MULTI-MODAL ANALYSIS OF COMPLEX NETWORK
Point Stimulus Response Depending on its Location in the Network

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Abstract: We report a new method of diagnosis of a node in a network by “Point Stimulus Response”. The “Point Stimulus Response” corresponds to the impulse response of the network, that is, the state temporal variation in the Markov transition with the delta-function of initial state. We can evaluate the reaction of the system against a point stimulus such as a point failure. In this report, for the first, we summarize our mathematical platform for analysing complex network system using the adjacency matrix as the transition matrix in Markov transition approximation. On this basis, we formulate the point stimulus response. The location dependence of the point stimulus response is demonstrated in Tokyo Metropolitan Railway Network System. For a concrete example, the total amount of suffered passengers and time response of recovery from a point failure will be discussed depending on the location of point failure in the network system. It can be said that a way to find a point for effective stimulus response is one of key approaches for knowledge discovery. However, the real indication or meaning of the point stimulus is in the stage of speculation.

1 INTRODUCTION

Knowledge Discovery is an interdisciplinary area focusing upon methodologies for identifying valid, novel, potentially useful and meaningful patterns from data, often based on underlying large data sets. Our mathematical platform is aiming extraction and analysis of knowledge from the mutual interaction patterns, obtained by such network log data (Onnela, 2008). The mutual interaction pattern is described as the adjacency matrix in the Markov process approximation (Ozeki, 2010).

Brin and Page reported, in their first paper on “Google”(Page, 1990), that it was a great surprise the PageRank is obtained purely mechanically from the pattern of mutual page links. That is the surprise of discovery that the pattern is entangled with the real world. The “Google” approximates a Web surfer as a random walker in the Markov process and combines the dominant eigenvector with the list of coincidence as the PageRank .

The “Google”, however, uses only the dominant eigenmode because the eigenvectors of higher-order modes are not positive valued so that the probability finding the Web surfer at a page cannot be defined for the higher-order modes (Langville, 2006).

Here, we have proposed a mathematical platform for analysing the network pattern in multi-modal scheme (Ozeki, 2009). Each mode corresponds to a substructure of the pattern. Various pattern dependant behaviours can be analysed for knowledge discovery.

In this paper, we would like to report a new method for the diagnosis of various objectives, such as security and activation, of a network system by a “Point Stimulus Response”.

The “Point Stimulus Response” corresponds to the impulse response of the network system, that is, the state variation in the Markov transition with the delta-function of initial state. We can evaluate the system activity against the point stimulus.

It can be said that a way to find a point of effective stimulus response or “tsubo” is one of the key approaches of “Knowledge Discovery”.

In Japan, “Shiatsu” is a popular therapy by pressing “shiatsu point” to enhance the body’s

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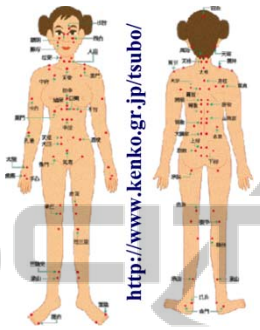
natural healing ability and prevent the progression of disease. Shiatsu points are called “tsubo”, in Japanese and their locations and effects are based on understanding of modern anatomy and physiology. The concept of “tsubo” is our stimulus point of the network system.

The point stimulus has been used as a way of reactivation of an old city (Horiike, 2002).

This report is believed as the first theoretical approval of locating stimulus points of a network system.



2 MATHEMATICAL PLATFORM

In this session, we would like to summarize our mathematical platform for network system analysis, briefly.

2.1 Adjacency Matrix

The adjacency matrix $A_{i,j}$ of a network can be used as a Markov-transition matrix to simulate the evolution of states: $(\hat{q})_{n+1} = A\cdot(\hat{q})_n$ where $(\hat{q})_n$ is the probability amplitude vector of the state at the nth transition step. The probability amplitude is normalized with respect to the Euclidean norm after each transition step by application of $\sum_{i=0}^{N-1}|(q_i)_n|^2=1$,where $(q_i)_n$ is the i^{th} component of $(\hat{q})_n$. The probability $(p_i)_n$ of finding a random walker at the node “i “ is given by $(p_i)_n = |(q_i)_n|^2$. The eigen-equation is $A\cdot\phi_i^{(m)} = \lambda_m\cdot\phi_i^{(m)}$ where λ_m is the eigenvalue of mode “ m ” and $\phi_i^{(m)}$ is its eigenvector; the eigenvectors form a complete orthogonal basis under the assumption of a symmetric adjacency matrix. This fact is the reason of using the adjacency matrix as the transition matrix in the Markov transition.

