasoline. The total fleet fuel consumption for the given solution is 3.81 gallons of gasoline. The results are presented graphically in Figures 3 and 4.

We next reformulate the problem to take into account fuel economy, i.e. distinguishing in our optimization between highway and city driving, and treating the two vehicles separately, in recognition of their distinct characteristics. The optimal solution for this problem is presented in Figures 5 and 6.

Although the VMT increases (from 97.2 to 101.7), the fuel consumption decreases from 3.81 to 3.72 gallons. Note that the optimal routes have changed (see Figure 4). The route served by the Taurus covers customers 1, 2, 7, and 8 with a total length of 63.3 miles, but of this 54.4 miles is highway driving and only 8.9 miles is city driving. Conversely, the Escape Hybrid is assigned to a route that serves customers 5, 6, 3, and 4, using only city driving (for a total length of 38.4 miles).

Table 2 provides a comparative summary of the two solutions. In addition to the economical impact, this demonstrates how a reduction in fuel consumption also leads to a reduction in environmental impact from the fleet operations. First of all, the reduction of fuel consumption reduces the CO_2 emissions. In addition, the second solution shifts the time the Taurus spends in the city routes to the Escape Hybrid. As is shown in Figure 1, the Taurus has a lower EPA Air Pollution Score than the Escape Hybrid. Consequently the second solution also reduces health-damaging and smog-forming airborne pollutants along populated areas. We could also capture this explicitly in the objective function of our formulation, either by specifying constraints on the total Air Pollution Score (possibly weighted by the location of the route arcs), or by introducing weights in the objective function.

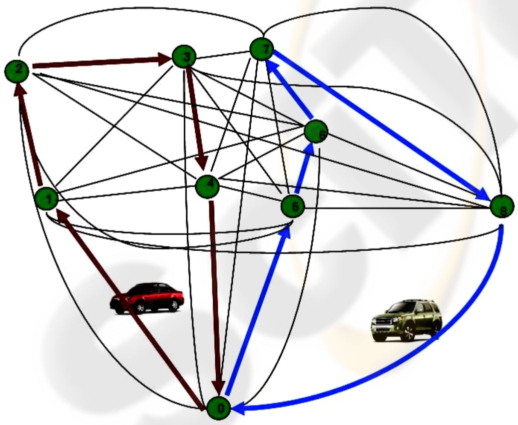

Figure 3: Solution 1 Graph.

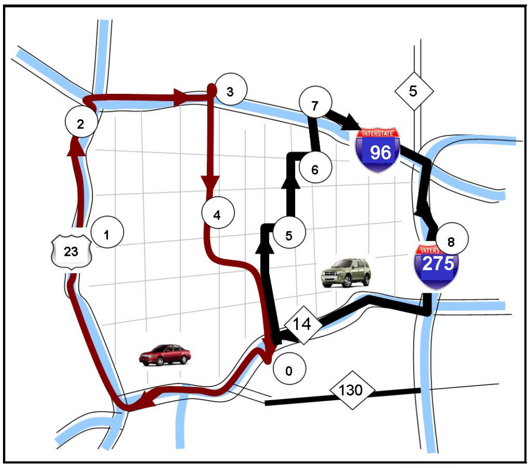

Figure 4: Solution 1 Map.

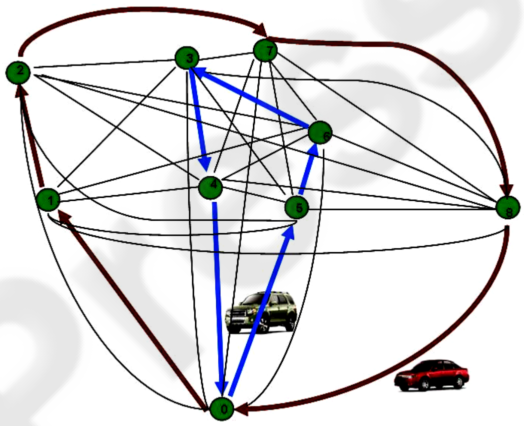

Figure 5: Solution 2 Graph.

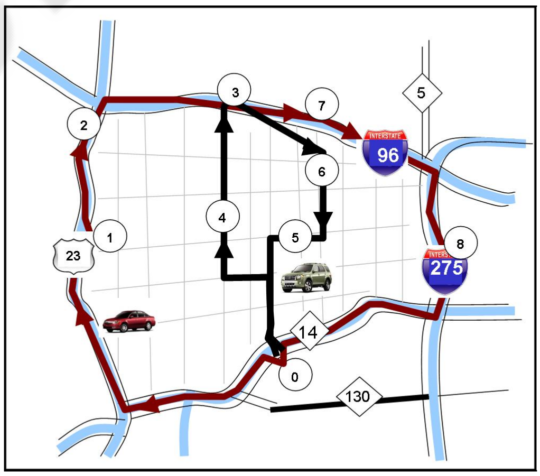

Figure 6: Solution 2 Map.

Table 2: Comparative summary of two VRT solutions.