It should be noted that using a Markov process normalized by the Euclidean norm makes it possible to describe the network states in a multi-modal way. Previously, in such systems as the Google search engine (Langville, 2006) using a stochastic transition matrix normalized by the 1-norm, higher order modes cannot define the probability of finding a random walker because the components of eigenvectors are not always positive.

2.2 Non-linear Markov Transition

To examine multi-modal dynamics of the network, we define a Markov transition with weak non-linearity; a non-linear Markov process can be formulated as follows: the transition coefficient from node ”j “ to node ”i” is *affected* by the probability amplitude $(q_k)_n$ of node ”k” linked to node ”i”. Such a non-linear Markov transition is given by

$$(q_i)_{n+1} = \sum_j A_{i,j}\cdot(q_j)_n + \sum_{j,k}\nu\cdot A_{j,i}\cdot A_{k,i}(q_j)_n\cdot(q_k)_n \quad (1)$$

, where ν is a measure of the strength of the non-linearity. Since the Markov property states that the probability distribution for the system at the next step depends only on the current state of the system, the non-linear state transition given by equation (1) indeed defines a Markov process. It is possible to define higher-order non-linear interactions in a similar way (Ozeki,2009). Since we have a complete basis of orthogonal eigenvectors, the mode amplitudes $(a_m)_n = \sum_{i=0}^{N-1}(q_i)_n\cdot\varphi_i^{(m)}$ can describe the mode evolution of the system (Haken,1987).

2.3 Node, Mode and Network Entropies

The entropy may be efficient measure of network optimazation. We define three kinds of entropies based on the Shanonn entropy (Shanonn,1948) using the probability finding a random walker at each node. The node entropy NE_i is defined by $NE_i = -\sum_m(\varphi_i^m)^2\ln((\varphi_i^m)^2)$ that is the sum of the Shanonn entropy $-(\phi_i^m)^2\ln((\phi_i^m)^2$ of node i over all of mode m.The mode entropy ME_m is defined by $ME_m = -\sum_i(\phi_i^m)^2\ln((\phi_i^m)^2)$ that is the sum of Shanonn entropy of the mode m over all of node i. The network entropy GE is defined by $GE = \sum_i NE_i = \sum_m NE_m$.

3 POINT STIMULUS

3.1 Formulation of "Point Stimulus Response"

The point stimulus response is the impulse response in the electronic circuit system: that is, the temporal response stimulated by a delta-function provides the network system characteristics. The point stimulus response is defined by the temporal response in the non-linear Markov transition for the positive point stimulus; $PPS_i = \delta(i, p)$, where node "p" is a location of stimulus. We found that the inverse or negative delta function is more effective in some network with particular symmetric nature. In a kind of network having skew degeneracy (Ozeki,2010), a negative point stimulus, $NPS_i = -\delta(i, p)$ is effective to stimulated the mode competition among the skew degenerate modes. In the following, for the first, the positive point stimulus response is discussed by a concrete network example and in later the negative point stimulus response is discussed. The stimulation of the mode competition between the modes close to quasi-skew degeneracy is interesting related to the potential activity or development of nodes.

3.2 Diagnosis of Tokyo Railway System

Fig.1 denotes the complexity of a central part of Tokyo Railway System including subways. The adjacency matrix is assumed to be symmetric and the total number of stations (nodes) is truncated to 736 (Rail Map of Tokyo Area, 2004). A distorted hexagonal in Fig.1 is "Yamanote Circular Line" which includes several well-known stations such as Tokyo, Akihabara, Ikebukuro, Shinjuku, Shibuya and etc. Before the detail analysis of point stimulus response, it seems better to summarize the mode structure of the network. The list of eigenmode naming and eigenvalue is shown the top of Fig.2.
The probability amplitude distributions of the important four modes are shown in Fig.2. The dominant mode with the largest positive eigenvalue is named mode #2 of which probability amplitude is positive. The mode # 0 has the largest negative eigenvalue and its mode amplitude is similar with that of mode #3, that is the mode with the second largest positive eigenvalue. These mode relations are important to understand the mode competition. It is our surprise that the probability distribution of the

Figure1: Tokyo Metropolitan Railway Network System.

	0
0	-4.271
1	-3.947
2	4.738
3	4.423

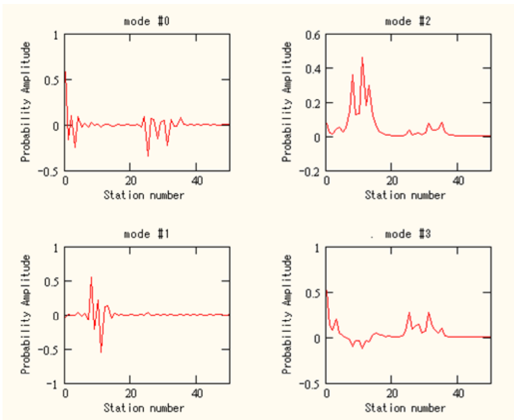


Figure 2: Eigenmode naming, eigenvalue and eigenvectors.

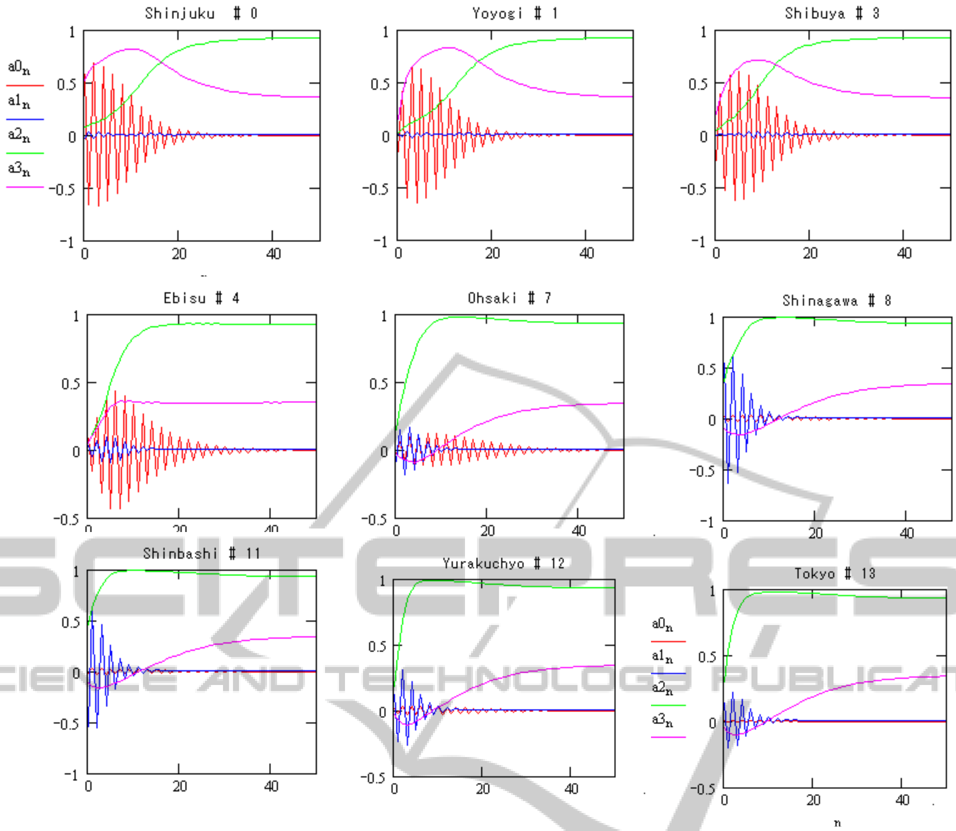


Figure 4: Location Dependence of Positive Point Stimulus Response.