Minimum Distance Optimization									
	Distance (miles)			Fuel (gallons)			Time (minutes)		
	Hwy	City	Total	Hwy	City	Total	Hwy	City	Total
	24.4	22.9	47.3	0.9	0.8	1.7	21	54	75
	28.0	21.9	49.9	1.0	1.2	2.2	25	53	78
Summary	*52.4*	*44.8*	***97.2***	*1.9*	*2.0*	***3.9***	*46*	*107*	*153*
Minimum Fuel Optimization									
	Distance (miles)			Fuel (gallons)			Time (minutes)		
	Hwy	City	Total	Hwy	City	Total	Hwy	City	Total
	54.4	8.9	63.3	1.9	0.5	2.4	48	26	74
		38.4	38.4		1.3	1.3		84	84
Summary	*54.4*	*47.3*	***101.7***	*1.9*	*1.8*	***3.7***	*48*	*110*	*158*

5 CONCLUSIONS

In this paper we discuss the importance of the heterogeneous fleet vehicle routing problem based on fuel consumption rather than just distance traveled. We describe the complex function that determines how much fuel a given route consumes, and argue that distance is an inadequate surrogate when multiple fleet types, especially varying across different technologies, are used. We provide a simple example to illustrate how minimizing total miles traveled can yield a very different solution than minimizing fuel consumption. We also discuss solution techniques, specifically based on the use of composite variable modeling, to solve this computationally challenging problem.

In the future, we propose to consider emissions as well as explicit fuel costs (which implicitly capture CO_2 emissions but not NO_x). We also suggest extending VRPMF to include variations and extensions such as those studied in the basic VRP, such as balancing routes, satisfying time windows, etc.

REFERENCES

Chan, C. C. and Chau, K.T.; Modern Electric Vehicle Technology. Oxford University Press Nov. 15, 2001.

Brundell-Freij, K. and Ericsson, E.; Influence of street characteristics, driver category and car performance on urban driving patterns, Transportation Research Part D 10, 2005, pp. 213-229.

Tavares, G., Zsigraiova, Z., Semiao, V., Carvalho, M.G , Optimization of MSW collection routes for minimum fuel consumption using 3D GIS modeling, Waste Management 29, 2009, pp. 1176–1185.

PSAT Overview, January 2008, http:// www.transportation.anl.gov/pdfs/HV/412.pdf

Rose-Hulman Institute of Technology; Rose-Hulman Institute of Technology Students Design Hybrid Vehicle Powertrain with Simulink and SimDriveLine, 2005; http:// www.rose-hulman.edu/ challengex/ downloads/Rose-Hulman_user_story.pdf

PTV AG, VISSIM – State-of-the-Art Multi-Modal Simulation; http:// www. ptvag. com/ fileadmin / files_ptvag.com/download/traffic/Broschures_Flyer/VISSIM/VISSIM_Brochure_e_2009_HiRes.pdf, 2009.

R. Wiedemann, U. Reiter, Wiedemann Model-Microscopic Traffic Simulation The Simulation System Mission, 1991; http:// www.ptvamerica.com/fileadmin/files_ptvamerica.com/library/1970s%20Wiedemann%20VISSIM%20car%20following.pdf

Roshan Busawon, M. David Checkel, CALMOB6: A Fuel Economy and Emissions Tool for Transportation Planners, presented to the Best Practices in Urban Transportation Planning session of the 2006 Annual Conference of the Transportation Association of Canada, Charlettetown, Price Edward Island, 2006.

Robert B. Noland, Mohammed A. Quddus, Flow improvements and vehicle emissions: Effects of trip generation and emission control technology, Transportation Research Part D 11, 2006, pp. 1-14.

Roberto Baldacci, Maria Battarra, and Daniele Vigo, Routing a Heterogeneous Fleet of Vehicles in B. Golden, et al. (Eds), The Vehicle Routing Problem: Latest Advances and New Challenges, Springer 2008, pp. 3-27.

Cohn, A. “Composite Variable Modeling for Large-Scale Problems in Transportation and Logistics,” doctoral dissertation in Operations Research, Massachusetts Institute of Technology, advised by Dr. Cynthia Barnhart, April 2002.

Barlatt A.,Cohn A.,FradkinY., Gusikhin O.; Morford C., Using composite variable modeling to achieve realism and tractability in production planning: An example from automotive stamping, IIE Transactions, Volume 41, Issue 5, 2009 , pp. 421 – 436.

Desaulniers, G., J. Desrosiers, and M. Solomon, editors. “A Primer in Column Generation.” Springer US, 2005.

Taillard E. D., A heuristic column generation method for the heterogeneous VRP, Operations Research -- Recherche opérationnelle 33 (1), 1999, pp. 1-14. (Publication CRT-96-03, Centre de recherche sur les transports, Université de Montréal, 1996. Revision 11.2005)

Desrochers, M. and F. Soumis. A Generalization Generalized Permanent Labeling Algorithm for the Shortest Path Problem with Time Windows. INFOR 1988 26:191 – 212.