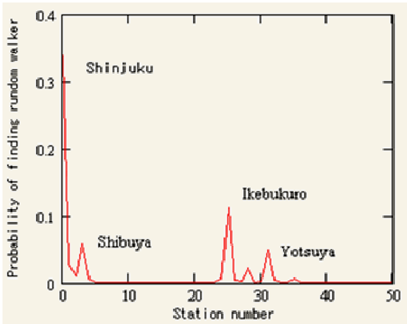


Figure 3: Probability distribution of mode #0.

mode having the largest negative eigenvalue shown in Fig.3 extract the world largest three stations from viewpoint of number of passengers without any passenger statistics. In Google-like matrix, the dominant mode provides only the degree vectors.

3.3 Location-Dependent Positive Point-Stimulus

We set the point stimulus on from Shinsjuku to Tokyo, along the Yamanote-line in CCW. The point stimulus responses of these stations calculated by the non-linear Markov process are shown in panels of Fig.4 with station name and code number. The mode amplitude $a0_n, a1_n, a2_n, a3_n$ correspond to the mode #0,#1,#2 and #3, being shown in Fig.2. The point stimulus responses of from Shinjuku #0 to Ebisu #4 dominantly consist of damped oscillation of mode #0 (red) and a quick build-up of mode #2 (green). The damped oscillation amplitudes decrease toward Ebisu #4. On the other hand, in the point stimulus responses of from Ohsaki#7 to Tokyo #13, damped oscillation of the mode #1 denoted by blue, becomes dominant, and the damped oscillation amplitudes reach at the peak around Shinagawa #8 and Shinbashi #11.

3.4 Location-Dependent Negative Point-Stimulus

Fig.5 denotes the negative point stimulus responses For typical three stations: Shinjuku, Shibuya and Tokyo. The bottom panels of Fig.5 show the probability amplitude distribution $sp_{n,i}$ finding a random walker, calculated by the superposition of

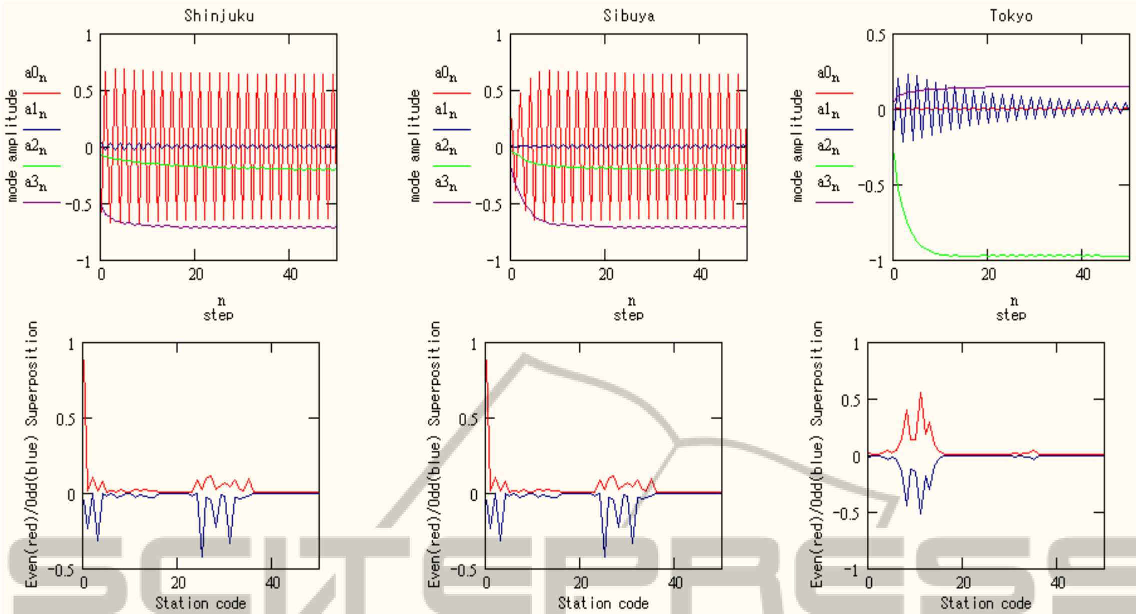


Figure 5: Location Dependant Point Stimulus Response.

modes using Eq.2. In the case of Shinjuku and Shibuya, since the sustainable oscillation of mode #0 is observed, the probability distribution of finding random walker also oscillates between the in-phase superposition and the out-of-phase superposition, just as shown in the bottom panels. The red line denotes the in-phase superposition and the blue line denotes the out-of-phase superposition. (Here, we should note that the sign of out-of-phase superposition is inverted for clear understanding.)
The distance between nodes included in red and blue lines is only one link distance: For example, Shinjuku (code #0) and Ikebukuro(code #25) in one-link distance due to the Saikyo-line, so that the random walker can transit between red/blue station-groups within one step. In the case of Tokyo, the superposition of modes of Eq.2 shows no temporal variation after damped oscillation is vanished.

$$sp_{n,i} = \sum_m (a_m)_n \cdot \phi_i^m \tag{2}$$

3.5 Categorization of Point Stimulus Response and Response Time

It is convenient to categorize the point stimulus response into the following two: The point stimulus response with the sustainable oscillation is named “the infinite response point”. The point stimulus response with the finite response is named “ the finite response point”. The categorization of stations within the Yamanote circular line is shown in the bottom panels of Shinjuku and Shibuya, in Fig.5, that is, the stations with larger probability amplitude, such as Shinjuku, Yoyogi, Harajuku, Shibuya, Ikebukuro, Shinohkubo and Yotsuya, are the infinite response nodes. These are the stations within one-link distance of Shinjuku and can be said as satellite stations: The others are the finite response nodes.

It should be noted that the build-up time of the dominant mode #2 takes longer steps to reach the stationary state due to the mode competition with mode #0, in the case that the positive point stimulus is applied to from Shinjuku to Ebisu, as shown inFig.4. The response time of nodes in the network is mainly determined by this mode competition. For further study, the recovery time from the point failure of the Tokyo Railway Network will be analysed from these viewpoints.

4 POINT FAILURE OF NETWORK SYSTEM

The point failure of the station in the Tokyo Metropolitan Railway Network System is one of concrete image of the point stimulus. We can estimate the total suffered passengers as shown in Fig.6 using the following;

$$S_i = \sum_m stimulus_m \cdot MA_m \cdot \phi_i^m \tag{3}$$

where $stimulus_m$ is the projection of the positive point stimulus PPS_i on the eigenvector ϕ_i^m . The total number of suffered passengers denoted by red line is rather independent of the location of point failure compared with larger variation in the number of passengers.

We feel that the number of suffered passenger calculated seems rather larger than the reported figures. Tokyo metropolitan railway system has a lot of redundancy in it structure for reliable operation, but we define the link topologically, that is, multiple duplication of trucks between adjacent stations is neglected. It is necessary to improve the accuracy of the adjacency matrix expression.

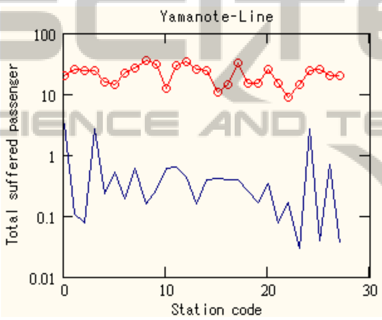


Figure 6: Total number of Suffered Passengers.

5 CONCLUSIONS AND FUTURE WORKS

We discuss on the multi-modal analysis method for discovery of knowledge from pattern information.

We proposed a diagnosis tool of the point stimulus response and demonstrated it in Tokyo Metropolitan Railway Network system. The point stimulus is effective to find interesting nodes to characterise the system, such as the excitation sustainable oscillation. It is not verified by physical data yet, but seems to be a way of an approval of “Tsubo” in “shiatsu therapy”.

As for future work, we would like to discuss on the knowledge discovery based on pattern structure embedded in data, automatically collected in the network systems. It is believed that the adjacency matrix obtained automatically, in such Facebook, gives us interesting chances to analyse the social substructures and their stability, using these new knowledge discovery technology.

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